

Contents

1. Kinematics	1
1.1 Configuration and Deformation.....	1
1.1.1 Change of Reference Configuration	4
1.2 Strain and Rotation	4
1.3 Linear Strain Tensors	8
1.4 Motion	13
1.4.1 Material and Spatial Descriptions	14
1.5 Relative Deformation	17
1.6 Rate of Deformation	20
1.7 Change of Frame and Objective Tensors	22
1.7.1 Transformation Property of Motion	25
1.7.2 Property of Some Kinematic Quantities	26
2. Balance Laws	31
2.1 General Balance Equation	31
2.1.1 Field Equation and Jump Condition	35
2.1.2 Balance Equations in Material Coordinates	36
2.2 Conservation of Mass	38
2.3 Laws of Dynamics	41
2.3.1 Forces and Moments	42
2.3.2 Stress Tensor	43
2.3.3 Conservation of Linear and Angular Momenta	50
2.4 Conservation of Energy	51
2.5 Summary of Basic Equations.....	54
2.5.1 Basic Equations in Material Coordinates	56
2.5.2 Boundary Conditions of a Material Body	57
2.6 Field Equations in Arbitrary Frames	58
3. Basic Principles of Constitutive Theories	63
3.1 Constitutive Relation	63
3.2 Principle of Material Objectivity	65
3.2.1 In Referential Description	68
3.2.2 An Example: a Particular Class of Materials	70
3.3 Simple Material Bodies.....	72

3.4	Reduced Constitutive Relations	75
3.5	Material Symmetry	77
3.5.1	Constitutive Equation for a Simple Solid Body	81
3.5.2	Constitutive Equation for a Simple Fluid	82
3.5.3	Fluid Crystal with an Intrinsic Direction	84
3.6	Isotropic Materials	86
3.6.1	Constitutive Equation of an Isotropic Material	88
3.7	Fading Memory	89
3.7.1	Linear Viscoelasticity	90
3.7.2	Boltzmann–Volterra Theory of Viscoelasticity	92
3.7.3	Linear Viscoelasticity of Rate Type	93
3.7.4	Remark on Material Objectivity of Linear Elasticity	94
4.	Representation of Constitutive Functions	97
4.1	Materials of Grade n	97
4.2	Isotropic Functions	98
4.2.1	Isotropic Elastic Materials and Linear Elasticity	107
4.2.2	Reiner–Rivlin Fluids and Navier–Stokes Fluids	109
4.2.3	Elastic Fluids	111
4.3	Representation of Isotropic Functions	112
4.3.1	Isotropic Thermoelastic Solids and Viscous Heat-Conducting Fluids	118
4.4	Hemitropic Invariants	119
4.5	Anisotropic Invariants	122
4.5.1	Transverse Isotropy and Orthotropy	124
4.5.2	On Irreducibility of Invariant Sets	126
5.	Entropy Principle	129
5.1	Entropy Inequality	129
5.2	Entropy Principle	131
5.3	Thermodynamics of Elastic Materials	132
5.3.1	Linear Thermoelasticity	135
5.4	Elastic Materials with Internal Constraints	139
5.5	Stability of Equilibrium	144
5.5.1	Thermodynamic Stability Criteria	148
5.6	Phase Equilibrium	149
6.	Isotropic Elastic Solids	153
6.1	Constitutive Equations	153
6.2	Boundary Value Problems in Elasticity	155
6.3	Homogeneous Stretch	157
6.3.1	Uniaxial Stretch	158
6.3.2	Biaxial Stretch	159
6.4	Symmetric Loading of a Square Sheet	160
6.4.1	Stability of a Square Sheet	162

6.5	Simple Shear	166
6.6	Pure Shear of a Square Block	169
6.7	Finite Deformation of Spherical Shells	173
6.7.1	Eversion of a Spherical Shell	175
6.7.2	Inflation of a Spherical Shell	176
6.8	Stability of Spherical Shells	179
6.8.1	Stability under Constant Pressures	180
6.8.2	Stability for an Enclosed Spherical Shell	181
7.	Thermodynamics with Lagrange Multipliers	183
7.1	Supply-Free Bodies	183
7.2	Viscous Heat-Conducting Fluid	184
7.2.1	General Results	186
7.2.2	Navier–Stokes–Fourier Fluids	188
7.3	Method of Lagrange Multipliers	189
7.3.1	An Algebraic Problem	190
7.3.2	Local Solvability	191
7.4	Relation Between Entropy Flux and Heat Flux	194
7.4.1	Theorem of Parallel Isotropic Vector Functions	194
8.	Rational Extended Thermodynamics	199
8.1	Introduction	199
8.2	Formal Structure of System of Balance Equations	200
8.2.1	Symmetric Hyperbolic System	201
8.2.2	Galilean Invariance	204
8.3	System of Moment Equations	207
8.4	Closure Problem	213
8.4.1	Entropy Principle	214
8.4.2	Formal Procedures	216
8.5	Thirteen-Moment Theory of Viscous Heat-Conducting Fluid	217
8.5.1	Field Equations	223
8.5.2	Entropy and Entropy Flux	225
8.6	Monatomic Ideal Gases	226
8.6.1	Thirteen-Moment Theory	227
8.6.2	Constitutive Equations	228
8.7	Stationary Heat Conduction in Ideal Gases	228
8.7.1	Fourier’s Law and Heat Conduction	229
8.7.2	Heat Conduction in Thirteen-Moment Theory	229
8.7.3	Remark on Boundary Value Problems	232

A. Elementary Tensor Analysis	233
A.1 Linear Algebra	233
A.1.1 Inner Product	234
A.1.2 Dual Bases	235
A.1.3 Tensor Product	238
A.1.4 Transformation Rules for Components	243
A.1.5 Determinant and Trace	245
A.1.6 Exterior Product and Vector Product	251
A.1.7 Second-Order Tensors	254
A.1.8 Some Theorems of Linear Algebra	256
A.2 Tensor Calculus	262
A.2.1 Euclidean Point Space	262
A.2.2 Differentiation	263
A.2.3 Coordinate System	272
A.2.4 Covariant Derivatives	275
A.2.5 Other Differential Operators	277
A.2.6 Physical Components	281
A.2.7 Orthogonal Coordinate Systems	282
References	289
Solutions of the Exercises	293
Index	321

Index

- Absolute temperature, 53, 130, 188
- Acceleration, 13, 15
- Angular momentum, 41
- Angular velocity tensor, 26
- Anisotropic invariant, 122
- Anisotropic material, 87
- Availability, 148
 - for biaxial stretching, 163
 - for spherical shell, 180, 182
- Axial vector, 25, 253

- Balance equation
 - formal structure, 200
 - general, 31
 - of moments, 207
- Balance of
 - internal energy, 53
 - kinetic energy, 53
 - linear momentum, 50
 - mass, 39
- Biaxial stretching, 159
- Body force, 42
 - apparent, 61
 - inertial, 61
- Boundary conditions, 57, 155

- Cauchy’s fundamental theorem, 45
- Cauchy’s law, 50
- Cauchy’s postulate, 43
- Cauchy–Green tensor, 5
- Cayley–Hamilton theorem, 103, 257
- Centrifugal force, 61
- Chain rule, 269
- Christoffel symbols, 275, 277
 - transformation rule, 277
- Clausius–Duhem inequality, 130, 132
- Closure problem
 - formal procedure, 216
 - in extended thermodynamics, 217
- Coefficient of thermal expansion, 147
- Commutation theorem, 258
- Compatibility condition, 13
 - kinematic, 37
- Components
 - contravariant, 237
 - covariant, 237
 - transformation rules, 243, 274
- Configuration, 1
 - reference, 1
- Conservation of
 - angular momentum, 50
 - energy, 52
 - linear momentum, 50
 - mass, 39
- Constitutive relation, 63
 - reduced, 75
- Constrained materials, 139
- Controllable deformation, 156
- Convected time derivative, 30
- Coordinate curve, 273
- Coordinate system, 272
 - change of, 274
 - metric tensor, 273
 - natural basis, 273
 - orthogonal, 281
- Coriolis force, 61
- Corotational time derivative, 29
- Covariant derivatives, 275, 277
- Curl, 278

- Deformation, 1
 - relative, 18
- Deformation gradient, 2
 - relative, 18
- Description
 - material, 14
 - spatial, 14
- Determinant, 245
 - of linear transformation, 247, 268
- Deviatoric part, 110, 189
- Dilatation, 11
- Directional derivative, 265
- Displacement, 9
 - gradient, 9, 108

- Divergence, 277
 - of tensor field, 278, 286
- Divergence theorem, 280
- Dual basis, 236
- Elastic fluid, 111, 203
- Elastic material
 - thermodynamics of, 132
- Elastic solid
 - isotropic, 107
 - orthotropic, 125
 - transversely isotropic, 124
- Elasticity
 - linear, 93
- Elasticity tensor, 93, 108
 - dynamic, 94
- Energy
 - free, 132
 - internal, 51
 - kinetic, 51
- Energy flux, 52
- Entropy inequality, 129
- Entropy principle, 131, 185, 192
 - in extended thermodynamics, 214
- Entropy production, 129
 - density, 223
- Entropy–entropy flux
 - integrability relation, 200, 216
 - pair, 201
- Equation
 - Navier–Stokes, 111
 - of continuity, 40
 - of motion, 50
 - Rankine–Hugoniot, 36
- Equivalent motion, 68
- Eshelby tensor, 150
- Euclidean point space, 262
- Euclidean transformation, 23
- Euclidean vector space, 234
- Euler equations, 112, 203
- Euler’s laws, 41
- Extended thermodynamics, 199
 - closure problem, 213
 - of 13 moments, 217
 - of ideal gases, 228
- Exterior product, 252
- Fading memory, 89
- Field equation, 54
 - general, 35
 - in arbitrary frame, 60
 - in material coordinates, 37
- First law of thermodynamics, 52
- Fluid, 80
 - elastic, 111
 - Navier–Stokes, 110, 119, 139, 143
 - Newtonian, 110
 - Reiner–Rivlin, 109
 - Stokes, 110
 - viscous heat-conducting, 98, 118
- Fluid crystal, 80
- Flux, 31
- Force, 41, 42
 - body, 42
 - contact, 42
 - moment of, 42
- Fourier’s law, 119, 133
 - heat conduction, 229
- Frame of reference, 1
 - change of, 22, 65
- Frame-indifference, 24, 58
- Free energy, 132, 150
 - Gibbs, 151
- Free enthalpy, 135, 151, 203
- Galilean invariance
 - of balance equation, 204
 - of dynamic laws, 42
- Galilean transformation, 26
- General balance equation
 - at a regular point, 35
 - at a singular point, 36
 - in integral form, 31
 - in material coordinates, 36
- General linear group, 255
- Generator function
 - hybrid, 216
 - of balance equation, 201
- Gibbs relation, 134, 188
 - for elastic solids, 135
- Gradient
 - of tensor field, 265
 - of tensor function, 266
- Heat conduction
 - 13-moment theory, 230
 - equation of, 138
 - Fourier’s law, 229
- Heat flux, 52
 - material, 56, 76
- Hemitropic invariant, 119
- Homogeneous deformation, 157
- Hooke’s law, 108
- Hyperbolic system
 - of balance equations, 200
 - symmetric, 201
- Hyperelastic material, 74, 134

- Ideal gas
 - classical, 226
 - monatomic, 226
- Incompressibility, 142
- Incompressible materials, 142
- Index notation, 279
- Inertial force, 61
- Inertial frame, 41, 42, 58
- Inextensibility, 143
- Inner product, 234
 - of tensors, 255
- Internal constraint, 139
 - incompressibility, 142
 - inextensibility, 143
- Internal energy, 51
- Invariant
 - anisotropic, 112, 122
 - functional basis, 113
 - hemitropic, 119
 - isotropic, 99, 112
- Inverse, 255
- Isothermal compressibility, 146, 147
- Isotropic elastic material, 107
- Isotropic function, 98
 - representation theorem, 99, 100, 105
- Isotropic invariant, 99
- Isotropic material, 86
 - reduced constitutive relation, 88
- Isotropic solid, 87
 - thermoelastic, 98, 118

- Jump condition, 54
 - entropy, 130
 - general, 36
 - in material coordinates, 37
- Jump discontinuity, 33

- Kelvin effect, 168
- Kinematic compatibility condition, 37
- Kinetic energy, 51
- Kronecker delta, 236

- Lagrange multiplier, 185, 200
 - lemma, existence of, 190
 - method of, 189
- Lamé elastic moduli, 108
- Laws of dynamics, 41
- Linear elasticity, 93
 - equation, 109
 - isotropic, 108
- Linear momentum, 41
- Linear thermoelasticity, 135
 - anisotropic, 136
 - isotropic, 137

- Linear transformation, 238
 - gradient of, 265
- Linear viscoelasticity, 90
 - of rate type, 93
- Local speed of propagation, 36

- Mass density, 39
- Material
 - of grade n , 97
 - with fading memory, 90
- Material heat flux, 56
- Material objectivity
 - condition, 70, 73, 75
 - principle, 65, 68
- Material symmetry, 77
 - group, 79
 - transformation, 78
- Material time derivative, 15
- Mechanical power, 51
- Metric tensor, 273
- Moment density, 208
 - internal, 208
- Moment equation, 207
 - system of, 210
- Moment of momentum, 41
- Monatomic ideal gas, 226
 - 13-moment theory, 226
 - classical, 226
- Mooney–Rivlin material, 154
- Motion, 13
 - incompressible, 34
 - rigid, 17

- Natural basis, 273
- Navier–Stokes equation, 111
- Navier–Stokes fluid, 110
- Navier–Stokes–Fourier fluid, 119, 188
- Neo-Hookean material, 155
- Noll’s rule, 80
- Norm, 234
 - of tensors, 255
- Normal stress effect, 168

- Objective tensor, 22, 24
- Orientation, 250
- Orthogonal group, 256
 - proper, 256
- Orthogonal transformation, 256
 - proper, 256
- Orthotropy, 87, 124

- Path, 13
- Permutation symbol, 250

- Phase equilibrium, 150
- Physical components, 281
- Piola–Kirchhoff stress tensor, 56
 - second, 76, 124
- Poisson’s ratio, 108
- Polar decomposition, 5
 - theorem, 260
- Positive definite tensor, 259
 - square root, 259
- Poynting effect
 - pure shear, 172
 - simple shear, 168
- Principal invariant, 103, 257
- Principal stretch, 5
- Principle of
 - determinism, 64, 141
 - material objectivity, 65, 68, 184
- Product
 - exterior, 252
 - inner, 234
 - tensor, 238
 - vector, 254
- Product basis, 239
- Product rule, 270
- Pure shear, 172, 173

- Rankine–Hugoniot equation, 36
- Rate of strain tensor, 21
- Reaction stress, 140
- Reduced constitutive relation, 75
 - anisotropic solid, 81
 - fluid, 83
 - isotropic solid, 88
 - simple solid, 81
- Reference configuration, 1
 - change of, 4
- Response function, 64
- Rigid motion, 17
- Rivlin–Ericksen tensor, 21, 97
- Rotation tensor, 5
 - infinitesimal, 10
- Rule of equipresence, 75

- Second law of thermodynamics, 129
- Second-order tensor, 239, 254
- Shear
 - pure, 48, 172
 - simple, 6, 166
- Shear modulus, 108, 167
- Simple fluid, 80
 - reduced constitutive relation, 83
- Simple material body, 73
- Simple shear, 166

- Simple solid, 80
 - reduced constitutive relation, 81
- Singular surface, 33, 149
- Solid, 80
 - isotropic, 87
- Special linear group, 256
- Specific heat
 - at constant pressure, 147
 - at constant volume, 136, 146, 148
- Spectral theorem, 258
- Spherical shell, 173
 - eversion, 175
 - inflation, 176
- Spin tensor, 21
 - of frame, 26
- Square root
 - positive definite tensor, 259
- Stability
 - criterion, 148
 - of equilibrium, 144
 - of spherical shell, 179
 - of square sheet, 162
 - thermodynamic criteria, 149
- Stokes fluid, 110
- Stored energy function, 74, 134, 139
- Strain tensor
 - Almansi–Hamel, 8
 - Green–St. Venant, 8
 - infinitesimal, 10, 108
 - linear, 8
- Stress
 - normal, 47
 - shear, 47
- Stress tensor, 43
 - Cauchy, 47
 - Piola–Kirchhoff, 56
- Stretch
 - biaxial, 159
 - uniaxial, 158
- Stretch tensors, 5
- Summation convention, 235
- Surface traction, 42
- Symmetric hyperbolic system, 201
 - of balance equations, 202
- System of moment equations, 207
 - hierarchy, 213

- Tangent space, 262
- Temperature, 53
 - absolute, 188
- Temperature gradient, 72
 - referential, 72, 82
 - relative, 82
 - spatial, 74, 82

- Tensor
 - contravariant, 240
 - covariant, 240
 - skew-symmetric, 242
 - symmetric, 242
- Tensor product, 238
 - space, 239
- Thermal conductivity, 225, 231
 - tensor, 133
- Thermodynamic process, 131
- Thermoelasticity
 - linear, 135
- Time derivative
 - convected, 30
 - corotational, 29
 - material, 15
- Trace, 248, 268
- Traceless part, 110, 189, 211
- Trajectory, 13
- Transformation
 - Euclidean, 23
 - Galilean, 26
 - rigid, 26
- Translation space, 262
- Transport theorem, 32
 - with a singular surface, 34
- Transpose, 241
- Transverse isotropy, 87, 124
- Unimodular group, 256
- Unimodular transformation, 79, 256
- Universal relation, 155
 - pure shear, 171
 - simple shear, 167
- Universal solution, 157
- Vector product, 254
- Velocity, 13
 - gradient, 15
- Viscoelasticity
 - Boltzmann–Volterra theory, 92
 - finite linear, 92
 - linear, 90
- Viscosity
 - bulk, 110
 - coefficients, 110
 - shear, 110, 225
- Viscosity coefficients, 189
- Viscous heat-conducting fluid, 98, 118, 184
 - 13-moment theory, 217
- Volume element, 250
- Vorticity vector, 21
- Young’s modulus, 108