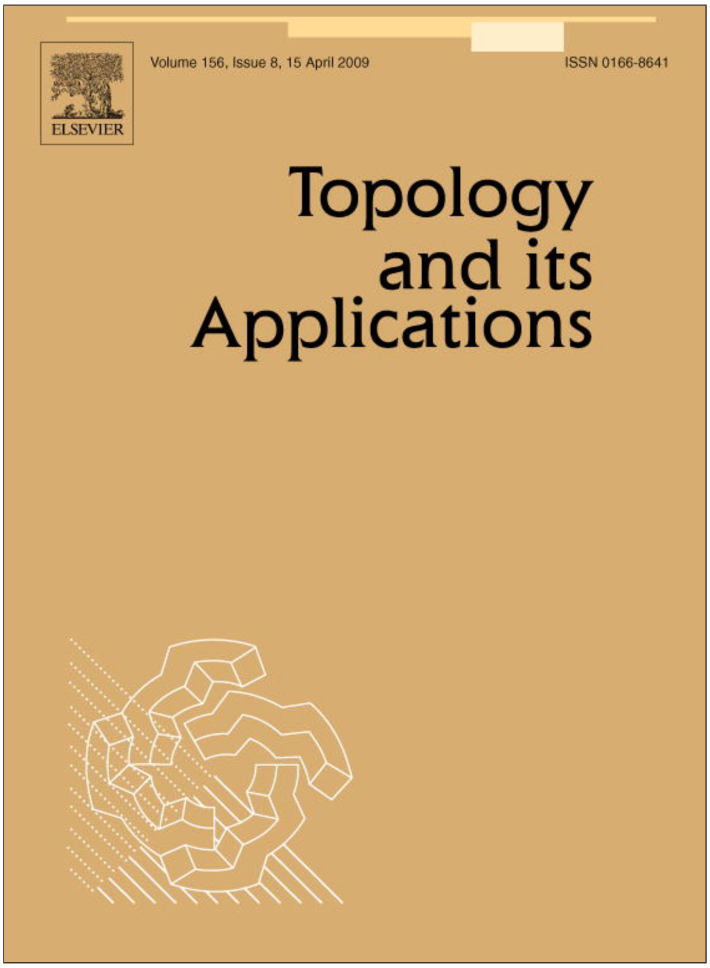
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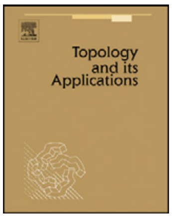

A λ-lemma for foliations

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ABSTRACT

We show that a C^0 codimension one foliation with C^1 leaves $\mathcal{F}$ of a closed manifold is minimal if there are a foliation $\mathcal{G}$ transverse to $\mathcal{F}$, and a diffeomorphism f preserving both foliations, such that every leaf of $\mathcal{F}$ intersects every leaf of $\mathcal{G}$ and f expands $\mathcal{G}$. We use this result to study of Anosov actions on closed manifolds.

1. Introduction

There are relatively few conditions ensuring the minimality of a foliation. For instance, we have the classical result by Sacksteder [18] which asserts that the orbit foliation of a C^2 locally free action without compact orbits of $\mathbb{R}^n$ on a closed $(n+1)$-manifold is minimal for all $n \geqslant 1$ (but this is false in the C^1 class by the Denjoy example in the 2-torus). On the other hand, more results about the minimality of a foliation usually arise from extra structures on the foliation as, for instance, be the stable (or unstable) foliation of an Anosov system. In fact, the codimension one invariant foliation of a codimension one Anosov diffeomorphism (or flow in dimension greather than three [19]) on a closed manifold is minimal [9]. Moreover, the strong (un)stable foliation of an Anosov flow on a compact manifold is minimal unless the flow is the constant-time suspension of an Anosov diffeomorphism [13]. Finally, we would like to mention a recent result [17] which gives a sufficient condition for the strong stable foliation $\mathcal{F}_f^{ss}$ of a partially hyperbolic C^1-diffeomorphism f on a closed manifold to be robustly minimal (i.e. the continuation $\mathcal{F}_g^{ss}$ of $\mathcal{F}_f^{ss}$ is minimal for all C^1 perturbation g of f).

In this paper we give another condition which will work for C^0 codimension one foliations $\mathcal{F}$ with C^1 leaves on closed manifolds. Indeed we require the existence of a foliation $\mathcal{G}$ transverse to $\mathcal{F}$, and a diffeomorphism f preserving these foliations, such that every leaf of $\mathcal{F}$ intersects every leaf of $\mathcal{G}$ and f expands $\mathcal{G}$. We call it *λ-lemma for foliations* for its proof resembles that of the classical λ-lemma in hyperbolic dynamics [3].

We use this lemma to analyze the dynamics of Anosov actions with a central Anosov element (or *central Anosov actions* for short). Indeed, we prove that a codimension one central Anosov action of a closed manifold is transitive when every stable leaf intersects every unstable leaf. Afterward, we discuss the existence of central Anosov actions of n-dimensional

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groups on closed $(n+1)$-manifolds, $n \geqslant 2$. We prove in particular that all such actions (if they exist) are minimal and the fundamental group of the supporting manifolds have exponential growth. As a corollary we obtain that central Anosov actions of amenable unimodular n-dimensional groups on closed $(n+1)$-manifolds do not exist.

2. λ-lemma for foliations

By a *closed manifold* we shall mean a compact connected boundaryless manifold. Let $\mathcal{F}$ be a C^0 foliation with C^1 leaves of a closed manifold M. Denote by $T\mathcal{F}$ the subbundle tangent to $\mathcal{F}$ (note that we are not assuming that $T\mathcal{F}$ is continuous). Given $x \in M$ we denote by $\mathcal{F}(x)$ the leaf of $\mathcal{F}$ containing x. A subset $\mu \subset M$ is *invariant* (for $\mathcal{F}$) if it contains every leaf of $\mathcal{F}$ intersecting it. We say that μ is a *minimal set* if it is non-empty closed invariant and does not contain a non-empty proper closed invariant subset. Since M is compact we have that $\mathcal{F}$ has a minimal set. The foliation $\mathcal{F}$ itself is called *minimal* if M is a minimal set of $\mathcal{F}$. A diffeomorphism f *preserves* $\mathcal{F}$ if it sends leaves of $\mathcal{F}$ into leaves of $\mathcal{F}$ and *expands* $\mathcal{F}$ if, in addition, there are constants $K, \lambda > 0$ such that

$$\left\| Df^n(x) \cdot v \right\| \geqslant K e^{\lambda n} \|v\|, \quad \forall x \in M,\ \forall v \in T_x\mathcal{F},\ \forall n \in \mathbb{N}.$$

Another C^0 foliation $\mathcal{G}$ with C^1 leaves is said to be *transverse* to $\mathcal{F}$ if every intersection between the leaves of $\mathcal{F}$ and $\mathcal{G}$ is transversal, i.e., $T_xM = T_x\mathcal{F}(x) \oplus T_x\mathcal{G}(x)$ for all $x \in M$. Note that every foliation transverse to a codimension one foliation is one-dimensional. With these definitions and notations we can state our λ-lemma for foliations.

Lemma 1. *Let $\mathcal{F}$ be a C^0 codimension one foliation with C^1 leaves of a closed manifold. If there are a foliation $\mathcal{G}$ transverse to $\mathcal{F}$ and a diffeomorphism preserving these foliations such that every leaf of $\mathcal{F}$ intersects every leaf of $\mathcal{G}$ and f expands $\mathcal{G}$, then $\mathcal{F}$ is minimal.*

Proof. We only need to prove that $\mathcal{F}$ has no proper minimal sets. For this we suppose by contradiction that $\mathcal{F}$ has a proper minimal set μ. Since μ is proper there are $x \in \mu$ and a transverse open arc J with x as boundary point such that

$$J \cap \mu = \emptyset.$$

We can assume without loss of generality that $J \subset \mathcal{G}(x)$. Since M is closed there is an integer sequence $n_k \to \infty$ such that $f^{n_k}(x)$ converges to some $y \in M$. Since $J \subset \mathcal{G}(x)$ and f expands $\mathcal{G}$ we have $|f^{n_k}(J)| \to \infty$ as $k \to \infty$ where $|\cdot|$ denotes the length operation. We claim that one of the two connected components of $\mathcal{G}(y) \setminus \{y\}$, say K, satisfies

$$K \cap \mathcal{F}(y) = \emptyset.$$

Indeed, suppose by contradiction that these components intersect $\mathcal{F}(y)$ in two points z_1, z_2 (one for each component). Note that such an intersection is transversal. Since $f^{n_k}(x) \to y$, f preserves $\mathcal{G}$ and $|f^{n_k}(J)| \to \infty$ we obtain from a tubular flow box around $\mathcal{G}(y)$ that the arc sequence $f^{n_k}(J)$ accumulates on one of these components. It follows again from tubular box arguments that $f^{n_k}(J)$ intersects $\mathcal{F}(y)$ transversally at some point z_3 close to either z_1 or z_2. However, if we take k large then $f^{n_k}(x)$ is close to y, and so, by considering the holonomy of $\mathcal{F}$ from y to z_3, we obtain that $f^{n_k}(J) \cap \mathcal{F}(f^{n_k}(x)) \neq \emptyset$ for k large. But $f^{n_k}(\mathcal{F}(x)) = \mathcal{F}(f^{n_k}(x))$ since f preserves $\mathcal{F}$ hence $f^{n_k}(J) \cap f^{n_k}(\mathcal{F}(x)) \neq \emptyset$. As f is a diffeomorphism we conclude that $J \cap \mathcal{F}(x) \neq \emptyset$. But $x \in \mu$ and μ is invariant so $J \cap \mu \neq \emptyset$ which contradicts $J \cap \mu = \emptyset$. This proves the claim. Now, take an accumulation point z of K. By hypothesis we have $\mathcal{G}(z) \cap \mathcal{F}(y) \neq \emptyset$, and so, $K \cap \mathcal{F}(y) \neq \emptyset$ by using a tubular flow box around z. This contradicts $K \cap \mathcal{F}(y) = \emptyset$ and the result follows. □

The arguments above imply that if a foliation $\mathcal{F}$ of arbitrary codimension satisfies the remainder hypotheses of the lemma, then it has no closed leaves.

3. Transitive Anosov actions

An *action* of a Lie group G on M is a C^1 map $\varphi : G \times M \to M$ such that $\varphi(e, x) = x$ and $\varphi(g, \varphi(h, x)) = \varphi(gh, x)$ for all $x \in M$ and all $g, h \in G$, where e is the neutral element of G. We shall use the customary notation $g(x) = \varphi(g, x)$.

The orbit and isotropy group of $x \in M$ are defined by $G(x) = \{g(x)\colon\ g \in G\}$ and $G_x = \{g \in G\colon\ g(x) = x\}$, respectively. We say that the action is *transitive* if it has a dense orbit; and *minimal* if every orbit is dense. Of course a minimal action is transitive but not conversely. The action is *locally free* if all its isotropy groups are discrete. In such a case the orbits of the action form a C^1 foliation of M of dimension $\dim(G)$, the dimension of G. Throughout such a foliation is called the *orbit foliation* of the action. In the locally free case the minimality of the action is equivalent to the minimality of the orbit foliation.

A locally free action is *Anosov* if there is $f \in G$ (throughout called *Anosov element*) such that f is *normally hyperbolic* to the orbit foliation $\mathcal{F}$. By this we mean that there are a continuous f-invariant splitting $TM = E^u \oplus T\mathcal{F} \oplus E^s$ and positive constants K, λ such that if $m(\cdot)$ denotes the co-norm operator, then the following inequalities hold $\forall n \in \mathbb{N}$, $\forall x \in M$:

$$\left\| Df^n(x)/E^s_x \right\|, \left\| Df^{-n}(x)/E^u_x \right\|, \frac{\|Df^n(x)/E^s_x\|}{m(Df^n(x)/T_x\mathcal{F})}, \frac{\|Df^n(x)/T_x\mathcal{F}\|}{m(Df^n(x)/E^u_x)} \leqslant K e^{-\lambda n}.$$

By the Invariant Manifold Theory [6] through each $x \in M$ passes a C^1 submanifold $W^{uu}(x)$ tangent to E^u at x. Moreover, $W^{uu} = \{W^{uu}(x)\colon\ x \in M\}$ is a C^0 foliation with C^1 leaves that is expanded by f. Analogously there is a C^1 submanifold $W^{ss}(x)$ tangent to E^s_x at x and $W^{ss} = \{W^{ss}(x)\colon\ x \in M\}$ is a C^0 foliation with C^1 leaves that is expanded by f^{-1}. Define

$$W^s(x) = \bigcup_{y \in G(x)} W^{ss}(y) \quad \text{and} \quad W^u(x) = \bigcup_{y \in G(x)} W^{uu}(y)$$

for all $x \in M$. It follows that the sets $W^s = \{W^s(x)\colon\ x \in M\}$ and $W^u = \{W^u(x)\colon\ x \in M\}$ are C^0 foliations with C^1 leaves $W^s(x)$, $W^u(x)$ tangent to $E^s_x \oplus T_xG(x), T_xG(x) \oplus E^u_x$ respectively for all $x \in M$. These foliations are usually called *stable* and *unstable* foliations, respectively. A *central Anosov action* is an Anosov action where the Anosov element belongs to the centralizer of the group (in such a case the element will be refereed to as a *central Anosov element*). This centralizer condition was first considered in [16]. A basic property of central Anosov actions of G on M is that $g(W^{uu}(x)) = W^{uu}(g(x))$ and $g(W^{ss}(x)) = W^{ss}(g(x))$ for all $g \in G$ and all $x \in M$. The class of central Anosov actions is broad enough to include many interesting examples as Anosov actions of abelian groups (e.g. $\mathbb{R}^k$ of $\mathbb{Z}^k$). Nevertheless there are examples of Anosov actions which are not central (e.g. [5] or Example 5 below).

The following proposition (which is classical for rank one actions) uses stable and unstable foliations to prove the transitivity of a central Anosov action. A related result can be found in [15].

Proposition 2. *A central Anosov action of a closed manifold is transitive if either W^s or W^u is minimal.*

Proof. We shall prove the result for W^s since the proof for W^u is analogous. Consider an Anosov action of a Lie group G on a closed manifold M such that W^s is minimal. We claim that for all open sets $U, V \subset M$ there is $g \in G$ such that $g(U) \cap V \neq \emptyset$. Indeed, fix $x \in V$ and an Anosov element f in the center of G. Since M is compact we can choose a sequence $n_k \to \infty$ such that $f^{-n_k}(x) \to x'$ for some $x' \in M$. We have that $W^s(x')$ is dense in M since W^s is minimal, then, $W^s(x') \cap U \neq \emptyset$ since U is open. So, there is $g' \in G$ such that $W^{ss}(g'(x')) \cap U \neq \emptyset$. On the other hand, $f^{-n_k}(g'(x)) \to g'(x')$ since f is central and g' is continuous. As $W^{ss}(g'(x')) \cap U \neq \emptyset$ and U is open we can fix a real number $b > 0$ such that for all k large there is a disk $D_k \subset W^{ss}(f^{-n_k}(g'(x)))$ centered at $f^{-n_k}(g'(x))$ of radius at most b such that $D_k \cap U \neq \emptyset$. Since $(g')^{-1}$ is continuous and $x \in V$ we can fix a neighborhood Q of $g'(x)$ such that $(g')^{-1}(Q) \subset V$. But f^{-1} expands W^{ss} and D_k is a disk centered at $f^{-n_k}(g'(x))$ of radius at most b (independent on k). So, for all k large we have that $f^{n_k}(D_k) \subset Q$. Then, $(g')^{-1} f^{n_k}(D_k) \subset V$ by the property of Q. Now pick k large and define $g = (g')^{-1} f^{n_k}$. As $D_k \cap U \subset U$ we have $g(D_k \cap U) \subset g(U)$. Additionally, $g(D_k \cap U) \subset (g')^{-1} f^{n_k}(D_k) \subset V$. Then, $g(U) \cap V \supset g(D_k \cap U)$ and so $g(U) \cap V \neq \emptyset$ since $D_k \cap U \neq \emptyset$. This proves the claim. Then, the result follows from the claim and well-known properties of group actions (e.g. [2]). □

An Anosov action is *codimension one* if the splitting associated to one of its Anosov elements satisfies either $\dim(E^u) = 1$ or $\dim(E^s) = 1$. We can assume that $\dim(E^u) = 1$ for, otherwise, we use the time reversed action $(g, x) \to g^{-1}(x)$. The following lemma gives a sufficient condition for the stable foliation W^s of a codimension one central Anosov action to be minimal.

Lemma 3. *The stable foliation W^s of a codimension one central Anosov action on a closed manifold is minimal if every leaf of W^s intersects every leaf of W^{uu}.*

Proof. Let f be a central Anosov element of the action. Then, f preserves both W^s and W^{uu} and, furthermore, it expands W^{uu}. Since $\dim(E^u) = 1$ we have that W^s is a codimension one foliation. Then, the result follows from the λ-lemma for foliations. □

The classical Spectral Decomposition Theorem [3] implies that an Anosov flow or diffeomorphism on a closed manifold is transitive if every leaf of W^u intersects every leaf of W^s. We would like to extend this transitivity condition to all Anosov actions, but there is no spectral decomposition theorem for general Anosov actions. However, our results allow us to extend at least the transitivity condition to the *codimension one central Anosov actions*, namely, Anosov actions which are both central and codimension one.

Corollary 4. *A codimension one central Anosov action on a closed manifold is transitive if every leaf of W^u intersects every leaf of W^s.*

Proof. We claim that every leaf of W^s intersects every leaf of W^{uu}. Indeed, fix $x, y \in M$ and $z \in W^u(x) \cap W^s(y)$ (this is possible by the hypothesis). Hence $z \in W^{uu}(g(x))$ for some $g \in G$ by definition. But $W^{uu}(g(x)) = g(W^{uu}(x))$ so $z \in g(W^{uu}(x)) \cap W^s(y)$ and then $g^{-1}(z) \in W^{uu}(x) \cap W^s(y)$ since $g(W^s(y)) = W^s(y)$. Therefore $W^{uu}(x) \cap W^s(y) \neq \emptyset$. Since x, y are arbitrary we obtain the claim. The claim and Lemma 3 imply that W^s is minimal, so, the action is transitive by Proposition 2. □

4. Anosov actions of n-dimensional groups on $(n+1)$-manifolds

Consider $n \geqslant 1$. By an *n-manifold* we mean a manifold of dimension n and by an *n-dimensional Lie group* we mean a Lie group of dimension n. In this section we investigate the existence (or not) of central Anosov actions of n-dimensional groups on closed $(n+1)$-manifolds, $n \geqslant 1$. Since such actions do not exist for $n = 1$ we restrict ourself to the case $n \geqslant 2$. As a motivation we present the following example in which T^k is the k-dimensional torus and $S^1 = T^1$ is the circle.

Example 5. We know from Example (8) in p. 1022 of [5] that there is a closed 3-manifold M^3 supporting an Anosov action of a 2-dimensional Lie group G_0 (note that $M^3 \neq T^3$ since $\pi_1(M^3)$ is not abelian, see Theorem 6). Now, consider the locally free transitive action $\mathbb{R} \times S^1 \to S^1$ given by the obvious flow. Using it we obtain from Example (5) in [5] that the closed $(k+3)$-manifold $M^3 \times T^k$ supports an Anosov action of $G_0 \times \mathbb{R}^k$ which is a $(k+2)$-dimensional Lie group, $\forall k \geqslant 1$. We conclude that for all $n \geqslant 2$ there is an Anosov action of a n-dimensional Lie group G (e.g. $G_0 \times \mathbb{R}^{n-2}$) on a certain closed $(n+1)$-manifold (e.g. $M^3 \times T^{n-2}$).

We observe that the group G in the above example has trivial center hence none of the actions in such an example is central. This fact suggests the following question: *Is there a* central *Anosov action of an n-dimensional Lie group on a closed* $(n+1)$*-manifold for some (or all)* $n \geqslant 2$*?* Although we do not have an answer for this question yet, it is possible to give properties of the corresponding actions (if they exist). For this we need some short definitions and facts. Recall that the fundamental group $\pi_1(M)$ of a manifold M has *exponential growth* (cf. [14]) if there are constants $A, B > 0$ such that the set $\Sigma(r) = \{h \in \pi_1(M)\colon h$ is represented by a curve γ of length $|\gamma| \leqslant r\}$ has cardinality $\#\Sigma(r) \geqslant Ae^{Br}$, $\forall r \geqslant 0$. Otherwise one says that $\pi_1(M)$ has *subexponential growth*. With this definition we have the following property.

Theorem 6. *The fundamental group of a closed* $(n+1)$*-manifold supporting an Anosov action of a connected n-dimensional Lie group has exponential growth.*

Proof. Let M a closed $(n+1)$-manifold and $G \times M \to M$ be an Anosov action of a connected n-dimensional group G on M. Fix a central Anosov element f together with the associated splitting $TM = E^s \oplus T\mathcal{F} \oplus E^u$ where $\mathcal{F}$ is the orbit foliation. Since the action is locally free we have that $T\mathcal{F}$ is n-dimensional. Therefore, as $\dim(M) = n+1$, we have either $E^u = 0$ (and E^s is one-dimensional) or $E^s = 0$ (and E^u is one-dimensional). We can assume the first case for, otherwise, we consider the time-reversed action $(g, x) \in G \times M \to g^{-1}(x) \in M$. Since G is connected we see that the diffeomorphism f fixes the leaves of $\mathcal{F}$. It follows that the orbit foliation $\mathcal{F}$ is expansive in the sense of [7], and so, it has a resilient leaf by a result in [7]. But, as is well known, the orbit foliation $\mathcal{F}$ has no vanishing cycles (e.g. Lemma 3.1 in [12]), and so, $\mathcal{F}$ cannot have null homotopic closed transversals (e.g. Theorem 3.1, p. 143 in [4]). Therefore, $\pi_1(M)$ has exponential growth by [11] since resilient leaves have exponential growth (e.g. [4]). This concludes the proof. □

Another possible approach to prove this theorem is to use holonomy invariant measures [11,12]. Observe that an Anosov action of the type considered in the theorem is necessarily a codimension one Anosov action. This observation, along with the now classic work [14] suggest the possibility that the fundamental group of a closed manifold that supports codimension one Anosov actions of connected Lie groups has an exponential growth. The theorem above is merely a particular case of this belief. As an application we give the following.

Corollary 7. *There are no Anosov actions of a connected amenable unimodular n-dimensional Lie group on a closed* $(n+1)$*-manifold.*

Proof. Suppose by contradiction that such an action exists. Hence the fundamental group $\pi_1(M)$ of the supporting manifold M has exponential growth by Theorem 6. But M is compact so the action has a minimal set. Then [11, Corollary 9.3], [12] and the (already used) fact that the orbit foliation has no null homotopic closed transversals imply that $\pi_1(M)$ has subexponential growth. This is a contradiction which proves the result. □

Since abelian groups are both amenable and unimodular (e.g. [8]) we conclude from this corollary that Anosov actions of connected abelian Lie groups of dimension n on closed $(n+1)$-manifolds does not exist. This was mentioned in [1, p. 2] when the acting group is $\mathbb{R}^n$. Using this conclusion we give negative answer for our question when $n = 2$. Its proof follows from Corollary 7 since the sole noncommutative 2-dimensional Lie group is the group of affine transformations of the line which has trivial center.

Corollary 8. *There are no central Anosov actions of a connected* 2*-dimensional Lie group on a closed* 3*-manifold.*

Now we finish with a result about *minimal Anosov actions*, namely, Anosov actions where all orbits are dense. The motivation is a fact mentioned in the proof of Corollary 2.2, p. 451 in [10] (and based on an unpublished work of G. Duminy) according to which any C^2 locally free action of the group of affine transformations of the line on a closed 3-manifold is minimal. With this in mind we have the following.

Theorem 9. *Every central Anosov action of a n-dimensional Lie group on a closed $(n+1)$-manifold is minimal.*

Proof. Consider a central Anosov action of an n-dimensional Lie group G on a closed $(n+1)$-manifold M, $n \geqslant 2$. To prove that the action is minimal we need to prove that its orbit foliation $\mathcal{F}$ is. For this we proceed as follows. As in the proof of Theorem 6 we can assume that $E^s = 0$ and E^u is one-dimensional. It follows that the Anosov action is a codimension one Anosov action and $W^s = \mathcal{F}$, where $\mathcal{F}$ is the orbit foliation.

We claim that every leaf of $\mathcal{F}$ intersects every leaf of W^{uu}. Indeed, fix $x \in M$. Since the orbit foliation is codimension one we have that $W^u(x)$ is open in M. On the other hand, if z is a limit point of $W^u(x)$, then using a local product box around z we can find $y \in G(x)$ such that $W^{uu}(y) \cap G(z) \neq \emptyset$. Thus, there are $z' \in W^{uu}(y)$ and $g \in G$ such that $g(z') = z$ so $z = g(z') \in W^{uu}(g(y))$ (for the action is central). This implies that $z \in W^u(x)$ since $g(y) \in G(x)$ therefore $W^u(x)$ is closed. Since M is connected we conclude that $W^u(x) = M$. It follows that $W^{uu}(z) \cap G(x) \neq \emptyset$ for all $z \in M$ proving the claim.

Then, the result follows from the claim and Lemma 3 since $W^s = \mathcal{F}$. □

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Further reading

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