Programa

14:00 - 15:20 – Patrick Cattiaux (Toulouse)

*About some stochastic models on collective behaviour*

Several mathematical models for describing the collective behaviour of biological populations (cells, birds, ants ...) have been introduced more or less recently. In 1970 and 1971, Evelyn F. Keller and Lee A. Segel proposed two connected models for the chemotactic interaction of amoebae as mediated by acrasin: a macroscopic model describing the behaviour of the local density of cells, concentration of the chemo-attractant etc..., in terms of a system of coupled PDE’s, a microscopic one describing the microscopic (individual) behaviour of each cell interacting with the other ones in terms of a random system. In particular the Keller-Segel model describes the possible aggregation of cells depending on the parameters of the system. The macroscopic model has been extensively studied since this time, furnishing many difficult and interesting mathematical problems, and actually the situation is only well understood in two dimensions. The microscopic model has been much less studied. We shall discuss another stochastic microscopic model directly related to the macroscopic one. It is some kind of McKean-Vlasov interacting diffusions model, but with a singular attractive potential (with the opposite sign as in the Dyson Brownian motion introduced in random matrix theory). We shall see how the system feels the critical parameter yielding aggregation. If we have some time, we shall also introduce a stochastic version of the Cucker-Smale model of flocking. Here randomness is introduced to take into account some degree of freedom of each individual, but furnishes a negative answer to flocking.

The most common property of these two (as well as others) models is that their properties are almost completely unknown.

15:40 - 17:00 – Hubert Lacoin (IMPA)

*Disorder relevance for pinning of random surfaces*  
(joint work with G. Giacomin)

Disorder relevance is an important question in Statistical Mechanics. It can be formulated as follows: "If the Hamiltonian of model is modified by adding a small random perturbation, does it conserve a phase transition with the same characteristics as that of the pure model." A mathematical investigation of this matter is of course possible only for models for which the phase transition is rigorously understood in the pure setup, and our work concerns a very simple and tractable model of surfaces in interaction with a defect plane.

The surfaces is modeled by the graph of a Gaussian-Free-Field $\mathbb{Z}^d$, $d \geq 2$, and the interaction is given by an energy reward for each point of the graph whose height is in the interval $[-1, 1]$. The system undergoes a wetting transition from a localized phase to a delocalized one, when the mean energy of interaction varies. We investigate the modification of the free-energy curve induced by the introduction of inhomogeneity in the interaction. We show that in a certain sense the critical point is left invariant by the presence of homogeneity, but that the localization transition becomes much smoother.

17:00 – Discussão e lanche