THE STABILITY NUMBER OF THE 1-D COUPLED WAVE EQUATIONS WITH SECOND SOUND.

RENATO F. C. LOBATO¹, D.S. ALMEIDA JR.², AND M.L. SANTOS³

ABSTRACT. In this work, we consider the Timoshenko beam model with second sound. We introduce a new number $\chi_0$ that characterizes the exponential decay. We prove that the corresponding semigroup associated to the system is exponentially stable if and only if $\chi_0 = 0$. Otherwise there is a lack of exponential stability. In this case we prove that the semigroup decays as $t^{-1/2}$. Moreover we show that the rate is optimal.

We get the following hyperbolic system of differential equations

$$u_{tt} - c_1^2 u_{xx} + \alpha (u - v) + \beta (u_t - v_t) + \delta \theta_x = 0 \quad \text{in } ]0, l[ \times ]0, \infty[ \quad (1)$$

$$v_{tt} - c_2^2 v_{xx} + \alpha (v - u) + \beta (v_t - u_t) + \delta \theta_x = 0 \quad \text{in } ]0, l[ \times ]0, \infty[ \quad (2)$$

$$\rho_3 \theta_t + q_x + \delta u_{xt} + \delta v_{xt} = 0 \quad \text{in } ]0, l[ \times ]0, \infty[ \quad (3)$$

$$\tau q_t + \gamma q + \theta_x = 0 \quad \text{in } ]0, l[ \times ]0, \infty[ \quad (4)$$

The positive constants $\rho_3$, $\tau$, $\delta$ and $\gamma$ relate to hypotheses in thermoelasticity. Here we consider the following boundary conditions

$$u(0, t) = u(l, t) = v(0, t) = v(l, t) = \theta_x(0, t) = \theta_x(l, t) = 0, \forall t \geq 0 \quad (5)$$

and the following initial conditions

$$u(x, 0) = u_0(x), u_t(x, 0) = u_1(x), v(x, 0) = v_0(x), v_t(x, 0) = v_1(x) \quad (6)$$

$$\theta(x, 0) = \theta_0(x), q(x, 0) = q_0(x), \forall x \in ]0, l[ \quad (7)$$

REFERENCES


1 Department of Mathematics - UFPA. E-mail address: renatolobato@ufpa.br

2 Department of Mathematics - UFPA. E-mail address: dilberto@ufpa.br

3 Department of Mathematics - UFPA. E-mail address: ls@ufpa.br