Society Collapse through erroneous Annual Tax rates: Piketty Recipe

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Foreword

This work is not yet finished.

I have still many doubts (*).

I hope you can help me with new ideas.
Gráfico 2 - Brasil: IR total/ renda total(%) por faixa de rendimento

Fonte dos dados brutos: Valor Econômico, edição 10/08/2015
Very Simple Model

$N$ agents, with their own wealths $W_i(t)$, $i = 0, 1 \ldots N - 1$. Initial wealths are randomly tossed at $t = 0$, and normalized ($\sum_i W_i = 1$). Time $t$ is counted in years.
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step I (annual income)
During the year, agents are randomly tossed to increase their wealths, $W_i \rightarrow 2W_i$. There are $N$ annual tosses (the same agent may be tossed more than once, or none).
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renormalization step
As we are not interested in the global economic growth, the distribution is renormalized ($\sum_i W_i = 1$). Nevertheless, the global growing factor can be booked.
step II (annual taxes)

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Renormalization again, before next year. Annual global growing factor booked.
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In this case, the wealth becomes null (exactly zero on computers), the corresponding agent is artificially ruled out of the game. Therefore, the system size $N$ becomes meaningless.
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In order to keep always the same system size $N$, every time an agent reaches the minimum computer figure, it is replaced by a copy of another random agent.
Results

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For large enough positive values of $p$ however, the system remains forever alive, no collapse. There is a transition from society extinction on the absorbing state towards an active forever changing dynamics, by surpassing a certain positive critical threshold $p_c$. 
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In evolutionary biology this is called the founders effect.
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Nevertheless, the wealth evolution of each agent is independent of others, therefore, perhaps mean-field reasonings may be applied “spatially” among agents themselves, generating an analytical formulation. Evaldo Curado investigates this possibility.
Zipf distribution

$N = 10240$, $T = 10^8$

$p = 0$, $p = 0.1$, $p = 0.5$, $p = 0.9$
Collapse time \( (W_0 > 0.999999) \) (*)

\[ T_\infty(p = 0) = T_N + K \times N^{-\alpha} = 1910 \pm 63 \]
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\[
T_\infty(0.15) = T_N + K \cdot N^{-\alpha}
= (1.55 \pm 0.02) \times 10^5
\]

\(p = 0.15\)
$T_{\infty} \times p \ (*)$

Fit $p_c = p + K \times T_{\infty}(p)^{-1/\gamma}$

$\gamma = 7.2 \pm 0.4$

Collapse time $T_{\infty}(p)$

Tax parameter $p$
Zipf first moment (*)

\[ N = 10240 \quad T = 10^8 \]