General Equilibrium with Risk Loving, Friedman-Savage and other Preferences

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Introduction
Our Results

- We prove that with sufficient Aggregate Risk, equilibrium exists, even with a finite number of agents, for a very large class of preferences.
- For Rank-Dependent preferences, there is risk-sharing for these equilibria.
- We provide robust examples in which:
  1. The Risk-Loving decreases the volatility and improves the welfare.
  2. Regulation increases volatility and reduces welfare.

Content

Example in the Edgeworth Box

- Two states of nature.
- Utility: $U^i(x_1, x_2) = \frac{1}{2} u^i(x_1) + \frac{1}{2} u^i(x_2)$.
- Agent 1:
  - $u^1(x) = \ln x$,
  - $\omega^1 = (\omega_1^1, \omega_2^1) > 0$,
  - with an Arrow-Debreu constraint.
- Agent 2:
  - $u^2(x) = x^2$,
  - $\omega^2 = (\omega_1^2, \omega_2^2) > 0$,
  - with an Arrow-Debreu constraint.
- $\omega_1 := \omega_1^1 + \omega_1^2$ and $\omega_2 := \omega_2^1 + \omega_2^2$.

Since u^2 is convex, any optimal allocation must satisfy $x_1^2 = 0$ or $x_2^2 = 0$ for any price $(p, 1-p)$.

In fact $x_2^2 = 0 \iff p \leq 1/2$ and under the FOC and Market Clearing

$$p = \frac{\omega_2 + \omega_2^2}{\omega_1^1 + \omega_2 + \omega_2^2} \leq \frac{1}{2} \iff \omega_1^1 \geq \omega_2 + \omega_2^2,$$

and analogously we have $x_1^2 = 0 \iff \omega_2^1 \geq \omega_1 + \omega_1^2$.

Then

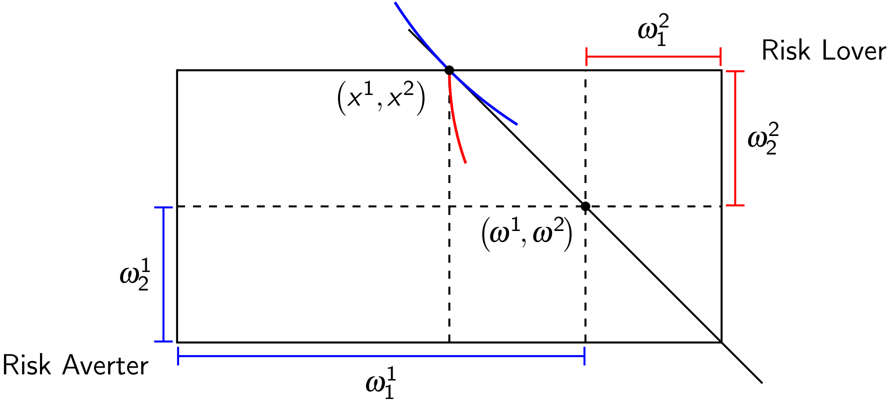
$$\text{Existence of Equilibrium} \iff \begin{array}{c} \omega_1^1 \geq \omega_2 + \omega_2^2 \\ \text{or} \\ \omega_2^1 \geq \omega_1 + \omega_1^2 \end{array} \iff \begin{array}{c} \omega_1 - \omega_2 \geq \omega_1^2 + \omega_2^2 \\ \text{or} \\ \omega_2 - \omega_1 \geq \omega_1^2 + \omega_2^2. \end{array}$$

Remark

There exists equilibrium if and only if

1. there is a difference of AT LEAST of $\omega_1^2 + \omega_2^2$ among the aggregate endowments i. e. there should be enough aggregate risk, or
2. the aggregate risk must be bigger or equal of the total wealth of the risk lover, or
3. the risk averter should be sufficiently rich in one of the state.

Edgeworth Box



Existence of Equilibrium
Expected Utility case

Let

- S states.
- Probability $\pi = (\pi_1, \ldots, \pi_S) \gg 0$,
- $I + J$ Expected Utility agents,
- I are Risk Averse,
- J are Risk Lovers.

Existence of Equilibrium
Risk Averters

- u^i is:
  - Strictly monotone,
  - Concave,
  - $C^1(0, \infty)$,
  - $\lim_{x \to \infty} u^{i\prime}(x) = 0$ and
- Endowments are $\omega_1^i, \ldots, \omega_S^i > 0$ for each i.
- With an AD constraint.

Existence of Equilibrium
Risk Lovers

- u^i is:
  - Strictly monotone and
  - Convex.
- Endowments are $\omega_1^i, \ldots, \omega_S^i > 0$ for each i.
- $\exists \lambda_s^i \in \left[0, \omega_s^i\right]$ a minimal consumption imposed in the state s.
- And with an AD constraint.

Existence of Equilibrium

Main Result

Theorem

Let U^i and $\{\omega_s^i\}_{i,r}$ fixed except in the state 1. If there is a $K > 0$ such that $\sum_{i \leq I} \omega_1^i \geq K$ then there is an equilibrium for the economy with $p \in \Delta_{++}^{S-1}$.

▸ Extension for more than one state ▸ Proof of Theorem

Lemma

Given a price p, all risk lovers will choose a consumption plan x^i such that

$$x_s^i = \begin{cases} \lambda_s^i & \text{for } s \neq s_0 \text{ (minimal consumption)}, \\ \frac{1}{p_{s_0}} \left[p\omega^i - \sum_{s \neq s_0} p_s \lambda_s^i \right] & \text{for some } s_0. \end{cases}$$

Existence of Equilibrium

Extensions to other preferences

- *Smooth Ambiguity Decision Makers*, Klibanoff, Marinacci and Mukerji, *Econometrica* (2005).
- *Choquet Expected Utility*, Schmeidler *Econometrica* (1989).
- *Variational Preferences*, Macheroni, Marinacci and Rustichini *Econometrica* (2006).
- *Friedman Savage Decision Makers*, Friedman and Savage *JPE* (1948). Friedman Savage case
- *Rank-Dependent Expected Utility* (RDEU) , Quiggin *Journal of Economic Behavior and Organization* (1982), *Kluwer Academic Publishers* (1993) and Yaari *Econometrica* (1987).
- With *rationing* on the amount of risk taken by the Risk/Ambiguity lovers.

RDEU model, Risk Sharing and Volatility

- Each agent distorts the prior π with f^i, where
  - f^i Continuous, $f^i(0) = 0$ and $f^i(1) = 1$.
  - u^i concave, f^i is convex $\Longrightarrow$ Ambiguity Averse (pessimism).
  - u^i convex, f^i is concave $\Longrightarrow$ Ambiguity Lovers (optimism).
- And the utility function is:

$$U^i(x) = (C)\int u^i \circ x \, df^i \circ \pi = \int_{-\infty}^{0} \left(f^i \circ \pi\left[u^i \circ x \geq t\right] - 1\right) dt + \int_0^{\infty} f^i \circ \pi\left[u^i \circ x \geq t\right] dt$$

Study of Volatility and Regulation
Implementation with complete markets

- **GOAL:** To evaluate the impact of Risk Loving on Volatility and Regulation.

- We interpret the AD equilibrium as a financial market equilibrium with two states with probability (π_1, π_2) with no consumption in $t = 0$.

Study of Volatility and Regulation
Implementation with complete markets

- Consider two assets:

- Constraints:
  - at $t = 0$, $q\alpha + \beta = 0$
  - at $t = 1$, $\omega_s^i + R_s\alpha + R\beta \geq \lambda_s^i$ for each s.

Volatility formula

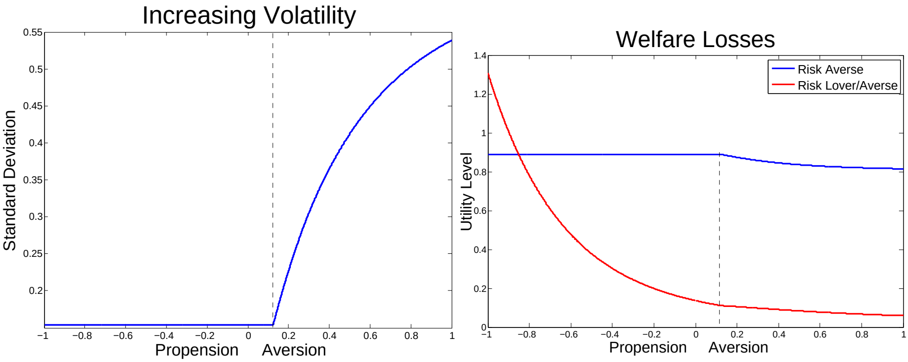
Risk-Loving Decreases Volatility

- Two states of nature,
- $f^i(x) = x$,
- $U^i(x) = \frac{1}{2}\, u^i(x_1) + \frac{1}{2}\, u^i(x_2)$ where

$$u^i(x) = \frac{1}{\rho^i}\left(1 - e^{-\rho^i x}\right)$$

Agent 1:	Agent 2:
$\rho^1 = 1$	$\rho^2 \in [-1, 1]$
$\omega^1 = (4, 1)$	$\omega^2 = (2, 1)$

Risk-Loving Decreases Volatility
Volatility and Welfare



In presence of Aggregate Risk, Risk Loving absorbs most of the risk reducing Volatility.

Also there is a reduction in Welfare when there is less Risk Loving in the economy.

Effects of Regulation

- Two states of nature,
- $f^i(x) = x$,
- $U^i(x) = \frac{1}{2}u^i(x_1) + \frac{1}{2}u^i(x_2)$ where

$$u^i(x) = \frac{1}{\rho^i}\left(1 - e^{-\rho^i x}\right)$$

Agent 1:	Agent 2:	Agent 3:
$\rho^1 = 1$ $\omega^1 = (2,1)$	$\rho^2 = 1.5$ $\omega^2 = (2,1)$	$\rho^3 = -1$ $\omega^3 = (1,1)$ $\lambda^3 \in [0,1]$

Effects of Regulation

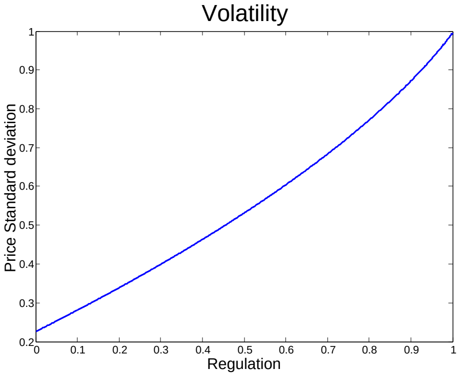
There is regulation on the risk lover's consumption

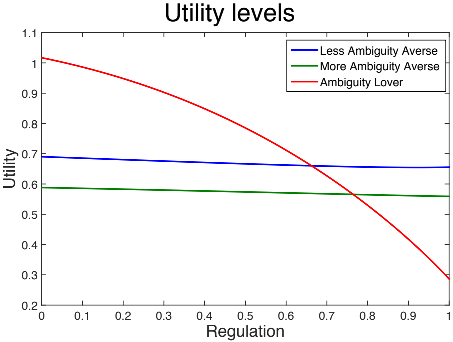
$$x_s^3 \geq \lambda^3 \in [0,1]$$

then

- $\lambda^3 = 0$ means no regulation.
- $\lambda^3 = 1$ means regulation impose the consumption to be $(1,1)$.

Effects of Regulation
Volatility and Welfare





$$\lambda^3 \text{ increases} \implies \text{Volatility increases}$$

Remark

Regulation increases Volatility and reduces Welfare in the economy.

What if there is no Risk Lover?
Example with Two Risk Averters

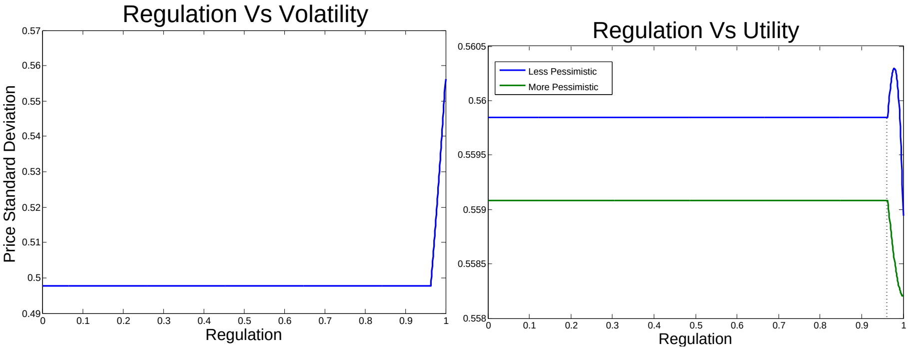
- New economy only with the two Risk Averse defined before.

- Now the two agents are under regulation.

$$x_s^i \geq \lambda^i = \lambda \in [0, 1]$$

What if there is no Risk Lover?

Effects of Regulation

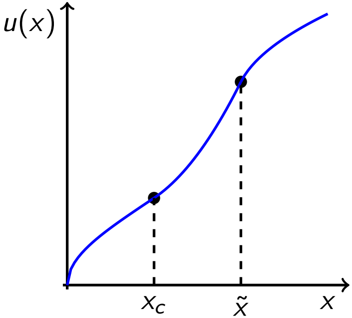


Remark

The regulation affects volatility and welfare only when it is unrealistically tight (i.e., $\lambda > 0.95$), since regulation is not binding for $\lambda < 0.95$.

Model with Friedman-Savage Decision Makers

Instead of Risk Lovers, consider Expected Utility agents with $u : [0, \infty) \to \mathbb{R}$ concave in $[0, x_c] \cup [\tilde{x}, infty)$ and convex in $[x_c, \tilde{x})$ where $x_c \geq 0$ and $\tilde{x} \geq x_c$.



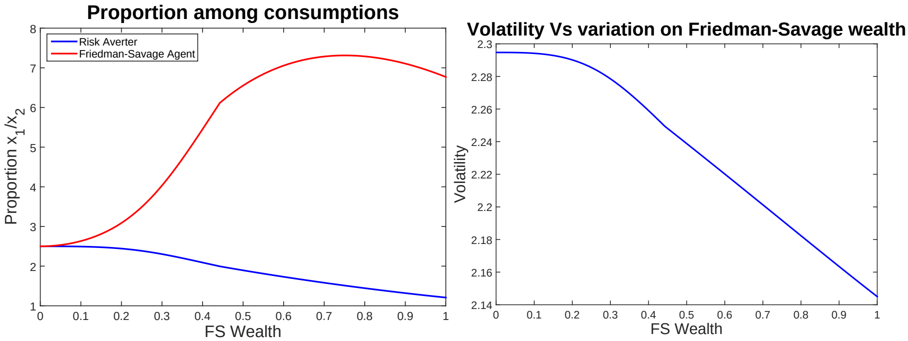
Proposition

If the aggregate endowment of risk averters is sufficiently large in $0 < S_1 < S$ *states compared with other states, there is an equilibrium for the economy with* $p \in \Delta_{++}^{S-1}$.

Friedman-Savage Case

Example and Volatility

- For the agent 1:
  - $u^1(x) = ln(x)$,
  - $\omega_1^1 = 5 - 2.5a$, $\omega_2^1 = 2 - a$.
- For the agent 2:
  - $u^2(x) = \begin{cases} \ln(x) + (1/2)x^2 & \text{if } x \leq 3/2, \\ 13/6(x - 3/2) + 9/8 + \log(3/2) & \text{if } x > 3/2. \end{cases}$
  - u^2 has an inflection point at $x_c = 1$,
  - $\omega_1^2 = 2.5a$, $\omega_2^2 = a$, where $a \in [0, 1]$.



Remark

FS Decision Maker behaves more as a Risk Lover and less as a Risk Averter when his wealth increases. This implies a reduction on volatility.

Back to extensions

Model with Friedman Savage and Prospect Theory

Remark

A FS Decision maker with $x_c = 0$ is consistent with **Kahneman and Tversky (1992)** with the weighting function as the identity when the second inflection point satisfies $\tilde{x} = \omega_s$ for all s.

Remark

A FS Decision maker is consistent with **Jullien and Salanié (2000)** with the weighting function as the identity when the the second inflection point satisfies $\tilde{x} = \omega_s$ for all s.

Proposition

For preferences mentioned above instead of Risk lovers, under the conditions mentioned in the proposition above, there is an equilibrium for the economy with $p \in \Delta_{++}^{S-1}$.

Model with Friedman Savage and Prospect Theory

Remark

For general distorsions of an objective probability. If the endowment distributions are such that $\omega_{s_1} \neq \omega_{s_2}$ for each pair of states s_1, s_2, there is an equilibrium for the economy with $p \in \Delta_{++}^{S-1}$ under the conditions mentioned in the proposition above.

Notice that this is not inconsistent with **Azevedo and Gottlieb (2012)** since, in our framework, there is only a finite number of states.

Thank you!

Remark

For preferences given by an Expected Utility agent with a **Friedman-Savage (1948)** utility index or a **Kahneman and Tversky (1992)** agent with a reference point $\tilde{x} = \omega_s^{I+j} > 0 \ \forall j$ and the capacity for losses is such that the functional $V(\cdot)$ is convex, there is an optimal solution for the consumer problem with an AD constrain in which there is **AT MOST** one state in which the agent consumes in the convex part of the utility index or value function.

Remark

Notice that if V is no convex for losses, the only possible form to ensure equilibrium is increasing, even more, the aggregate risk to ensure that all agents consume 0 for all given by the Prospect Theory if their consumption is in the losses part.

Existence of Equilibrium

Extension for more that one state

Proposition

Given $\left\{\omega_s^i\right\}_{s,i}$ *if there exist* R *states* $1 \leq s_1, \ldots, s_R \leq S$ *and* $0 < k < K$*, with* K *sufficiently big such that:*

1. $\pi_{s_1} = \cdots = \pi_{s_R}$,
2. $J = R\tilde{J}$ *with* $\tilde{J} \in \mathbb{N}$ *and* $\omega^{I+j_1} = \omega^{I+j_2}$ *for* $j_1 = \tilde{j}R + l_1$ *and* $j_2 = \tilde{j}R + l_2$ *where* $1 \leq l_1, l_2 \leq R$ *and* $0 \leq \tilde{j} < R$,
3. $\sum_{i \leq I} \omega_{s_r}^i \geq K$ *and* $\sum_{i > I} \omega_{s_r}^i \leq k$ *for all* $r = 1, \ldots, R$,
4. $\sum_i \omega_{s'}^i \leq k$ *for* $s_r \neq s'$ $\forall r = 1, \ldots, R$,
5. *there exists* $\alpha \in [0,1]$ *such that* $\lambda_s^i = \alpha \omega_s^i$ *for each* s *and* $i > I$.

Then there is an equilibrium for the economy with $p \in \Delta_{++}^{S-1}$.

Main Theorem

Existence of Equilibrium

Proof.

Define a modified generalized game with $I+J+1$ players.

For each Risk Averse, define a player as usual.

For each Risk Lover:

- **Utility:** $V^i(p,x) := x_1$.
- **Set of actions:** $x \in X^i := \{(x_1, \underbrace{\lambda_2^i, \ldots, \lambda_S^i}_{\text{minimal cons.}}) \;:\; \lambda_1^i \leq x_1 \leq 2\omega\}$.
- **Restriction:** $B^i(p) := \{x \in X^i \;:\; px \leq p\omega^i\}$.

Existence of Equilibrium

Proof. (Cont)

The last player is the traditional market:

- **Utility:** $V^i(p,x) := \sum_i \left(px^i - p\omega^i\right).$
- **Set of actions:** $p \in \Delta_+^{S-1}$.
- **Restriction:** Δ_+^{S-1}.

We have Existence of Nash Equilibrium $\left(\left(x^i\right)_{i=1}^{I+J}, p\right)$ which satisfies $\sum_i x^i = \sum_i \omega^i$ and optimization for the Risk Averse.

Missing:

Optimality of consumptions for Risk Lovers in the original economy.

Existence of Equilibrium

Proof. (Cont)

Using the First Order Conditions for the Risk Averse, $\lim_{x \to \infty} u^{i'}(x) = 0$ and $\lim_{x \to 0} u^{i'}(x) = \infty$ we have:

$$\sum_{i \leq I} \omega_1^i \geq K \text{ with } K \text{ big enough and } x_1^i > 0 \ \forall i = 1, \ldots, I \implies p_1 \approx 0.$$

And similarly, for each state $s \neq 1$ the previous condition implies that p_s must be bounded from below and far away from zero.

Existence of Equilibrium

Proof. (Cont)

And as consequence of $p_1 \approx 0$ and p_s bounded and far away from zero for $s \neq 1$.

Then the Risk Lovers will specialize in state 1 and the Nash Equilibrium allocation will be also optimal for the Risk Lovers.

And finally the Nash Equilibrium allocation will be also an equilibrium for the economy. □

▸ Main Theorem

Formula for Volatility

Volatility of returns:

$$\sigma(q) = \pi_1 \left| \frac{R_1}{q} - \mu(q) \right| + (1 - \pi_1) \left| \frac{R_2}{q} - \mu(q) \right|,$$

where $\mu(q) = \pi_1 \frac{R_1}{q} + (1 - \pi_1) \frac{R_2}{q}$.

▸ Finantial Equilibrium