On the Model Building for Transmission Line Cables: a Bayesian Approach

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OUTLINE

- Introduction
- Transmission Line Cables
- Bayesian Inverse Analysis
- Model Selection
- Results
- Concluding Remarks

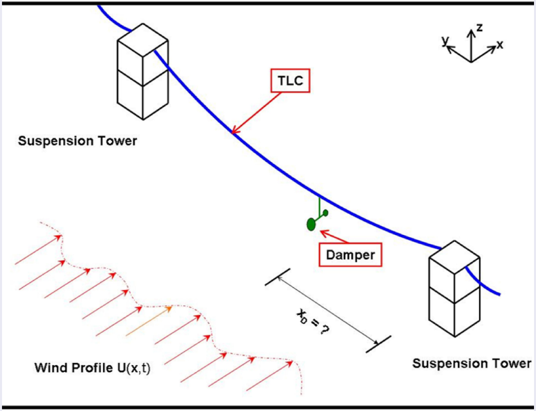
INTRODUCTION

Stranded Cables

- Used in cable-stayed bridges.
- Used in high-voltage transmission lines.
- Stranded cables of electric transmission lines are subjected to wind-induced vibrations caused by the vortex-shedding phenomenon.

INTRODUCTION

Electric transmission lines



INTRODUCTION

Motivation

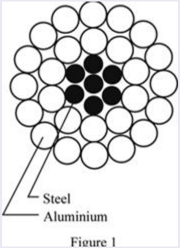
- Determining accurate **equivalent homogeneous models** to describe the mechanical behavior of stranded cables may lead to safer transmission line systems.

Goal

- To provide an approach to calibrate transmission line models using a **Bayesian Framework**.

Mathematical Modelling

Governing equation ?



Mathematical Modelling

Governing equation: equivalent homogeneous model

$$\mu\frac{\partial^2 w(x,t)}{\partial t^2}+\alpha\frac{\partial w(x,t)}{\partial t}+\frac{\partial^2 M(x,t)}{\partial x^2}-\frac{\partial}{\partial x}\left(T\frac{\partial w(x,t)}{\partial x}\right)=F(x,t) \tag{1}$$

$$M(x,t)=\int_A -y\ \sigma(x,y,t)\ dA \tag{2}$$

$$\varepsilon(x,y,t)=\frac{T}{EA}-y\frac{\partial^2 w(x,t)}{\partial x^2} \tag{3}$$

Mathematical Modelling

Constitutive equation: Kelvin-Voigt

$$\sigma^{KVM}(x,y,t) = E\varepsilon(x,y,t) + \xi \frac{d\varepsilon(x,y,t)}{dt} \quad (4)$$

$$\boldsymbol{\theta}_{KVM} = \{E,\xi\}^T \quad (5)$$

Mathematical Modelling

Governing equation with Kelvin-Voigt

$$\mu \frac{\partial^2 w}{\partial t^2} + \alpha \frac{\partial w}{\partial t} + EI \frac{\partial^4 w}{\partial x^4} + \xi I \frac{\partial}{\partial t} \left(\frac{\partial^4 w}{\partial x^4} \right) - T \frac{\partial^2 w}{\partial x^2} = F(x,t). \quad (6)$$

where $w = w(x,t)$

Mathematical Modelling

Constitutive equation: fractional Kelvin-Voigt

$$\sigma^{FDM}(x,y,t) = E\varepsilon(x,y,t) + \xi \frac{d^{\beta}\varepsilon(x,y,t)}{dt^{\beta}} \quad (7)$$

$$\boldsymbol{\theta}_{FDM} = \{E, \xi, \beta\}^{T} \quad (8)$$

$$\text{where } \beta \in [0,1] \quad (9)$$

Mathematical Modelling

Rieman-Liouville Fractional derivative of order $\nu > 0$

$$\frac{d^\beta}{dt^\beta}[f(t)] \triangleq \begin{cases} \frac{1}{\Gamma(m-\beta)}\frac{d^m}{dt^m}\int_0^t \frac{f(\tau)}{(t-\tau)^{\beta+1-m}}d\tau, & \text{if} \quad (m-1) < \beta < m \\ \frac{d^m}{dt^m}f(t), & \text{if} \quad \beta = m \end{cases} \tag{10}$$

- $m \in \mathbb{N}$
- Γ is the Gamma function;
- ν is the order of the fractional derivative operator

Mathematical Modelling

Caputo Fractional derivative of order $\nu > 0$

$$\frac{d^{\beta}}{dt^{\beta}}[f(t)] \triangleq \begin{cases} \frac{1}{\Gamma(m-\beta)} \int_0^t \frac{f^{(m)}(\tau)}{(t-\tau)^{\beta+1-m}} d\tau, & \text{if} \quad (m-1) < \beta < m \\ \frac{d^m}{dt^m} f(t), & \text{if} \quad \beta = m \end{cases} \tag{11}$$

- $m \in \mathbb{N}$
- Γ is the Gamma function;
- ν is the order of the fractional derivative operator

Mathematical Modelling

Governing equation: fractional Kelvin-Voigt (Caputo)

$$\mu \frac{\partial^2 w}{\partial t^2} + \alpha \frac{\partial w}{\partial t} + EI \frac{\partial^4 w}{\partial x^4} - T \frac{\partial^2 w}{\partial x^2} + $$
$$+ \frac{\xi I}{\Gamma(1-\beta)} \left\{ \int_0^t \frac{1}{(t-\tau)^\beta} \frac{\partial}{\partial \tau} \left(\frac{\partial^4 w(x,\tau)}{\partial x^4} \right) d\tau \right\} = F(x,t) \quad (12)$$

where $\beta \in [0,1]$

BAYESIAN INFERENCE: Framework

Bayesian paradigm for inverse problems

- All the variables are modeled as random variables.
- Their uncertainties are encoded in their pdfs.
- The solution is the exploration of the posterior probability density.

BAYESIAN INFERENCE: Framework

Bayes Rule

- Model parameters $\boldsymbol{\theta}$
- Observation Model

$$\mathbf{y} = \mathbf{A}(\boldsymbol{\theta}) + \mathbf{e}$$

- Bayes rule

$$\pi(\boldsymbol{\theta}|\mathbf{y}) = \frac{\pi(\mathbf{y}|\boldsymbol{\theta})\ \pi_{pr}(\boldsymbol{\theta})}{\pi(\mathbf{y})}$$

- Assuming that $\boldsymbol{\theta}$ and $\mathbf{e}$ are independent random vectors, the likelihood function casts as

$$\pi(\mathbf{y}|\boldsymbol{\theta}) = \pi_e(\mathbf{y} - \mathbf{A}(\boldsymbol{\theta}))$$

BAYESIAN INFERENCE: Framework

Bayes Rule

- Likelihood **function** $\pi(\mathbf{y}|\boldsymbol{\theta})$
- Prior $\pi_{pr}(\boldsymbol{\theta})$: **Subjective** information.
- Prior $\pi_{pr}(\boldsymbol{\theta})$: What are the **criticisms** ?
- $\pi(\mathbf{y})$: One would hardly be able to obtain this. Fortunately, it acts as a **scaling factor** when using sampling based strategies.

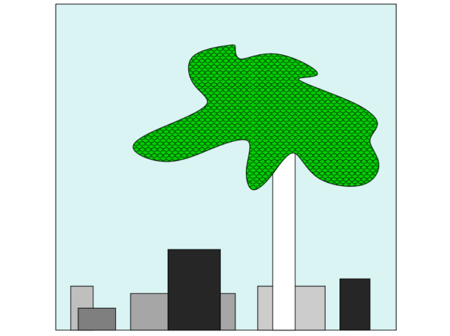
Model class selection

Thinkings

- What constitutes a good model class ?
- Quality of the data fitting.
- Nevertheless, one may also assess the complexity of the model class.

Model class selection

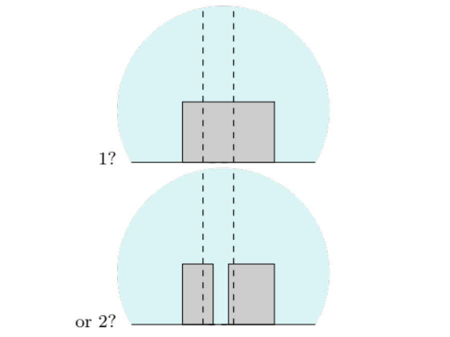
Model simplicity ... : How many boxes are behind the tree ?[1]



[1] MacKay DJC. Information theory, inference, and learning algorithms. Cambridge (UK): Cambridge University Press; 2003.

Model class selection

Is there 1 or are thre 2 ? [2]



[2]MacKay DJC. Information theory, inference, and learning algorithms. Cambridge (UK): Cambridge University Press; 2003.

Model class selection

Thinkings: Occam's razor and Parsimony

- The notion of model simplicity as a criterion for model selection dates back to the principle of parsimony suggested by Box and Jenkins, for whom the model class that adequately represents the observed data and with the smallest number of parameters should be selected.
- **Occam's razor** is the principle that states a preference for simple theories. 'Accept the simplest explanation that fits the data'.

Model class selection

Thinkings

- Bayesian Model Class Selection (BMCS) provides a rigorous framework to compare the performance of a set of candidate model classes in describing experiment data.

Model class selection

Theory

- Consider the set $\mathbf{M} = \{\mathrm{M}_1(\boldsymbol{\theta}_1), \mathrm{M}_2(\boldsymbol{\theta}_2), \cdots, \mathrm{M}_{N_M}(\boldsymbol{\theta}_{N_M})\}$ of N_M ($N_M > 1$) plausible/suitable model classes proposed to predict a response quantity of interest for the system under investigation.
- $\boldsymbol{\theta}_k$ is the random vector that characterizes model class M_k

Model class selection

Theory: Bayes

$$\pi(\mathrm{M}_i|\mathbf{Y},\mathbf{M}) = \frac{\pi(\mathbf{Y}|\mathrm{M}_i)\,\pi(\mathrm{M}_i|\mathbf{M})}{\sum_{k=1}^{N_M}\pi(\mathbf{Y}|\mathrm{M}_k)\,\pi(\mathrm{M}_k|\mathbf{M})} \tag{13}$$

- $\pi(\mathrm{M}_k|\mathbf{M}) = \pi_{pr}(\mathrm{M}_k|\mathbf{M})$ indicates the modeler's belief about initial relative plausibility of the model class M_i within the set $\mathbf{M}$.
- What about $\pi(\mathbf{Y}|\mathrm{M}_i)$?

Model class selection

Evidence $\pi(\mathbf{Y}|\mathrm{M}_i)$: the probability of data $\mathbf{Y}$ according to the model class M_i

$$\pi(\mathbf{Y}|\mathrm{M}_i) = \int_{\Theta} \pi(\mathbf{Y}|\boldsymbol{\theta}, \mathrm{M}_i)\, \pi(\boldsymbol{\theta}|\mathrm{M}_i) d\boldsymbol{\theta} \tag{14}$$

- No analytical expressions.
- Its computation requires computational methods such as MCMC.

Model class selection

Log-evidence

$$\ln \pi(\mathbf{Y}|\mathrm{M}_i) = \ln \pi(\mathbf{Y}|\boldsymbol{\theta}, \mathrm{M}_i) - \ln \frac{\pi(\boldsymbol{\theta}|\mathbf{Y}, \mathrm{M}_i)}{\pi(\boldsymbol{\theta}|\mathrm{M}_i)} \tag{15}$$

Model class selection

Expected value in $\pi(\boldsymbol{\theta}|\mathbf{Y}, \mathrm{M}_i)$

$$\int \ln \pi(\mathbf{Y}|\mathrm{M}_i) \times \pi(\boldsymbol{\theta}|\mathbf{Y}, \mathrm{M}_i) d\boldsymbol{\theta} = \ldots \quad (16)$$

$$\ln \pi(\mathbf{Y}|\mathrm{M}_i) = \mathbb{E}[\ln \pi(\mathbf{Y}|\boldsymbol{\theta}, \mathrm{M}_i)] - \mathbb{E}\left[\ln \frac{\pi(\boldsymbol{\theta}|\mathbf{Y}, \mathrm{M}_i)}{\pi(\boldsymbol{\theta}|\mathrm{M}_i)}\right] \quad (17)$$

Model class selection

Thinking ...

- $\mathbb{E}[\ln \pi(\mathbf{Y}|\boldsymbol{\theta}, \mathrm{M}_i)]$ is the posterior mean of the log-likelihood distribution and quantifies the degree to which a model class M_i fits the observed data

Model class selection

Thinking ...

- $\mathbb{E}\left[\ln \frac{\pi(\boldsymbol{\theta}|\mathbf{Y},\mathrm{M}_i)}{\pi(\boldsymbol{\theta}|\mathrm{M}_i)}\right]$ is the Kullback-Leibler divergence from posterior to the prior distributions.
- It reflects the amount of information extracted from observed data **Y** and is always non-negative.

Model class selection

Thinking ...

- Hence, the (log-)evidence trades off between data-fit and complexity of a model class !!

BAYESIAN INFERENCE: Sampling

Exploration of the posterior $\pi(\boldsymbol{\theta}|\mathbf{y})$

- Markov Chain Monte Carlo Methods.
- Expected values and variances of model parameters.
- Information for UQ analyses.
- Model validation analyses can take both measured and model uncertainties into account.

BAYESIAN INFERENCE: MCMC

MCMC

- A MCMC method for the simulation of a distribution $\bar{\pi}(\boldsymbol{\theta})$ method is any method that produces samples $\{\boldsymbol{\theta}^{(1)}, \ldots, \boldsymbol{\theta}^{(N)}\}$ whose stationary distribution is $\bar{\pi}(\boldsymbol{\theta})$.

- When using the Metropolis-Hastings (MH) algorithm **one should tune the proposal**. Too wide proposals lead to hardly accepted samples. On the other hand, too narrow proposals lead to high levels of acceptance and the exploration over the density gets too slow ...

- In addition, one could modify the accept/reject procedure for improving the MH efficiency...

DRAM Algorithm

Delayed Rejection Adaptive Metropolis (DRAM)

MH based algorihm which combines the **Adaptive Metropolis (AM)** and the **Delayed Rejection (DR)**.

Adaptive Metropolis (AM)

- The proposed distribution is tuned considering the Markov chain empirical covariance.

$$C_n = s_d Cov(\boldsymbol{\theta}^1, \boldsymbol{\theta}^2, ..., \boldsymbol{\theta}^{n-1}) + s_d \lambda I_d \quad (18)$$

where $s_d = \frac{2.39^2}{d}$, being d the problem dimension, λ is a constant with small magnitude, I_d is a identity matrix and $Cov(\boldsymbol{\theta}^1, \boldsymbol{\theta}^2, ..., \boldsymbol{\theta}^{n-1})$ is the empirical covariance.

DRAM Algorithm

Delayed Rejection (DR)

- By the MH, consecutive rejections may lead to poor estimates
- By the DR it may be bypassed by proposing other candidates.
- The acceptance/rejection rule takes into account all the candidates that have already been rejected for that specific candidate in order to maintain the ergodicity.
- For symmetric proposals

$$\alpha_i(\boldsymbol{\theta}_i|\boldsymbol{\theta},\boldsymbol{\theta}_1,...,\boldsymbol{\theta}_{i-1}) = max\left[1, \frac{min\left(0, \pi_{post}(\boldsymbol{\theta}_i|\mathbf{y}) - \pi_{post}(\boldsymbol{\theta}^*|\mathbf{y})\right)}{\pi_{post}(\boldsymbol{\theta}|\mathbf{y}) - \pi_{post}(\boldsymbol{\theta}^*|\mathbf{y})}\right]$$

where $\pi(\boldsymbol{\theta}^*|\mathbf{y})$ is the lowest probability density proposed and $\boldsymbol{\theta}^*$ is the respective state.

DRAM Algorithm

Local proposal adaptation

A local adaptation of the proposal may be performed.

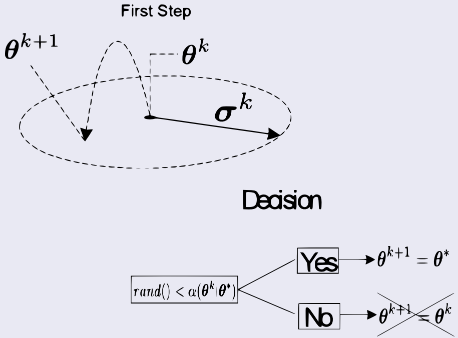
- On each i stage of the DR, a local proposal covariance modification may be conducted using

$$C_n^i = \gamma_i C_n \quad i = 2, 3 \ldots \tag{19}$$

where γ_i is a scaling factor for a i DR stage.

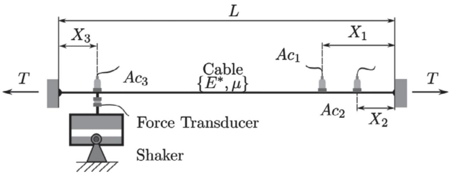
DRAM Algorithm

Sketch



Experimental Set-Up

$\mu = 1.3027$ kg/m, $D = 25.15$ mm and $L = 51.95$ m. The tests were performed for two different tensile loads, namely $T_1 = 16481$ N and $T_2 = 21778$ N .



Model Calibration

We considered pairs of sensors.

Tabela: Definition of subsets S_{ijk} used for model updating and model validation, $i, j, k \in \{1, 2, 3\}$ and $i \neq j \neq k$.

Subset	Model Updating	Model Validation
S_{123}	$\mathbf{Y} = \{\mathbf{H}_{AC1}\mathbf{H}_{AC2}\}^T$	$\mathbf{Y} = \mathbf{H}_{AC3}$
S_{132}	$\mathbf{Y} = \{\mathbf{H}_{AC1}\mathbf{H}_{AC3}\}^T$	$\mathbf{Y} = \mathbf{H}_{AC2}$
S_{231}	$\mathbf{Y} = \{\mathbf{H}_{AC2}\mathbf{H}_{AC3}\}^T$	$\mathbf{Y} = \mathbf{H}_{AC1}$

Model Calibration

Kelvin-Voigt

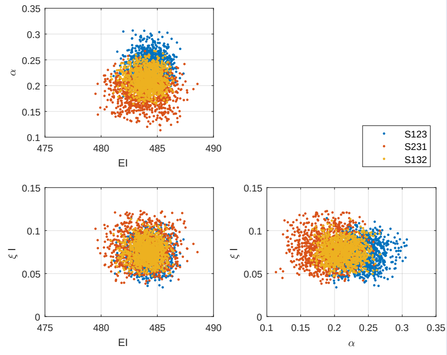


Figura: Posterior samples generated with the DRAM algorithm for the KVM at $T = 16481\,\mathrm{N}$.

Model Calibration

Fractional Kelvin-Voigt

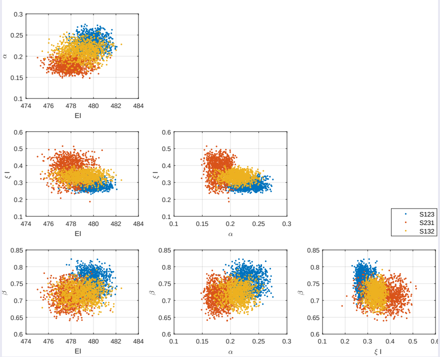
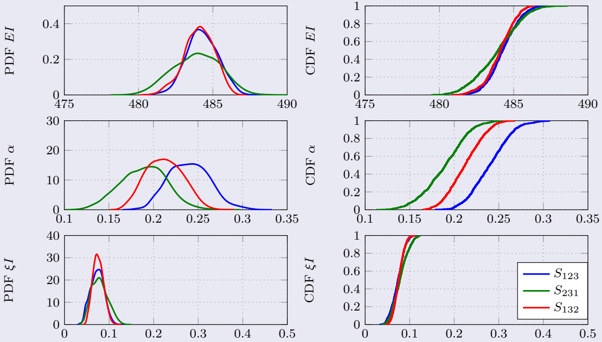


Figura: Posterior samples generated with the DRAM algorithm for the FDM at $T = 21778\,\mathrm{N}$.

Model Calibration

Kelvin-Voigt

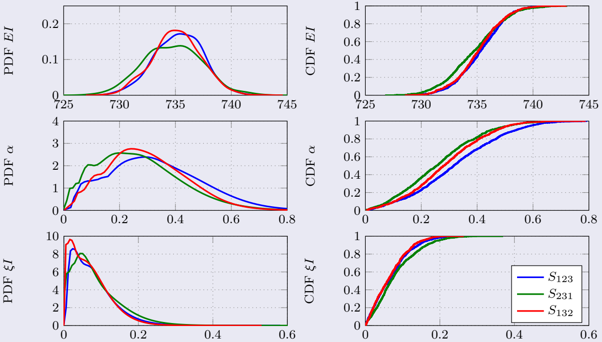


(a) $T_1 = 16481$ N

Figura: PDFs and CDFs for the KV model parameters.

Model Calibration

Kelvin-Voigt

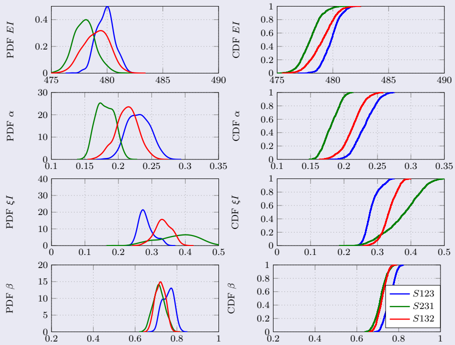


(a) $T_2 = 21778$ N

Figura: PDFs and CDFs for the KV model parameters.

Model Calibration

Fractional Kelvin-Voigt

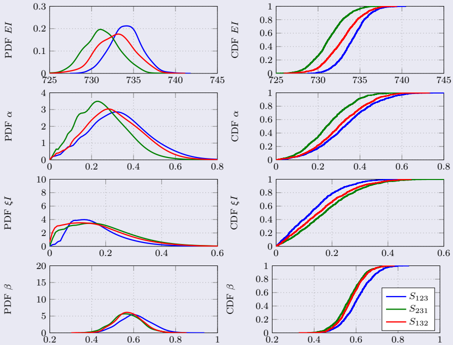


(a) $T_1 = 16481$ N

Figura: PDFs and CDFs for the FD model parameters.

Model Calibration

Fractional Kelvin-Voigt



(a) $T_2 = 21778$ N

Figura: PDFs and CDFs for the FD model parameters.

Bayesian model class selection

Integrals

$$\ln \pi(\mathbf{Y}|\mathrm{M}_i) =$$

$$= \int_{\boldsymbol{\Theta}} \pi(\boldsymbol{\theta}|\mathbf{Y}, \mathrm{M}_i) \left\{ \ln \left[\pi(\mathbf{Y}|\boldsymbol{\theta}, \mathrm{M}_i) \pi(\boldsymbol{\theta}|\mathrm{M}_i) \right] - \ln \pi(\boldsymbol{\theta}|\mathbf{Y}, \mathrm{M}_i) \right\} d\boldsymbol{\theta} =$$

$$= \int_{\boldsymbol{\Theta}} \pi(\boldsymbol{\theta}|\mathbf{Y}, \mathrm{M}_i) \ln \left[\pi(\mathbf{Y}|\boldsymbol{\theta}, \mathrm{M}_i) \pi(\boldsymbol{\theta}|\mathrm{M}_i) \right] \, d\boldsymbol{\theta} + S[\pi(\boldsymbol{\theta}|\mathbf{Y}, \mathrm{M}_i)] \quad (20)$$

Bayesian model class selection

Integrals

$$S[\pi(\boldsymbol{\theta}|\mathbf{Y},\mathrm{M}_i)] \equiv -\int_{\boldsymbol{\Theta}} \pi(\boldsymbol{\theta}|\mathbf{Y},\mathrm{M}_i)\, \ln \pi(\boldsymbol{\theta}|\mathbf{Y},\mathrm{M}_i)\; d\boldsymbol{\theta} \tag{21}$$

Bayesian model class selection

The (log-)evidence is thus approximately computed from posterior samples $\boldsymbol{\theta}^{(k)},\ k = 1, 2, \ldots, N_{se}$

$$\ln \pi(\mathbf{Y}|\mathrm{M}_i) \approx \frac{1}{N_{se}} \left\{ \sum_{k=1}^{N_{se}} \ln \left[\pi(\mathbf{Y}|\boldsymbol{\theta}^{(k)}, \mathrm{M}_i) \pi(\boldsymbol{\theta}^{(k)}|\mathrm{M}_i) \right] - \ln \pi(\boldsymbol{\theta}^{(k)}|\mathbf{Y}, \mathrm{M}_i) \right\}$$

Bayesian model class selection

Results

Tabela: Posterior probabilities for the two model classes (KVM and FDM).

Measurement subset	Tensile load 16481 [N]		Tensile load 21778 [N]	
	Model class		Model class	
	KVM	FDM	KVM	FDM
S123	0.387	0.613	0.361	0.639
S231	0.374	0.626	0.376	0.624
S132	0.410	0.590	0.327	0.673

Bayesian model class selection

Bayes Factor $\frac{P(FDM|\mathbf{Y})}{P(KVM|\mathbf{Y})}$ for $T = 16481$N

- S123: 1.6
- S231: 1.7
- S312: 1.5

Bayes Factor $\frac{P(FDM|\mathbf{Y})}{P(KVM|\mathbf{Y})}$ for $T = 21778$N

- S123: 1.8
- S231: 1.6
- S312: 2.0

Bayesian model class selection

Jeffrey's scale[3] for Bayes factor $B = \frac{P(M|\mathbf{Y})}{P(M_0|\mathbf{Y})}$ when $P_{PR}(M_0) = P_{PR}(M)$

- $B < 1$ indicates negative evidence for M
- $B \in [1, 3]$ indicates an evidence that is not worth more than a bare mention.
- $B \in [3, 20]$ indicates a positive evidence for M.
- $B \in [20, 150]$ indicates a strong evidence for M.
- $B > 150$ indicates a very strong evidence for M.

[1] Bradley Efron and Alan Gous. Scales of evidence for model selection: Fisher versus Jeffreys. IMS Lecture notes - Monograph Series. vol 38, 2001.

Model Validation

Forward Uncertainty Propagation and Model Validation

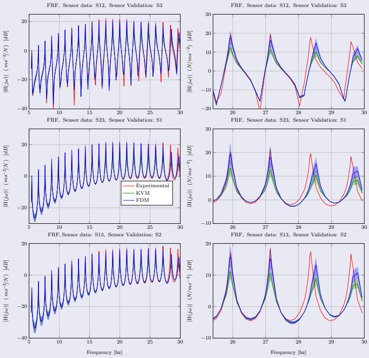


Figura: Uncertainty propagation when considering $\boldsymbol{\theta} \sim \pi(\boldsymbol{\theta}|\mathbf{Y})$. Tensile load $T_1 = 16481$ N. Green: 99% credibility interval for the KVM. Blue 99% credibility interval for the FDM

Model Validation

APCC

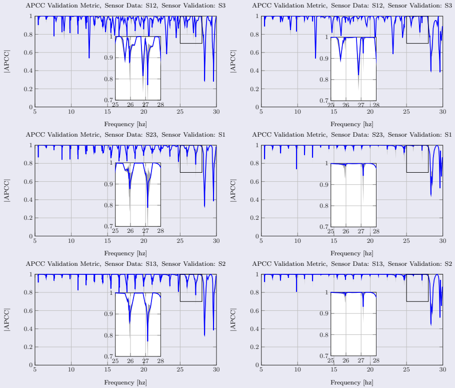


Figura: APCC Validation metric for the tensile $T_1 = 16481$ N. On the left: KVM. On the right: FDM.

Results

Results

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ABSTRACT

This work is aimed at building models to predict the bending vibrations of stranded cables used in high-voltage transmission lines. The present approach encompasses model calibration, validation and selection based on a statistical framework. Model calibration is tackled using a Bayesian framework and the Delayed Rejection Adaptive Metropolis (DRAM) sampling algorithm is employed to

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