Hymenoptera Sexual Behaviour

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Abstract

Hymenoptera females are heterozygote diploids (two different alleles at a single sexual locus). Fertile males are haploids (just one sexual allele). In reproducing, the female somehow chooses to use or not the sperm of a haploid male. If not, she produces a haploid male offspring simply by cloning one of her gametes.

Otherwise, she combines one of her own gametes with one male gamete, producing a diploid offspring. In case this offspring is heterozygote, a new female is born. However, in case the offspring is homozygote (the same allele twice at the sexual locus), it is an infertile male. Being a waste for the population, selection should avoid this event.

Mean Field Model

Consider an infinite, panmictic (ramdom mates) population. Females have probability p of choosing to produce a diploid offspring, $1-p$ a haploid offspring. The fraction of males is m_t at generation t, that of females is $1-m_t$. Among males, the fraction of haploids is h_t. The fixed number of sexual alleles is A at a single locus.

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At the next generation, $1-p$ is the fraction of haploid males produced, $\frac{A-1}{A} p h_t$ that of females, and $\frac{1}{N} p h_t$ that of infertile diploid males. Females having the bad luck of choosing a diploid male to mate do not produce any offspring, leading to a further term of $p(1-h_t)$ in order to sum-up unity.

Mean Field Map

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Therefore

$$h_{t+1} = \frac{1 - p}{1 - p(1 - \frac{1}{A} h_t)}$$

Fixed Point, Stabilisation

The last equation converges to

$$h^* = \frac{A}{2}\,\frac{1-p}{p}\left(\sqrt{1 + \frac{4}{A}\,\frac{p}{1-p}} - 1\right)$$

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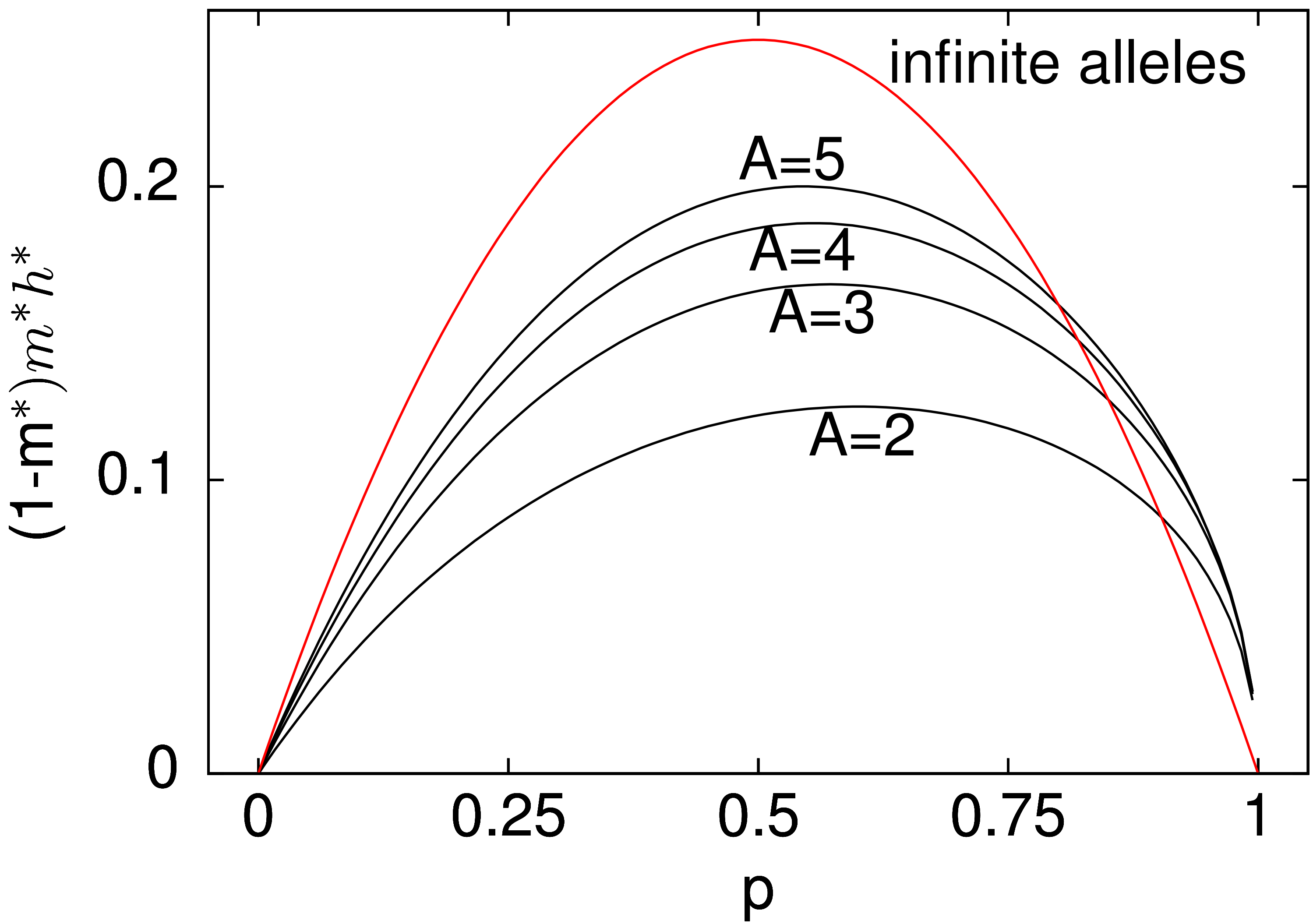
The interesting quantity is

$$(1-m^*)m^*h^*$$

corresponding to successful mates

Optimum



Agent's Model with Geography

$N = L^2$ heterozygous females are distributed over a $L \times L$ square lattice (one per site), each one with two randomly chosen single-locus sexual alleles tossed among A possibilities.

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Other, non-sexual genes are considered equally fitted, thus ignored.

Agent's Model (continuation)

One female is randomly chosen to reproduce, and decides first to use the genetic charge of the male at the same site, with probability p (say, $p = 50\%$). In this case, she produces a diploid offspring with the male allele and one randomly chosen of her two alleles. This diploid offspring maybe female (heterozygous) or diploid male (homozygous).

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Otherwise, with probability $1 - p$, she produces a haploid male offspring with one of her sexual alleles randomly chosen.

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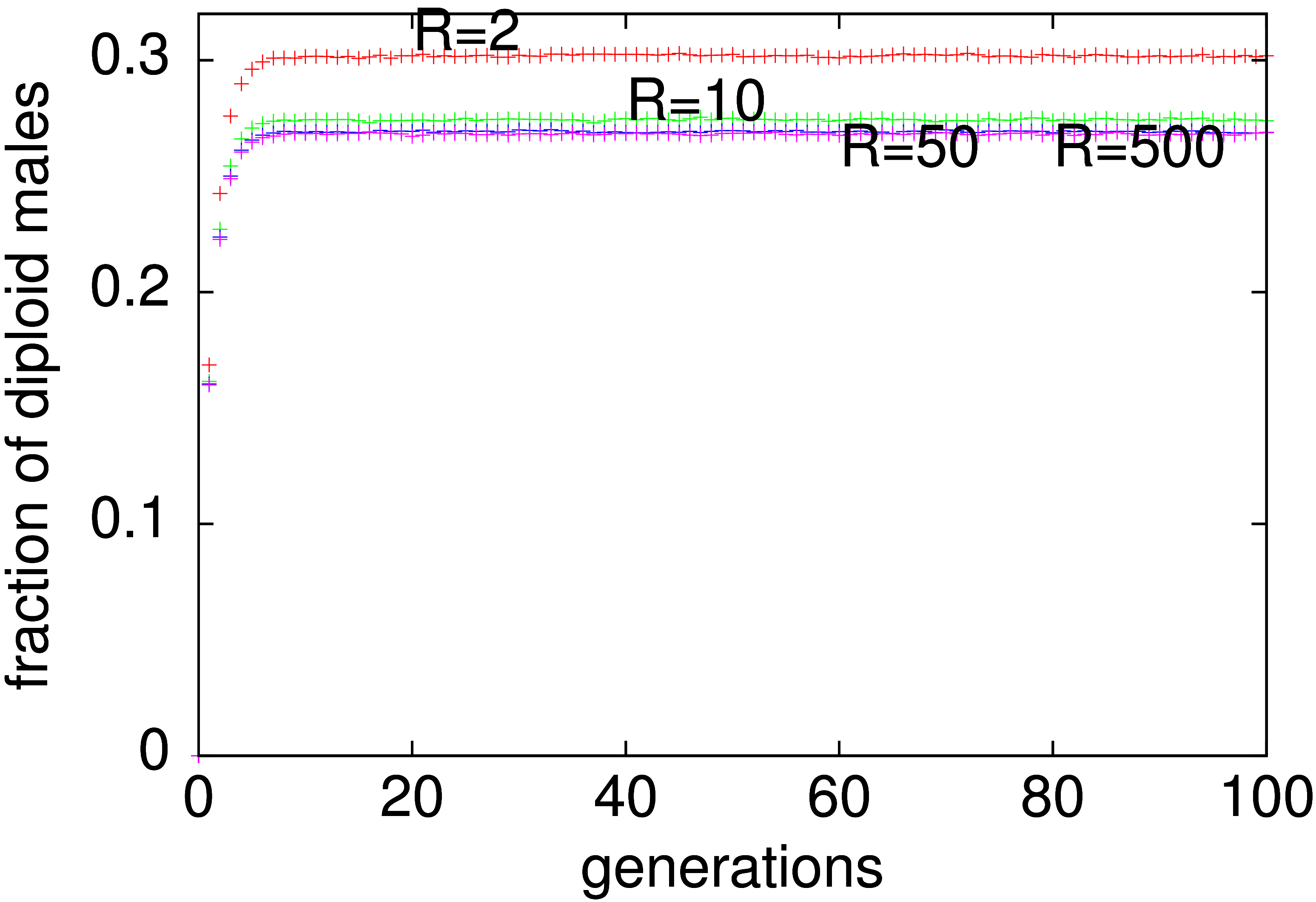
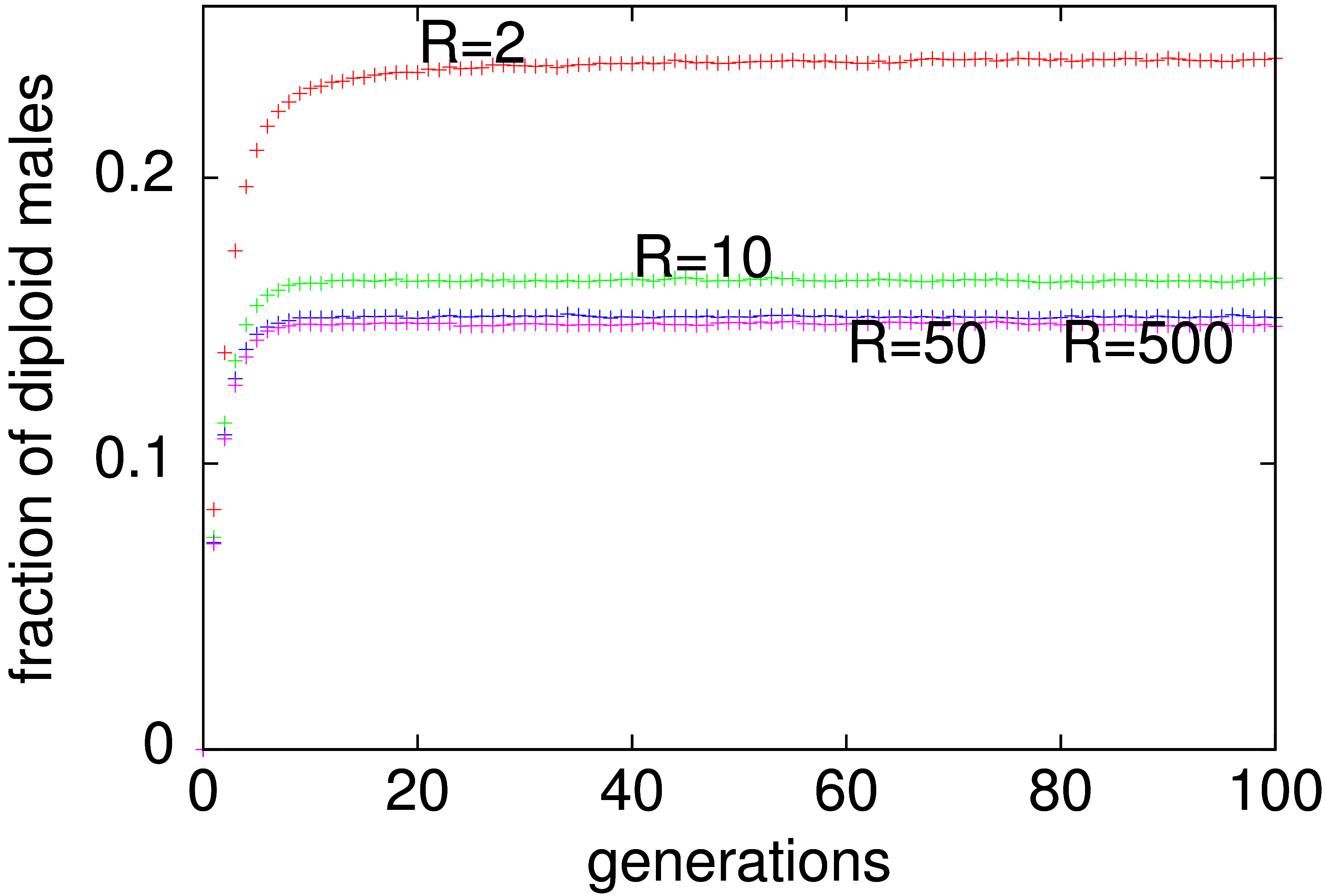
A homozygous male is infertile, in case a female decides to use his genetic charge, no offspring is produced.

A=2

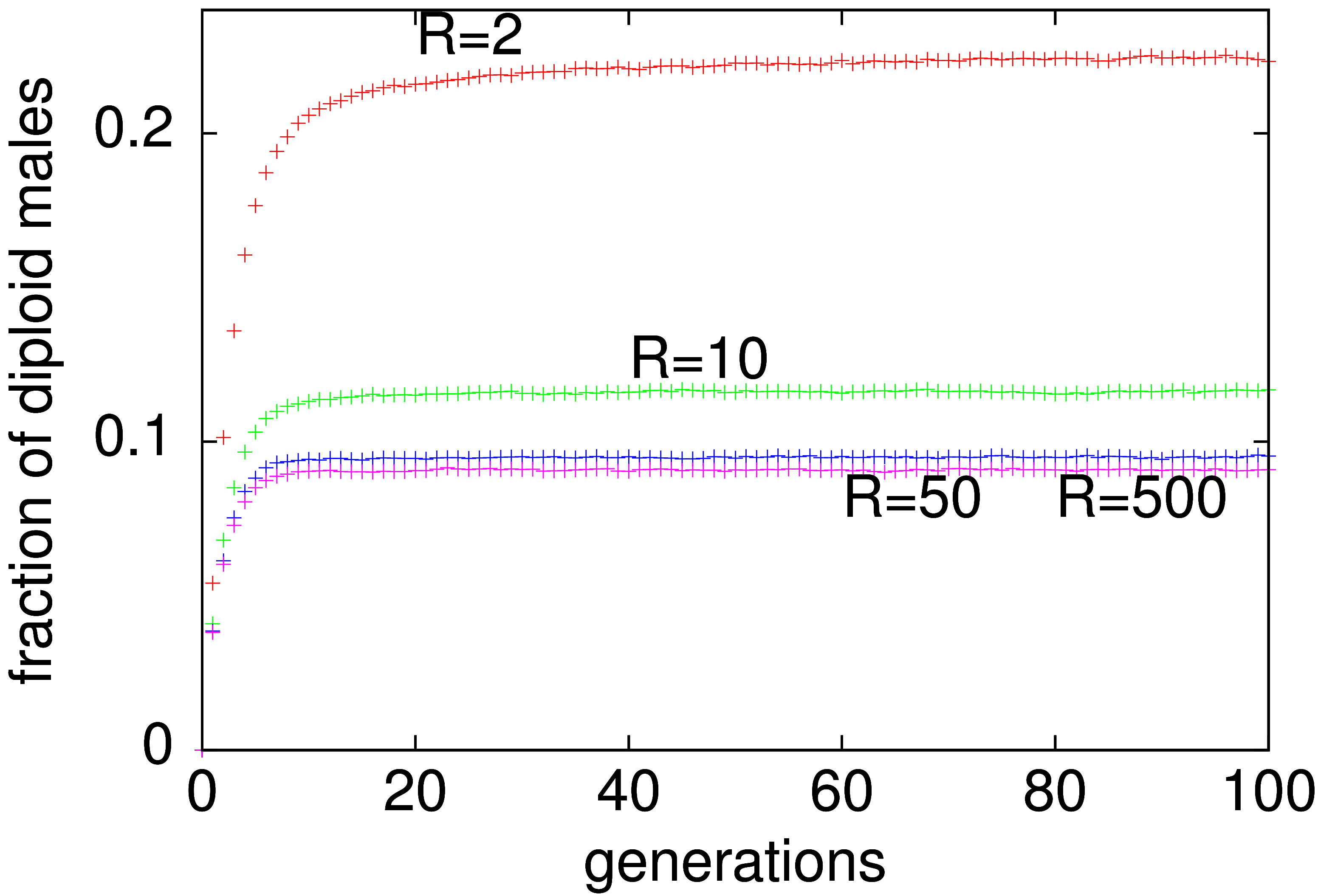


R=2
R=10
R=50
R=500
0.3
0.2
0.1
0
0
20
40
60
80
100
fraction of diploid males
generations

A=5


R=2
R=10
R=50
R=500
fraction of diploid males
generations
0
0.1
0.2
0
20
40
60
80
100

A=10


R=2
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Take a $\ell \times \ell$ sublattice and count the frequency of the $A(A-1)/2$ possible female genomes there.

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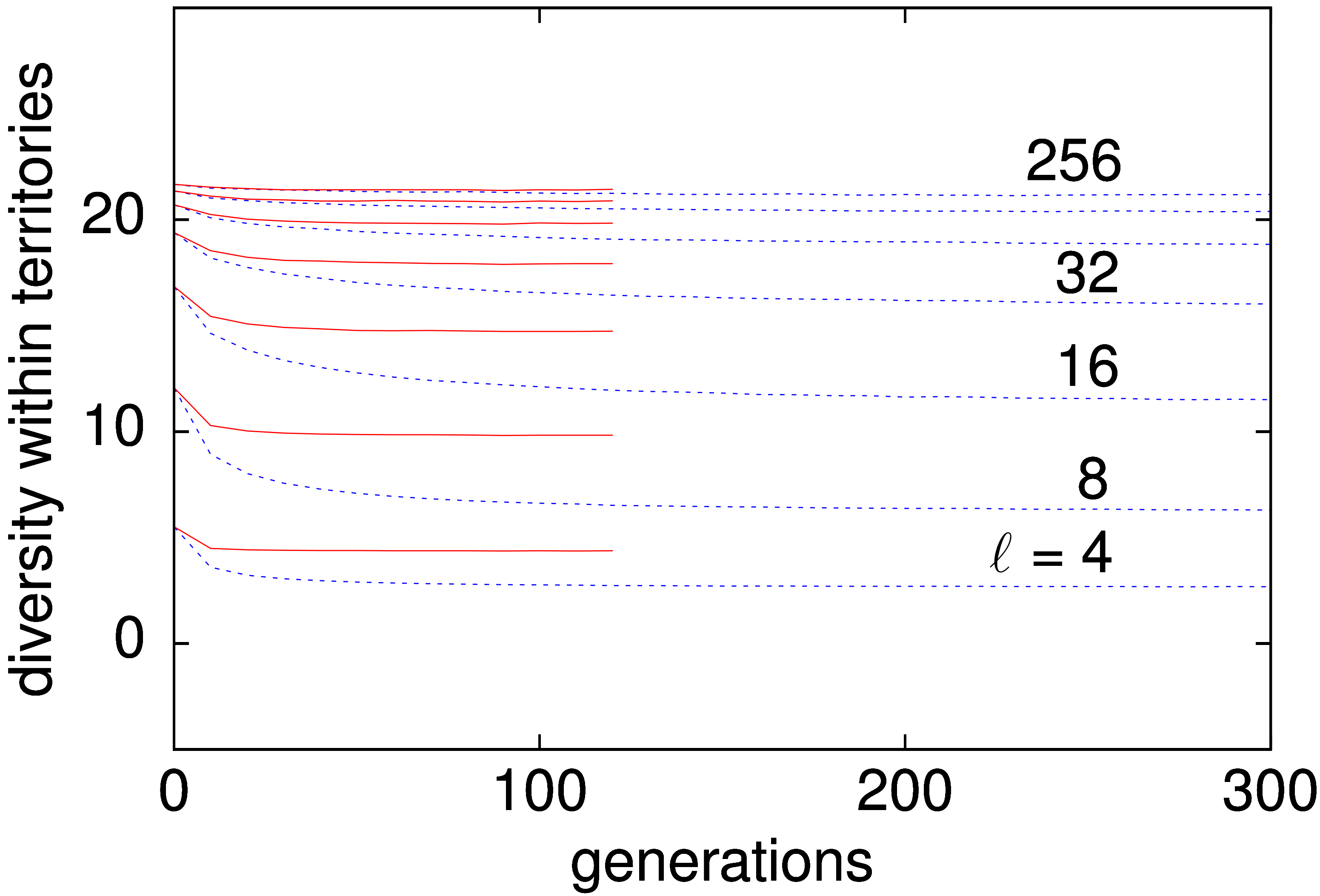
Put this distribution in decreasing order (Zipf plot), and computes its first moment D. It is a measure of the local diversity inside this territory.

$$A = 10, R = 2, 10$$


256
32
16
8
ℓ = 4
diversity within territories
20
10
0
0
100
200
300
generations