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colmea

Colóquio Interinstitucional - Modelos Estocásticos e Aplicações

IM/UFRJ - 1 DE JUNHO DE 2016

TRANSFORMAÇÃO INTEGRAL GENERALIZADA EM PROBLEMAS DIRETO E INVERSO DE CONVECÇÃO-DIFUSÃO NÃO-LINEAR EM DOMINIOS FISICOS COMPLEXOS

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Professor -Engenharia Mecânica e Eng. da Nanotecnologia
POLI & COPPE, CT, UFRJ

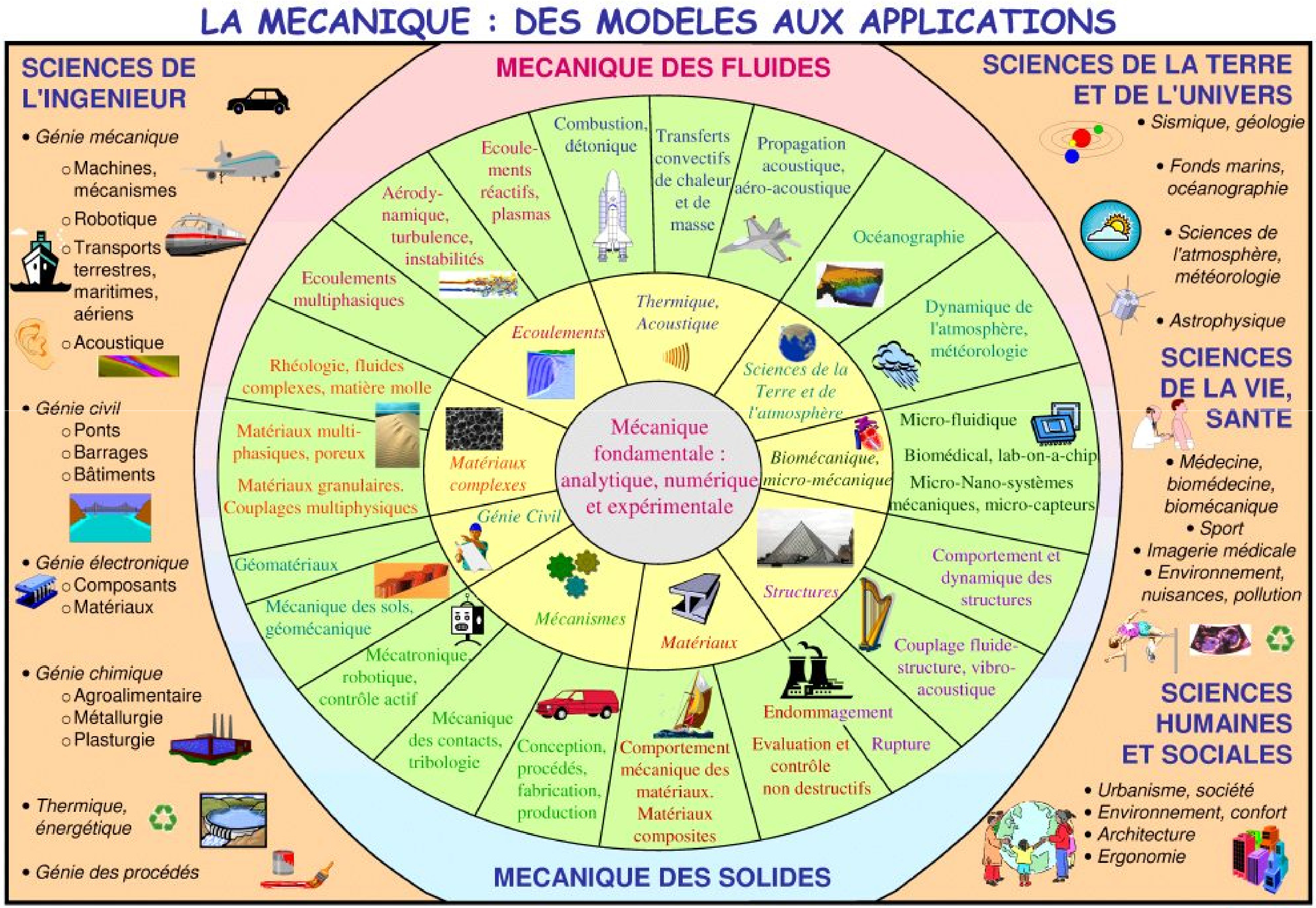
Interdisciplinary Nucleus of Fluid Dynamics
NIDF - CT-2 UFRJ

LabMEMS

Nano & Microfluidics & Microsystems

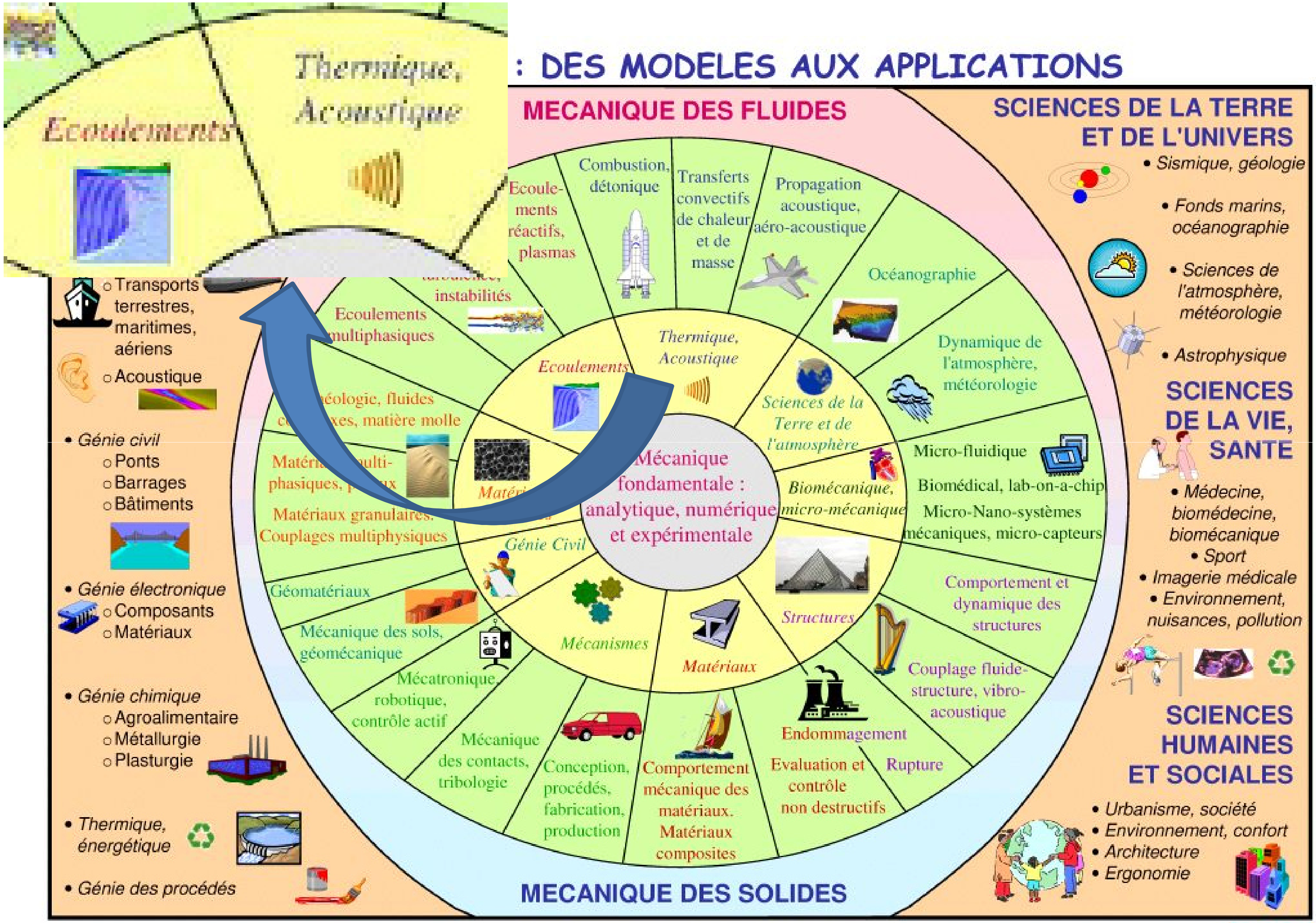
Túnel de Vento Climático

Mechanical Sciences



De l'extérieur vers le centre : les quatre champs d'activités humaines (saumon),
les deux grands pôles de la mécanique (rouge-bleu), **les domaines de la mécanique** (vert), **les secteurs d'activité** (jaune).
Réalisation du groupe "Activités Universitaires en Mécanique" de l'ASSOCIATION FRANÇAISE DE MÉCANIQUE (AFM)

Mechanical Sciences



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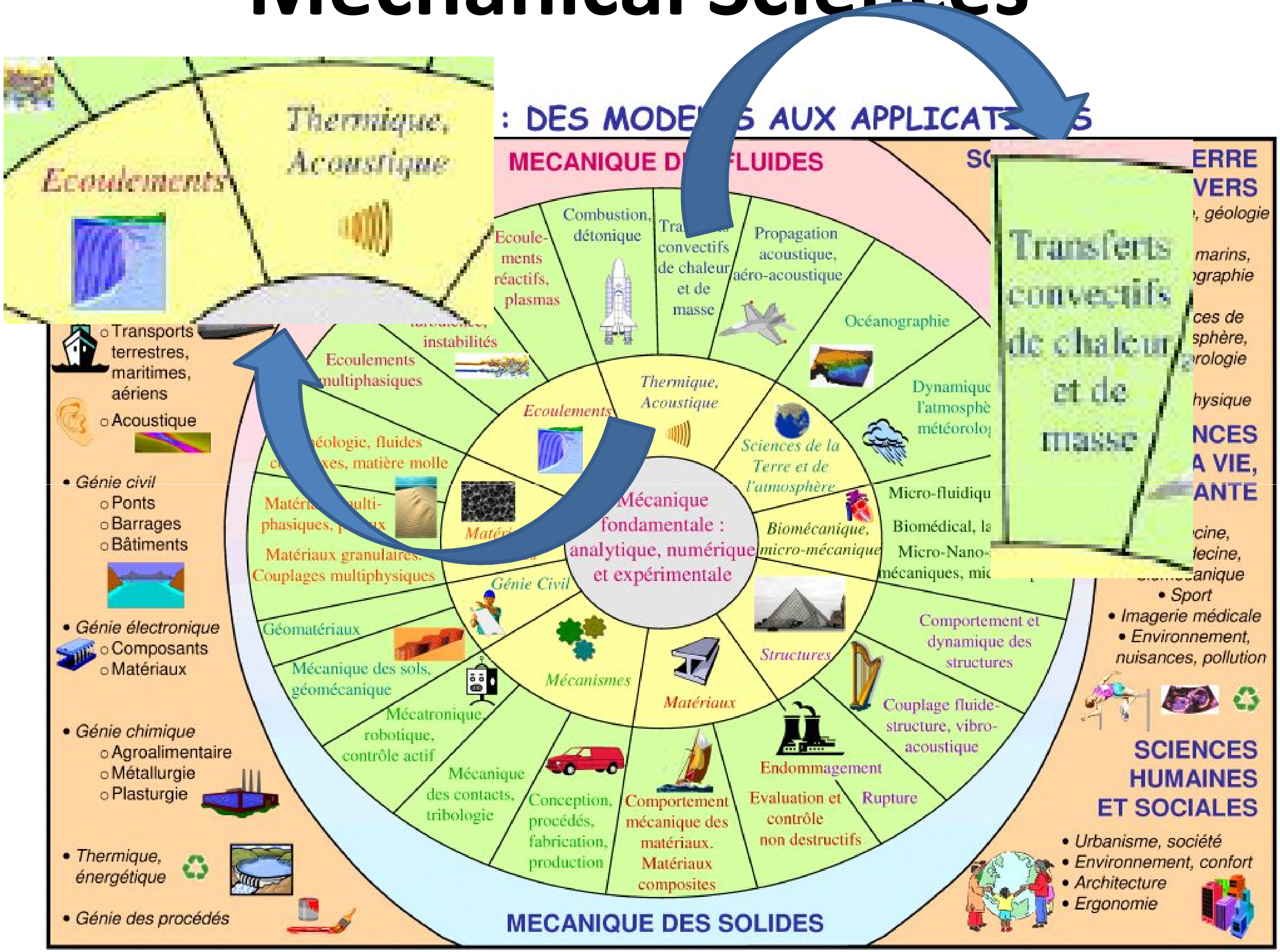

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Réalisation du groupe "Activités Universitaires en Mécanique" de l'ASSOCIATION FRANÇAISE DE MÉCANIQUE (AFM)

Heat and Mass Transfer





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Unified Integral Transforms

Application

Recent and Future Research

COPPE/UFRJ

Background

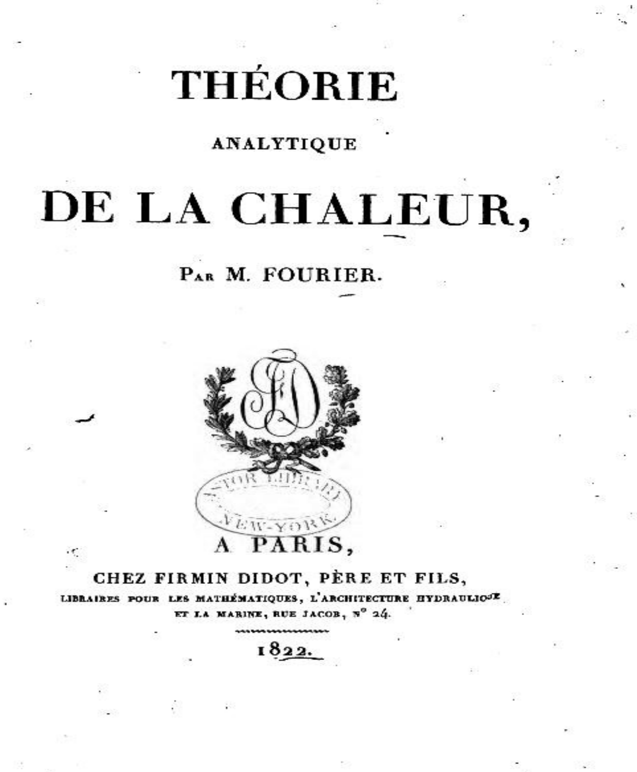
LTTC

Analytical Methods:

Separation of Variables

Joseph Baptiste Fourier (1768-1830)

1822


THÉORIE

ANALYTIQUE

DE LA CHALEUR,

PAR M. FOURIER.

A PARIS,

CHEZ FIRMIN DIDOT, PÈRE ET FILS,

LIBRAIRES POUR LES MATHÉMATIQUES, L'ARCHITECTURE HYDRAULIQUE ET LA MARINE, RUE JACOB, N° 24.

1822.

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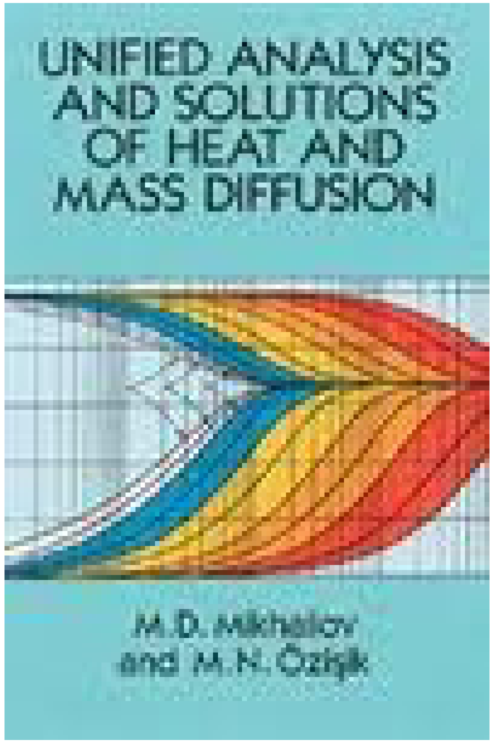
Classical Integral Transforms & Heat Transfer

LTTC

- J.B. Fourier, **Théorie Analytique de la Chaleur**, 1822. (Separation of Variables)
- N.S. Koshlyakov, **Fundamental Differential Equations of Mathematical Physics**, 1936.
- A.V. Luikov, **Analytical Heat Diffusion Theory**, 2nd ed., 1967.
- M.D. Mikhailov & M.N. Ozisik, **Unified Analysis and Solutions of Heat and Mass Diffusion**, 1984.



Background

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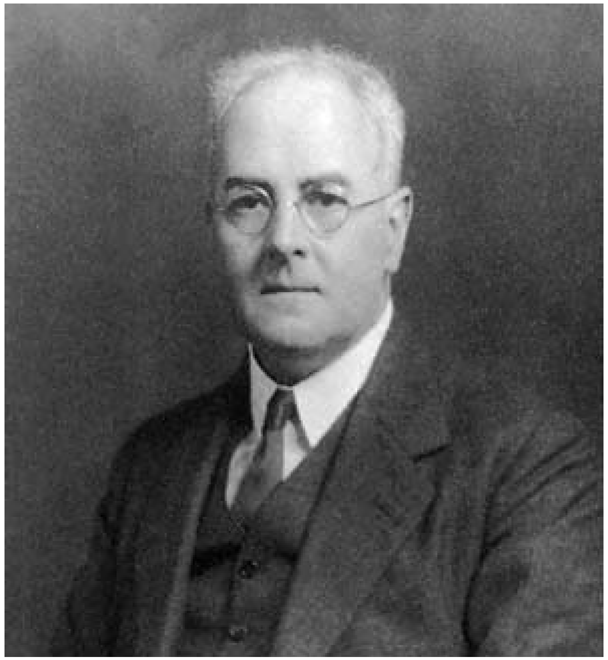
LTTC

Numerical Methods:

Finite Differences Method

Lewis Fry Richardson (1881-1953)

Fellow – Royal Society of London - 1926



IX. *The Approximate Arithmetical Solution by Finite Differences of Physical Problems involving Differential Equations, with an Application to the Stresses in a Masonry Dam.*

By L. F. Richardson, *King's College, Cambridge.*

Communicated by Dr. R. T. Glazebrook, *F.R.S.*

Received (in revised form) November 2, 1909,—Read January 13, 1910.

§ 1. Introduction.—§ 1·0. The object of this paper is to develop methods whereby the differential equations of physics may be applied more freely than hitherto in the approximate form of difference equations to problems concerning irregular bodies.

Though very different in method, it is in purpose a continuation of a former paper by the author, on a "Freehand Graphic Way of Determining Stream Lines and Equipotentials" ('Phil. Mag.,' February, 1908; also 'Proc. Physical Soc.,' London, vol. xxi.). And all that was there said, as to the need for new methods, may be taken to apply here also. In brief, analytical methods are the foundation of the whole subject, and in practice they are the most accurate when they will work, but in the integration of partial equations, with reference to irregular-shaped boundaries, their field of application is very limited.

Both for engineering and for many of the less exact sciences, such as biology, there is a demand for rapid methods, easy to be understood and applicable to unusual equations and irregular bodies. If they can be accurate, so much the better; but 1 per cent. would suffice for many purposes. It is hoped that the methods put forward in this paper will help to supply this demand.

The equations considered in any detail are only a few of the commoner ones occurring in physical mathematics, namely:—Laplace's equation $\nabla^2\phi = 0$; the oscillation equations $(\nabla^2+k^2)\phi = 0$ and $(\nabla^4-k^4)\phi = 0$; and the equation $\nabla^4\phi = 0$. But the methods employed are not limited to these equations.

The Number of Independent Variables.—In the examples treated in the paper this never exceeds two. The extension to three variables is, however, perfectly obvious. One has only to let the third variable be represented by the number of the page of a book of tracing paper. The operators are extended quite simply, and the same

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Background

Richardson, L.F., The approximate arithmetical solution by finite differences of physical problems involving differential equations, with an application to the stresses in a masonry dam, **Phil. Trans. Royal Soc., London** 210:307-357, **1911**.

Both for engineering and for many of the less exact sciences, such as biology, there is a demand for rapid methods, easy to be understood and applicable to unusual equations and irregular bodies. If they can be accurate, so much the better; but 1 per cent. would suffice for many purposes. It is hoped that the methods put forward in this paper will help to supply this demand.

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HYBRID METHODS (ANALYTICALLY BASED)

1. **Exact solutions:** computationally costless and within user prescribed accuracy, though might not be available when complexity increases;

2. **Trends and asymptotic behaviors**: minimal computational effort, parametric analysis and limiting behaviors identification;

3. **Reference benchmark results**: verification of numerical methods and codes;

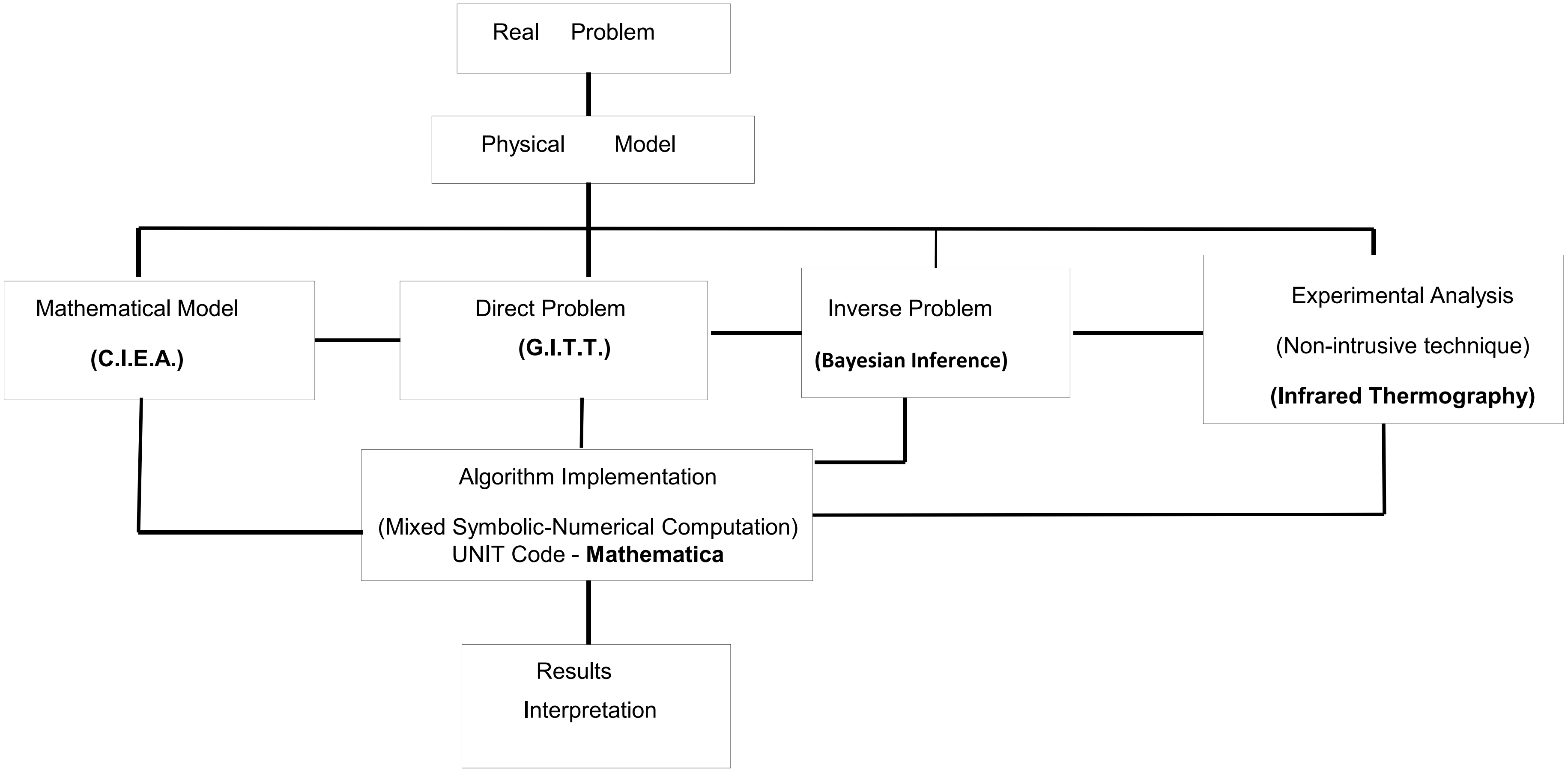
4. **More straightforward and feasible**: symbolic computation platforms;

5. **Intensive computational tasks:** optimization tasks, inverse problem analysis, and simulation under uncertainty;

6. **Bridge to build hybrid numerical-analytical approaches**: search for higher precision and robustness.

Hybrid Approach

THE SIMULATION PROCESS



Hybrid Direct-Inverse Analysis

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Parameters and Functions Identification

Direct Problem

C.I.E.A.
Integral TransformTechnique
C.I.T.T. / G.I.T.T.
UNIT Code

Inverse Problem

Bayesian Inference
MCMC

Experiments

Non-Intrusive
Measurements
IR-Thermography

Eigenfunction Expansions of the Functions to be Estimated

Integral Transformation of Experimental Data

Generalized Integral Transform Technique GITT

- Approximate analytical solutions to non-transformable problems

Ozisik, M.N., and Murray, R.L., **1974**, On the Solution of Linear Diffusion Problems with Variable Boundary Condition Parameters, **J. Heat Transfer**, vol.96c, pp.48-51.

Mikhailov, M.D., **1975**, On the Solution of the Heat Equation with Time Dependent Coefficient, **Int. J. Heat & Mass Transfer**, vol.18, pp.344-345.

- Numerical and analytical solutions of complete transformed system

- ❖Cotta, R.M., **1986**, Diffusion in Media with Prescribed Moving Boundaries: Application to Metals Oxidation at High Temperatures, Proc. of the II Latin American Congress of Heat & Mass Transfer, vol. 1, pp. 502-513, São Paulo, Brasil.
- ❖Cotta, R.M. and Ozisik, M.N., **1986**, Laminar Forced Convection in Ducts with Periodic Variation of Inlet Temperature, Int. J. Heat Mass & Transfer, vol. 29, pp. 1495-1501.

Generalized Integral Transform Technique - GITT

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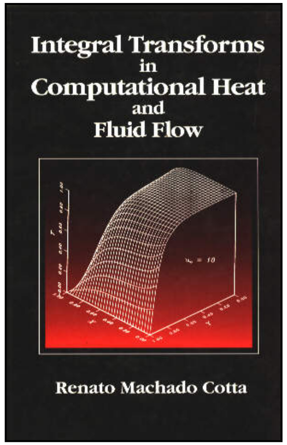
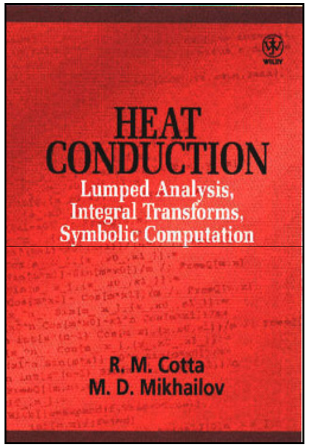
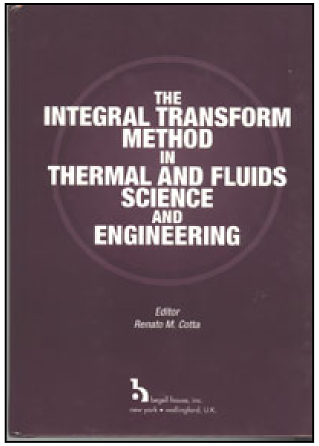
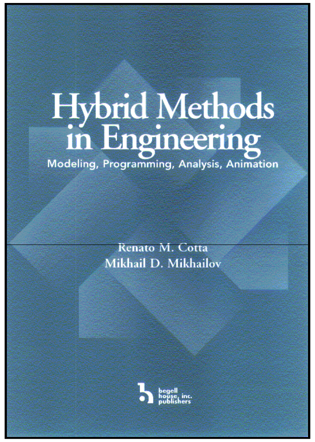
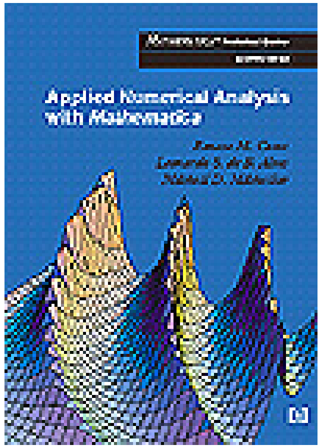
1993	1997	1998	1999	2000	2001

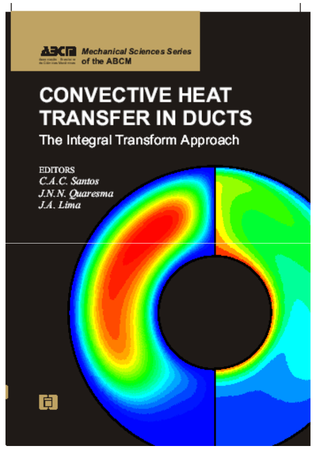


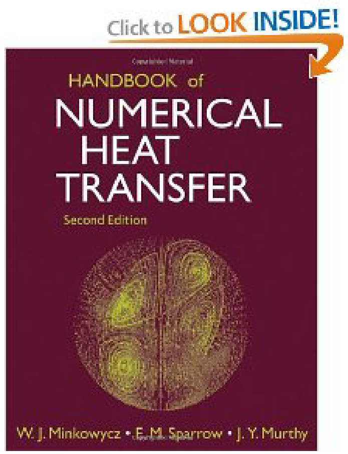












2006

Cotta, R.M., and M.D. Mikhailov, "Hybrid Methods and Symbolic Computations", in: **Handbook of Numerical Heat Transfer**, 2nd edition, Chapter 16, Eds. W.J. Minkowycz, E.M. Sparrow, and J.Y. Murthy, John Wiley, New York, pp.493-522, 2006.

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Classes of Problems

(Linear and Nonlinear)

Diffusion

Convection-Diffusion

Reaction-Convection-Diffusion

Eigenvalue Problems

Boundary Layer Equations

Navier-Stokes Equations

Stochastic Partial Differential Equations

Inverse Problem Analysis

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Advantages - GITT

1. Time-consuming numerical task is always in one single independent variable (ODEs).
2. Reasonably simple computational implementation (subroutines libraries).
3. Handles irregular domains and complex configurations directly .
4. Automatic global error control.
5. Mild increase in computational cost for increasing number of space variables.

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LTTC

STEPS in the GENERALIZED Integral Transform Technique – G.I.T.T.

1. Choose the associated eigenvalue problem.

2. Develop the integral transform pair.

3. Integral transform the original PDE.

4. Numerically (or analytically) solve the resulting coupled ODE system for the transformed potentials.

5. Recall the analytical inversion formula to reconstruct the hybrid solution of the desired potential.

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LTTC

General Formulation

Problem rewritten with **nonlinear source terms**

$$w_k(\boldsymbol{x})\frac{\partial T_k(\boldsymbol{x},t)}{\partial t} = \nabla . K_k(\boldsymbol{x})\nabla T_k(\boldsymbol{x},t) - d_k(\boldsymbol{x})T_k(\boldsymbol{x},t) + P_k(\boldsymbol{x},t,T_l),$$

$$\boldsymbol{x} \in V, \quad t>0, \quad k,\ell=1,2,...,M$$

$$T_k(\boldsymbol{x},0) = f_k(\boldsymbol{x}), \qquad \boldsymbol{x} \in V$$

$$\left[\alpha_k(\boldsymbol{x}) + \beta_k(\boldsymbol{x})K_k(\boldsymbol{x})\frac{\partial}{\partial \boldsymbol{n}}\right]T_k(\boldsymbol{x},t) = \phi_k(\boldsymbol{x},t,T_\ell), \qquad \boldsymbol{x} \in S, \qquad t>0$$

$w_k(\mathbf{x})$, $K_k(\mathbf{x})$, $d_k(\mathbf{x})$, $\alpha_k(\mathbf{x})$, $\beta_k(\mathbf{x})$ - characteristic linear coefficients.

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Generalized Integral Transform Technique
Eigenfunction Expansion

Eigenvalue Problem (step 1)

$$\nabla . k_k(\mathbf{x}) \nabla \psi_{k,i}(\mathbf{x}) + (\mu_{k,i}^2 w_k(\mathbf{x}) - d_k(\mathbf{x}))\psi_{k,i}(\mathbf{x}) = 0, \mathbf{x} \in V$$

$$\alpha_k(\mathbf{x}_l)\psi_{k,i}(\mathbf{x}) + \beta_k(\mathbf{x}) k_k(\mathbf{x}) \frac{\partial \psi_{k,i}(\mathbf{x})}{\partial \mathbf{n}} = 0, \quad \mathbf{x} \in S$$

Integral Transform Pair (step 2)

$$T_k(\mathbf{x},t) = \sum_{i=1}^{\infty} \tilde{\psi}_{k,i}(\mathbf{x}) \overline{T}_{k,i}(t), \qquad \textit{inverse}$$

$$\overline{T}_{k,i}(t) = \int_V w_k(\mathbf{x}) \tilde{\psi}_{k,i}(\mathbf{x}) T_k(\mathbf{x},t) dv, \qquad \textit{transform}$$

$$\tilde{\psi}_{k,i}(\mathbf{x}) = \frac{\psi_{k,i}(\mathbf{x})}{\sqrt{N_{k,i}}}$$

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Transformed System

Integral Transformation (step 3)

$$\int_V \tilde{\psi}_{k,i}(\mathbf{x}) - dv$$

Transformed ODE System (step 4)

$$\frac{d\overline{T}_{k,i}(t)}{dt} = \int_V \tilde{\psi}_{k,i}(\mathbf{x}) H_k(\mathbf{x}, t, T_l) dv, \qquad t > 0, i = 1, 2, \ldots$$

Transformed initial conditions (step 4)

$$\overline{T}_{k,i}(0) = \int_V w_k(\mathbf{x}) \tilde{\psi}_{k,i}(\mathbf{x}) f_k(\mathbf{x}) dv$$

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Convergence Acceleration and Testing

Convergence acceleration: Ex. Filtering solution

$$T_k(\mathbf{x},t) = T_k^*(\mathbf{x},t) + T_{k,f}(\mathbf{x};t)$$

Convergence test

$$\varepsilon = \max_{x\in V} \left| \frac{\sum_{i=N^*}^{N} \tilde{\psi}_{ki}(\boldsymbol{x})\overline{T}_{k,i}(t)}{T_{f,k}(\boldsymbol{x};t) + \sum_{i=1}^{N} \tilde{\psi}_{ki}(\boldsymbol{x})\overline{T}_{k,i}(t)} \right|$$

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Total Transformation

PARABOLIC & PARABOLIC-HYPERBOLIC:

1D- 3D PDE ⇩ ⟶ **System of ODE's (IVP)**

ELLIPTIC:

2D- 3D PDE ⇩ ⟶ **System of ODE's (BVP)**

Partial Transformation

PARABOLIC & PARABOLIC-HYPERBOLIC:

2D- 3D PDE ⇩ ⟶ **1D-System of PDE's**

ELLIPTIC:

3D PDE ⇩ ⟶ **2D -System of PDE's**

UNIT Code (2007-2011)

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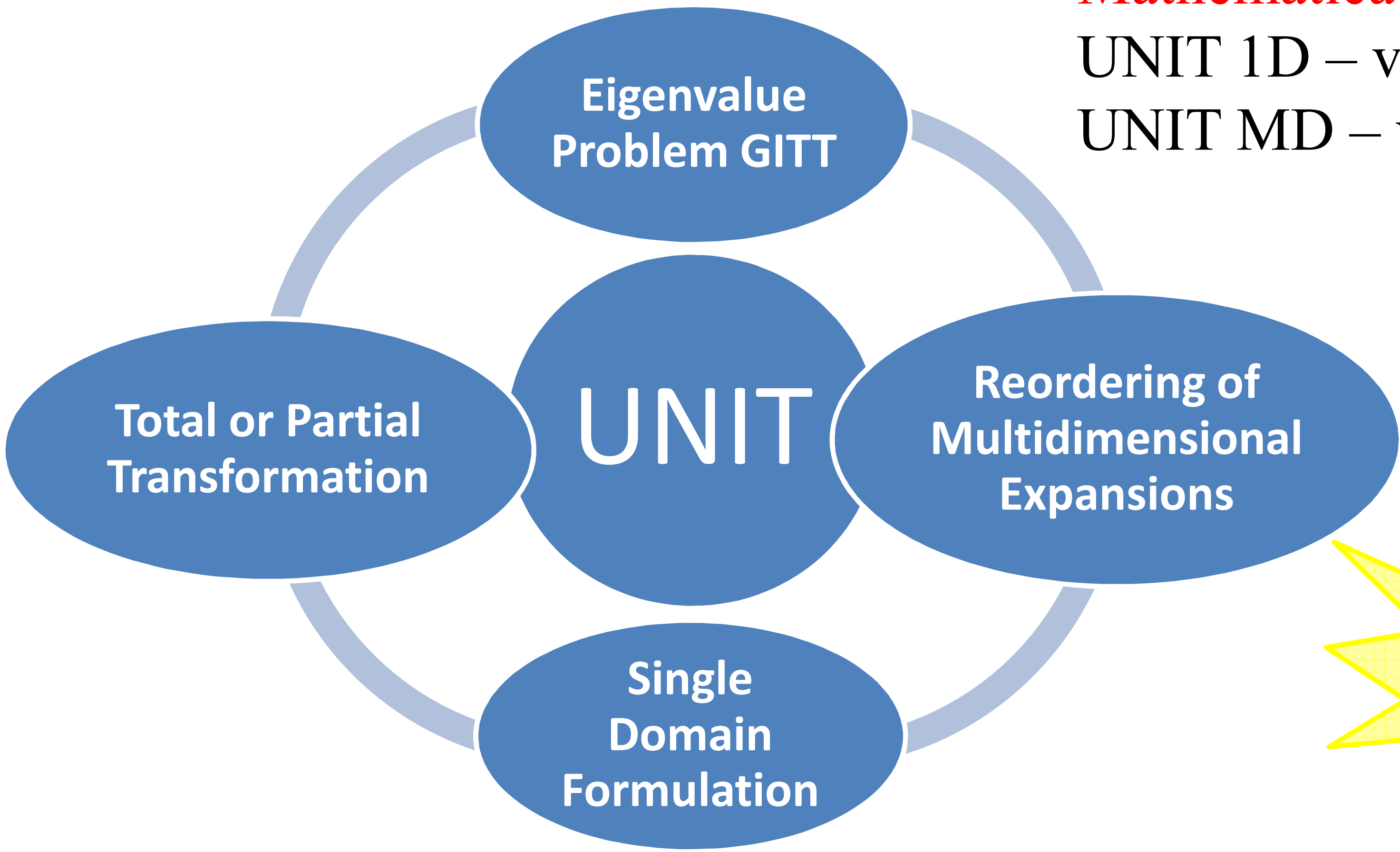
UNIT - Unified Integral Transforms

GITT - Generalized Integral Transform Technique

Mathematica 7.0 and up

UNIT 1D – v.1.6.5

UNIT MD – v.2.2.3



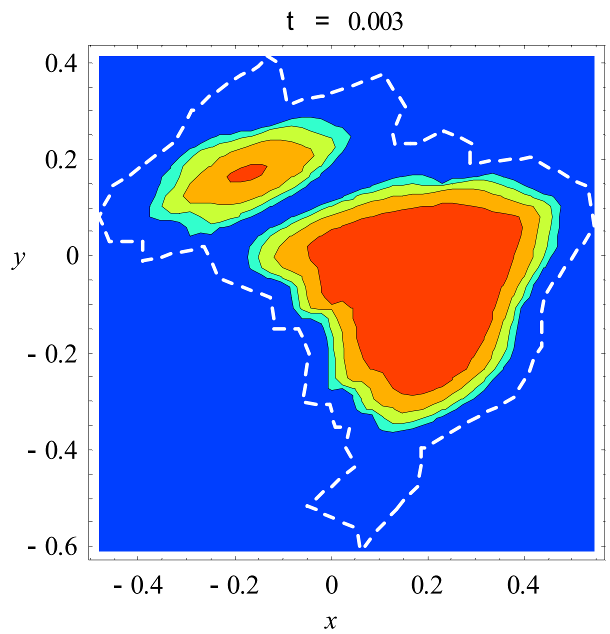
Open Source Code

Direct Integral Transformation

Irregular Domains

$$T(\mathbf{x},t)=\sum_{i=1}^{\infty}\frac{1}{N_i}\Psi_i(\mathbf{x})\,\bar{T}_i(t)$$

$$\bar{T}_i(t)=\int_{\mathcal{V}} w(\mathbf{x})\Psi_i(\mathbf{x})\,T(\mathbf{x},t)\,dv$$



;

$$x_0\le x\le x_1,\qquad y_0(x)\le y\le y_1(x),\qquad z_0(x,y)\le z\le z_1(x,y)$$

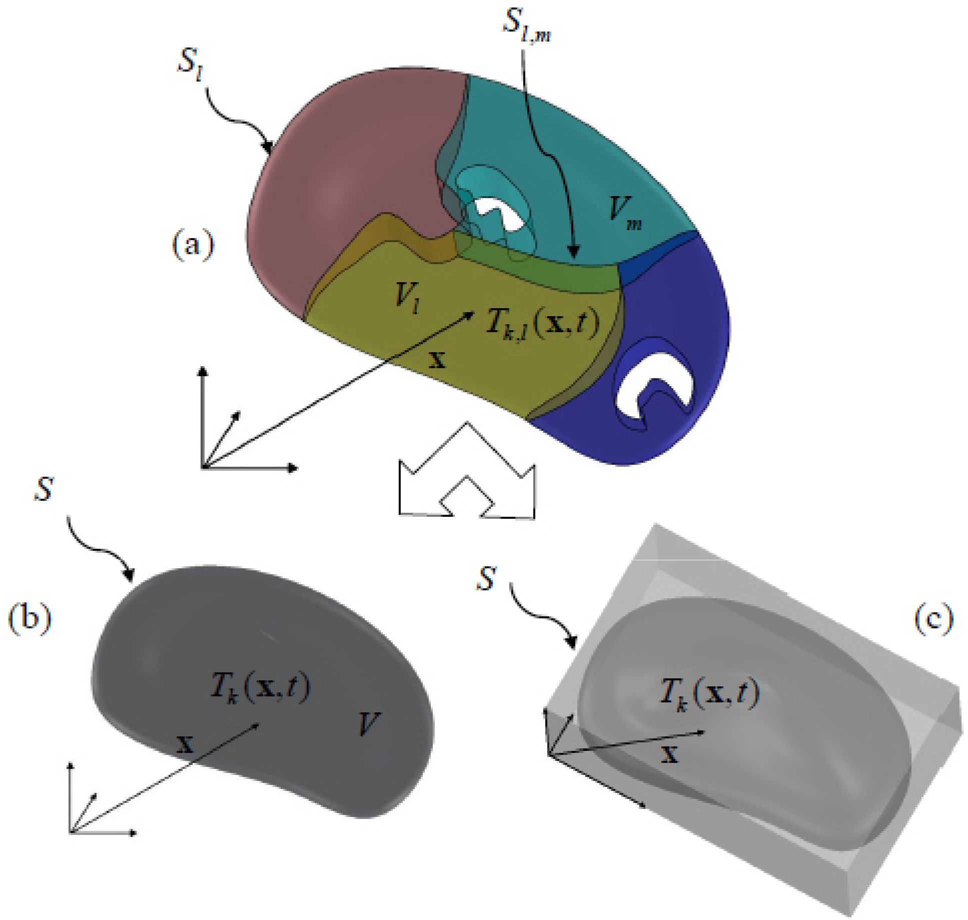
$$\int_{\mathcal{V}}\bullet\;dv=\int_{x_0}^{x_1}\int_{y_0(x)}^{y_1(x)}\int_{z_0(x,y)}^{z_1(x,y)}\bullet\;dz\,dy\,dx$$

$$\tilde{\Omega}_h(\mathbf{x})=\tilde{X}_j(x)\tilde{Y}_m(y;x)\tilde{Z}_p(z;x,y)$$

$$\text{inverse}\Longrightarrow\Psi(\mathbf{x})=\sum_{h=1}^{\infty}\tilde{\Omega}_h(\mathbf{x})\bar{\Psi}_h,$$

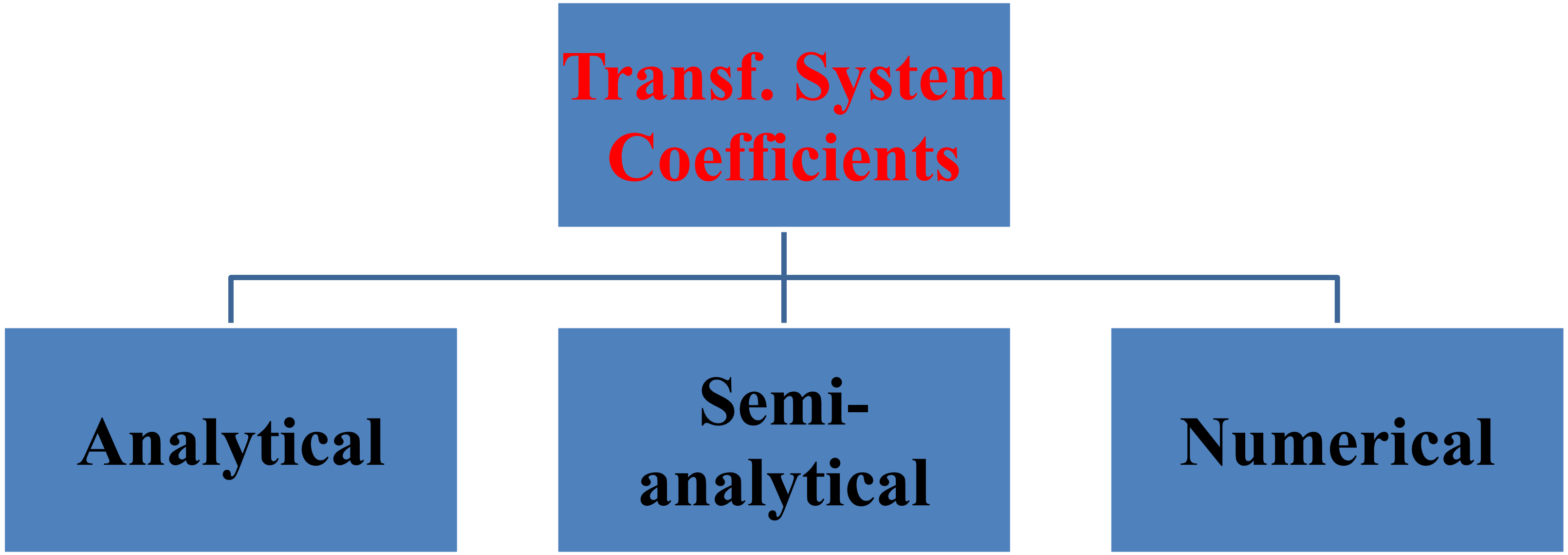
$$\text{transform}\Longrightarrow\bar{\Psi}_h=\int_{\mathcal{V}} w^*(\mathbf{x})\,\tilde{\Omega}_h(\mathbf{x})\Psi(\mathbf{x})\,dv$$

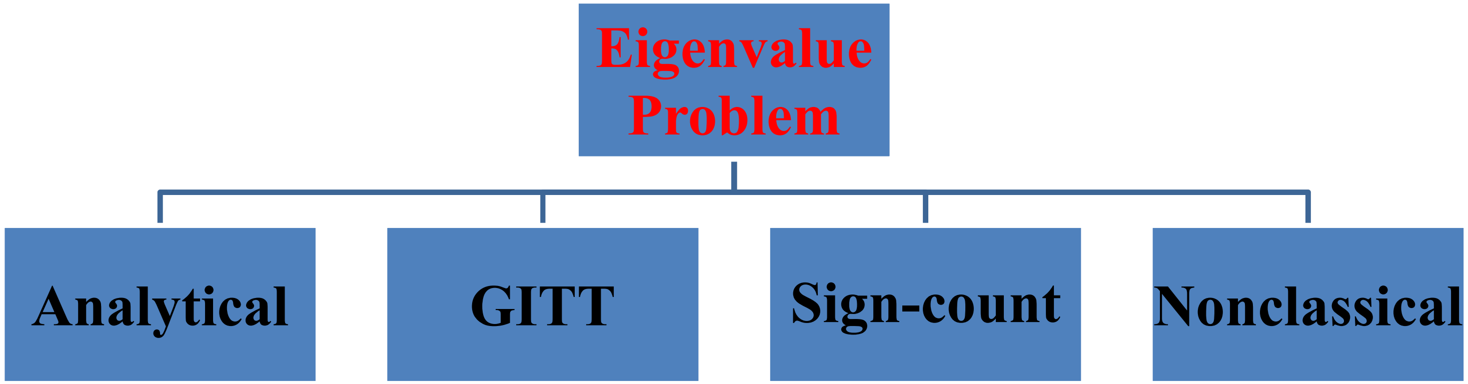
Single domain formulation



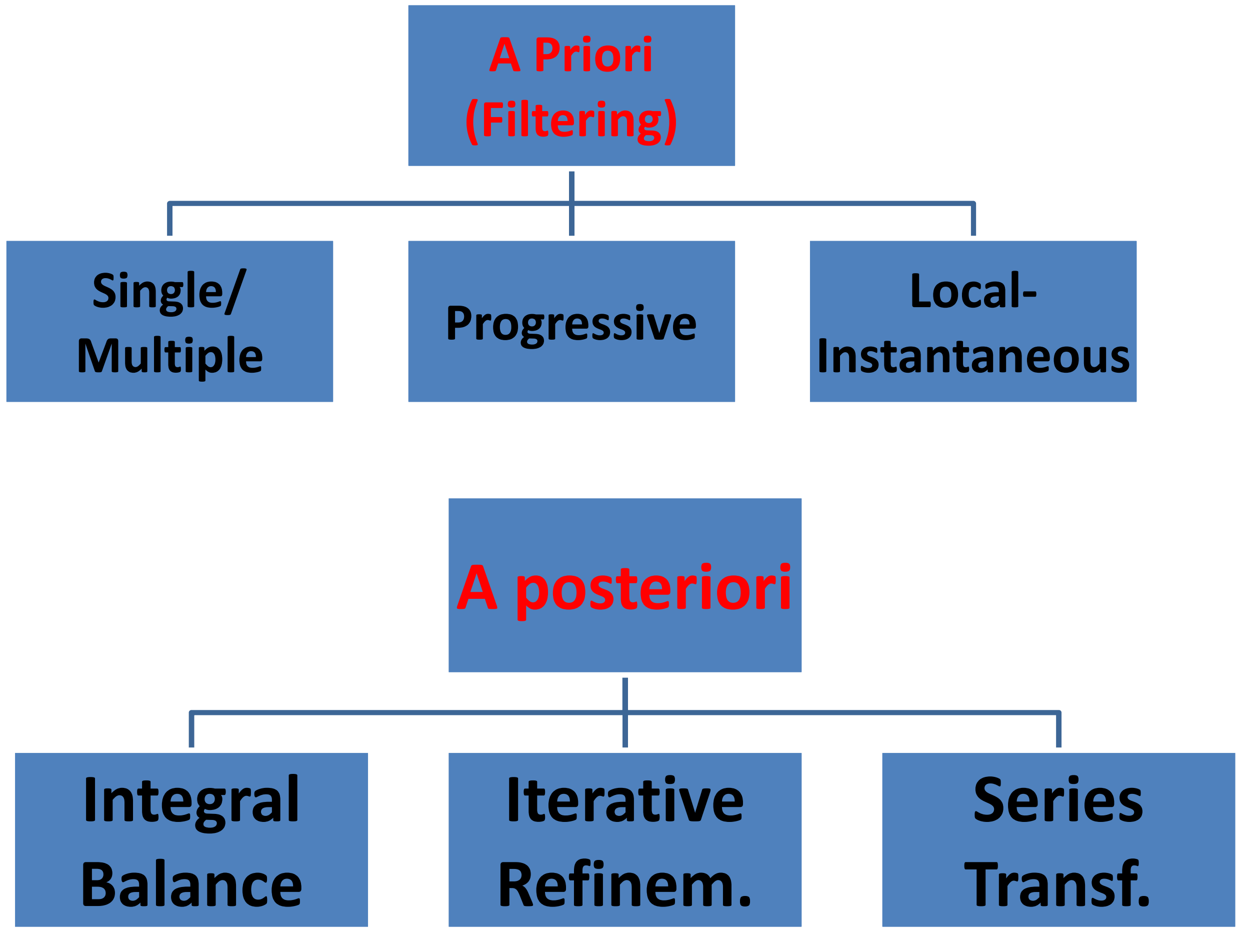
(a) Diffusion or convection-diffusion in a complex multidimensional configuration with n V sub-regions; (b) Single domain representation keeping the original overall domain; (c) Single domain representation considering a regular overall domain that envelopes the original one

Computational Integral Transforms
Implementation Algorithms





Convergence of Integral Transforms
Acceleration Techniques



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The Hybrid Approach

Unified Integral Transforms

Application

Recent and Future Research

Icing of aeronautical structures and sensors

Many accidents due to icing of aeronautical structures and sensors

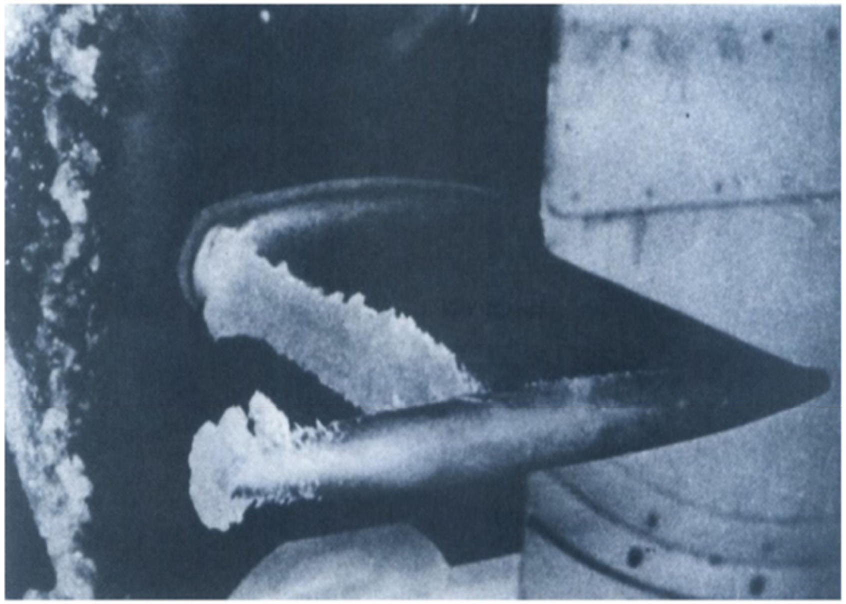
- American Eagle Flight 4184 in 1994 (ATR-72) – icing of ailerons
- Comair Flight 3272 in 1997 (EMB-120) – icing of wings
- AF 447 in 2009 (Airbus A-330) –icing of Pitot probes (31/05/09)

Icing of Pitot tubes

-No previous work on the open literature about icing of Pitot probes

-Plenty of work and knowledge on icing of airfoils

-Several previous incidents and accidents



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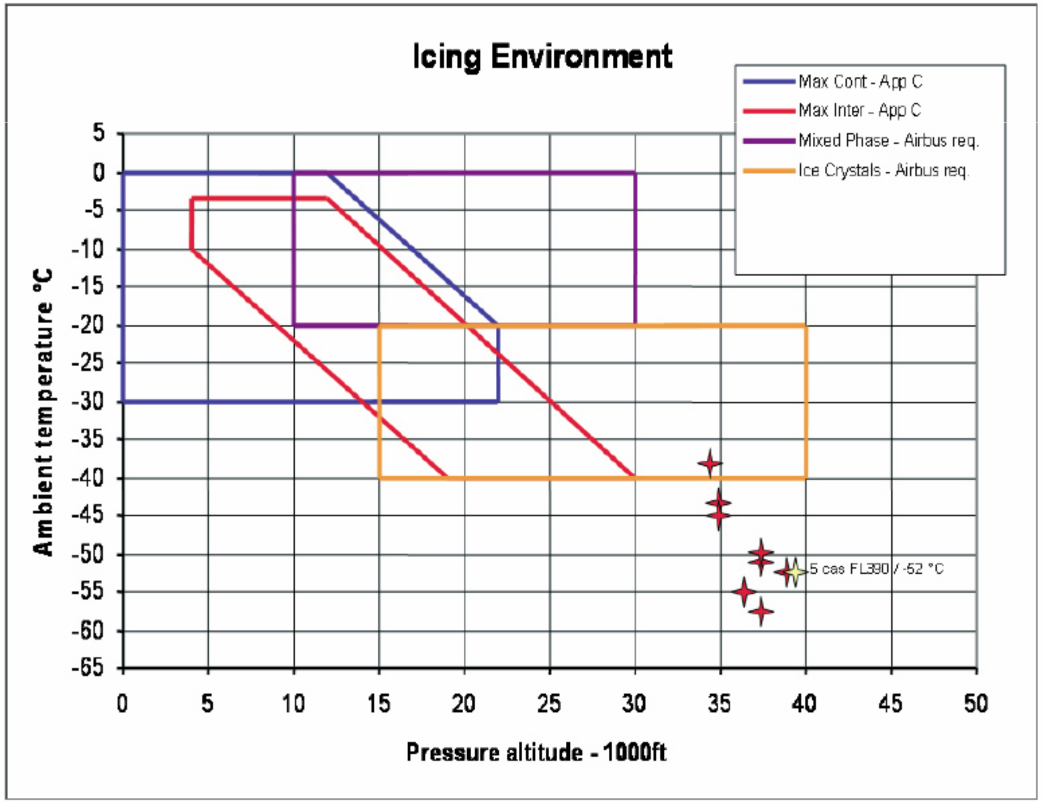
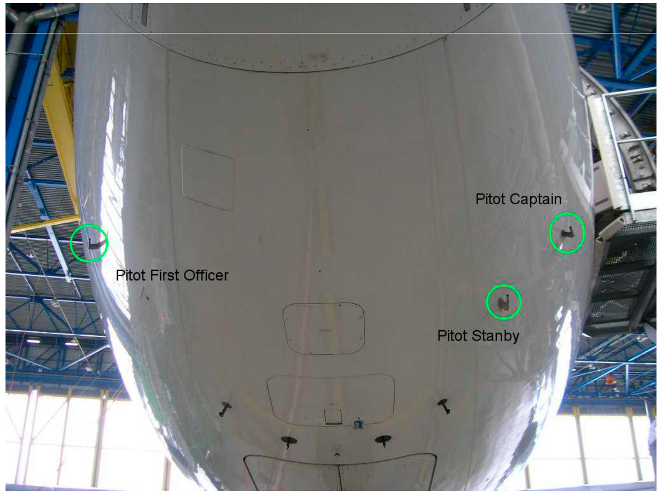
Icing of Pitot tubes

-Certification envelope for Pitot probes



Envelope de certification des sondes Pitot

annexe 4



Background: Conjugated heat transfer

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Convection-conduction formulation

Acad. A.V. Luikov, Inst. of Heat & Mass Transfer, Minsk, Belarus 1910-1974

- Pioneering work by Perelman and Luikov and coworkers
- Steady-state: no explicit analytical solution yet available
- Transient state: wall thermal capacitance effects can be important

Heat conduction – Pitot Probe

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Pitot Probe – A4 Skyhawk – Brazilian Navy

- **Aero-Instruments Company: Model - PH 510, 41 Ohms electrical**

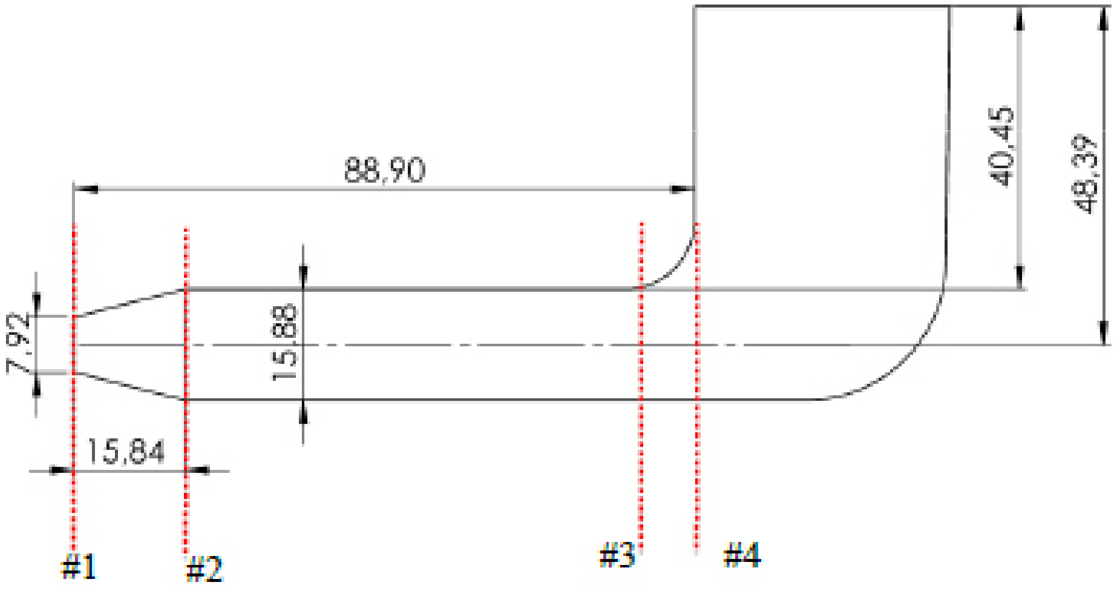
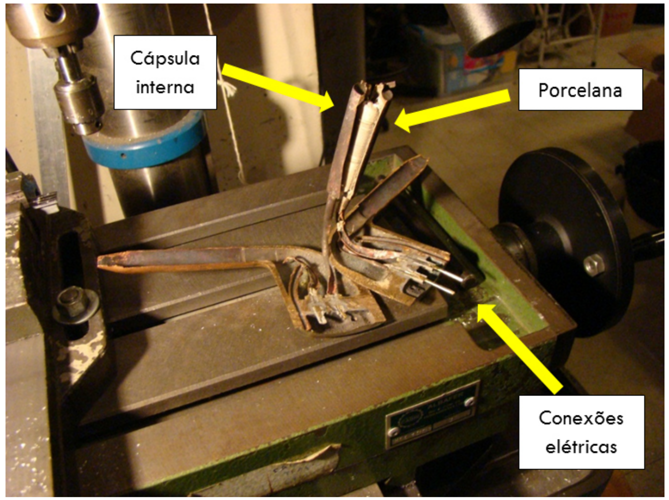


Figure 2b. Schematic drawing of the Pitot tube with main dimensions



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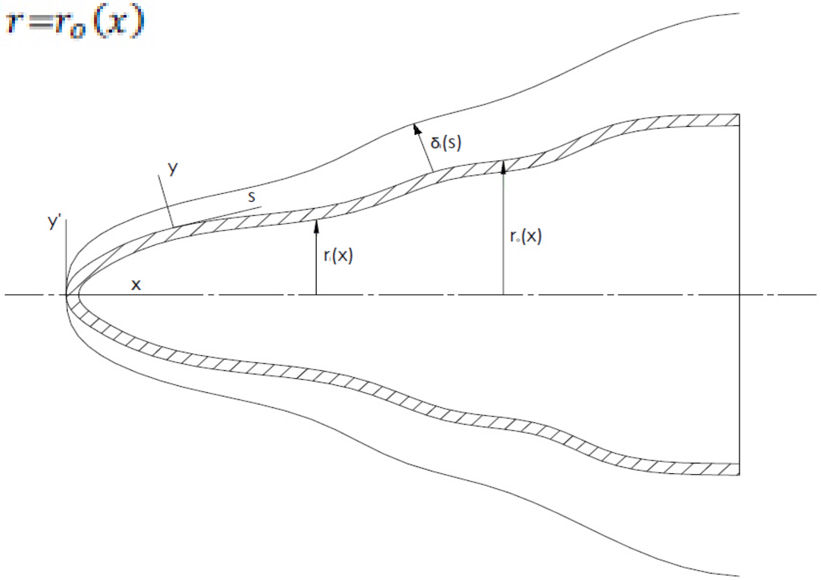
Heat conduction – Pitot Probe

$$w(x,r)\frac{\partial T_s}{\partial t}=\frac{\partial}{\partial x}\left(k(x,r)\frac{\partial T_s}{\partial x}\right)+\frac{1}{r}\frac{\partial}{\partial r}\left(k(x,r)r\frac{\partial T_s}{\partial r}\right)+g(x,r,t),\quad 0\le x\le L, r_i(x)\le r\le r_o(x)$$

$$T_s(x,r,0)=T_0(x,r)$$

$$h_eT_s(0,r,t)-k(0,r)\left.\frac{\partial T_s}{\partial x}\right|_{x=0}=h_eT_{aw};\qquad \left.\frac{\partial T_s}{\partial x}\right|_{x=L}=0$$

$$\left.\frac{\partial T_s}{\partial \boldsymbol{n}}\right|_{r=r_i(x)}=0;\quad h(x)T_s(x,r_o(x),t)+k(x)\left.\frac{\partial T_s}{\partial \boldsymbol{n}}\right|_{r=r_o(x)}=h(x)T_{aw}$$



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Lumped-Differential Formulation

$$T_{av}(x,t) = \frac{2}{r_o^2 - r_i^2}\int_{r_i(x)}^{r_o(x)} T_s(x,r,t)\,r\,dr$$

$$w_{av}(x)\frac{\partial T_{av}}{\partial t} = \frac{1}{A(x)}\frac{\partial}{\partial x}\left(k_{av}(x)A(x)\frac{\partial T_{av}}{\partial x}\right) + \frac{2k_{av}(x)r_o}{r_o^2 - r_i^2}\left.\frac{\partial T_s}{\partial r}\right|_{r=r_o(x)} + g_{av}(x,t), \qquad 0 \le x \le L$$

$$T_{av}(x,0) = T_{av_0}(x)$$

$$h_e T_{av}(0,t) - k_{av}(0)\left.\frac{\partial T_{av}}{\partial x}\right|_{x=0} = h_e T_{aw}; \qquad \left.\frac{\partial T_{av}}{\partial x}\right|_{x=L} = 0$$

$= f(T_{av})$

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Coupled Integral Equations Approach

CIEA – Hermite integration

$$\int_{x_{i-1}}^{x_i} y(x)\mathrm{d}x = \sum_{\nu=0}^{\alpha} C_\nu(\alpha,\beta) h_i^{\nu+1} y_{i-1}^{(\nu)} + \sum_{\nu=0}^{\beta} C_\nu(\beta,\alpha)(-1)^\nu h_i^{\nu+1} y_i^{(\nu)} + O\left(h_i^{\alpha+\beta+3}\right)$$

$$H_{0,0} \rightarrow \int_0^h y(x)dx \cong \frac{h}{2}\left(y(0)+y(h)\right)$$

$$H_{1,1} \rightarrow \int_0^h y(x)dx \cong \frac{h}{2}\left(y(0)+y(h)\right) + \frac{h^2}{12}\left(y'(0)-y'(h)\right)$$

$$E_{0,0} = -\frac{h^3}{12} y''(\eta)$$

$$E_{1,1} = +\frac{h^5}{720} y^{iv}(\xi)$$

$$T_{av}(x,t) = \frac{2}{r_o^2 - r_i^2} \int_{r_i(x)}^{r_o(x)} T_s(x,r,t) r dr$$

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CIEA – Hermite integration

$$\int_{x_{i-1}}^{x_i} y(x)\mathrm{d}x = \sum_{\nu=0}^{\alpha} C_\nu(\alpha,\beta) h_i^{\nu+1} y_{i-1}^{(\nu)} + \sum_{\nu=0}^{\beta} C_\nu(\beta,\alpha)(-1)^\nu h_i^{\nu+1} y_i^{(\nu)} + O\left(h_i^{\alpha+\beta+3}\right)$$

$$H_{0,0} \rightarrow \int_0^h y(x)dx \cong \frac{h}{2}\left(y(0)+y(h)\right)$$

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$$E_{0,0} = -\frac{h^3}{12} y''(\eta)$$

$$E_{1,1} = +\frac{h^5}{720} y^{iv}(\xi)$$

$$T_{av}(x,t) = \frac{T_s(x,r_o,t)r_o + T_s(x,r_i,t)r_i}{r_o+r_i} \rightarrow H_{0,0}$$

$$\int_{r_i(x)}^{r_o(x)} \frac{\partial T_s}{\partial r} dr = T_s(x,r_o,t) - T_s(x,r_i,t) = \frac{r_o-r_i}{2} \left.\frac{\partial T_s}{\partial r}\right|_{r=r_o(x)} \rightarrow H_{0,0}$$

$$T_{av}(x,t) = \frac{T_s(x,r_o,t)r_o + T_s(x,r_i,t)\left(\frac{r_o+5r_i}{6}\right)}{r_o+r_i} - \frac{r_o-r_i}{6(r_o+r_i)} \left.\frac{\partial (rT_s)}{\partial r}\right|_{r=r_o} \rightarrow H_{1,1}$$

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Lumped-Differential Formulation

- Classical lumped approach

$$T_s(x,r_o,t) \cong T_s(x,r_i,t) \cong T_{av}(x,t)$$

$$\left.\frac{\partial T_s}{\partial r}\right|_{r=r_o(x)} \cong \frac{h(x)}{k_{av}(x)}\left(T_{aw} - T_{av}(x,t)\right)$$

- Improved Lumping: Coupled integral equations approach (CIEA)

$$T_s(x,r_i,t) =$$

$$= \frac{12k_{av}(x)(r_o+r_i)T_{av}(x,t) + h(x)(r_o-r_i)[6(r_o+r_i)T_{av}(x,t) - (3r_o+r_i)T_{aw}]}{12k_{av}(x)(r_o+r_i) + h(x)(3r_o+5r_i)(r_o-r_i)}$$

$$T_s(x,r_o,t) = T_{aw} + \frac{12k_{av}(x)(r_o+r_i)(T_{av}(x,t) - T_{aw})}{12k_{av}(x)(r_o+r_i) + h(x)(3r_o+5r_i)(r_o-r_i)}$$

$$\left.\frac{\partial T_s}{\partial r}\right|_{r=r_o(x)} = -\frac{12h(x)(r_o+r_i)(T_{av}(x,t) - T_{aw})}{12k_{av}(x)(r_o+r_i) + h(x)(3r_o+5r_i)(r_o-r_i)}$$

Conjugated problem

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-Compressible boundary layer formulation (laminar and turbulent)
- Decoupled (correlations) versus Conjugated problem analysis

SOLID

$$w_{av}(x)\frac{\partial T_{av}}{\partial t}=\frac{1}{A(x)}\frac{\partial}{\partial x}\left(k_{av}(x)A(x)\frac{\partial T_{av}}{\partial x}\right)-\Omega(x)(T_{av}(x,t)-T_{aw})+g_{av}(x,t)$$

$$\Omega(x)=\begin{cases}\dfrac{24h(x)k_{av}(x)r_o}{12k_{av}(x)(r_o{}^2-r_i{}^2)+h(x)(r_o-r_i)^2(3r_o+5r_i)}, & 0\le x\le a\\[2ex] \dfrac{h(x)P(x)}{A(x)}, & a\le x\le L\end{cases}$$

FLUID

$$\overline{\rho u}\frac{\partial\bar{u}}{\partial s}+\overline{\rho v}\frac{\partial\bar{u}}{\partial y}=\rho_e u_e\frac{du_e}{ds}+\frac{1}{r}\frac{\partial}{\partial y}\left[(\mu+\mu_t)r\frac{\partial u}{\partial y}\right]$$

$$\bar{u}(0)=0,\quad \bar{u}(\infty)=u_e,\quad \left.\frac{\partial\bar{u}}{\partial y}\right|_{y=\infty}=0$$

$$\overline{\rho u}\frac{\partial\bar{H}}{\partial s}+\overline{\rho v}\frac{\partial\bar{H}}{\partial y}=\frac{1}{r}\frac{\partial}{\partial y}\left[\left(\frac{\mu}{Pr}+\frac{\mu_t}{Pr_t}\right)r\frac{\partial\bar{H}}{\partial y}\right]$$
$$+\frac{1}{r}\frac{\partial}{\partial y}\left[\mu r\left(1-\frac{1}{Pr}\right)+\mu_t r\left(1-\frac{1}{Pr_t}\right)\right]\frac{\partial}{\partial y}\left(\frac{\bar{u}^2}{2}\right)$$

$$\bar{H}(0)=c_pT_w,\quad \bar{H}(\infty)=H_e,\quad \left.\frac{\partial\bar{H}}{\partial y}\right|_{y=\infty}=0$$

Thermal analysis- Wing section

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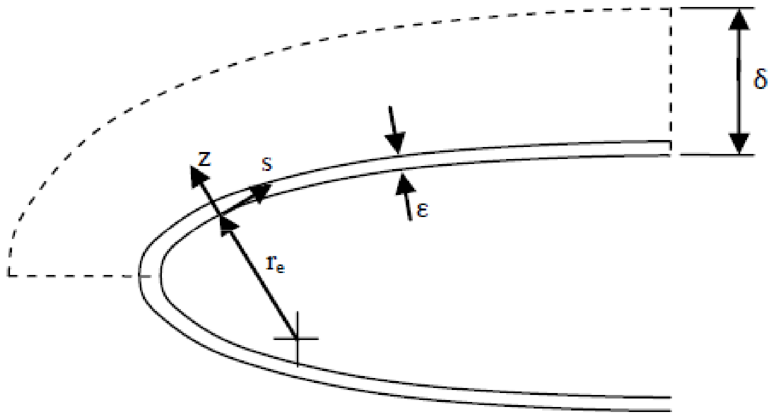
- Heat conduction – local formulation
- Compressible boundary layer formulation (laminar and turbulent)
- Conjugated problem analysis



SOLID

$$k_{sol}\left(\frac{\partial^2 T_1}{\partial s^2}+\frac{\partial^2 T_1}{\partial z^2}\right)+G(s,z)=0$$

$$\left.\frac{\partial T_1}{\partial s}\right|_{s=0}=0;\ \left.\frac{\partial T_1}{\partial s}\right|_{s=L}=0$$

$$-k_{sol}\left.\frac{\partial T_1}{\partial z}\right|_{z=0}=q_w;\ -k_{sol}\left.\frac{\partial T_1}{\partial z}\right|_{z=\varepsilon}=-k_f\left.\frac{\partial T_2}{\partial z}\right|_{z=\varepsilon}$$

FLUID

$$w_f u(s,z)\frac{\partial T_2}{\partial s}+w_f v(s,z)\frac{\partial T_2}{\partial z}=k_f\left(\frac{\partial^2 T_2}{\partial s^2}+\frac{\partial^2 T_2}{\partial z^2}\right)+w_f\frac{\partial}{\partial z}\left[\frac{l^2}{Pr_t}\frac{\partial u}{\partial z}\frac{\partial T_2}{\partial z}\right]$$

$$\left.\frac{\partial T_2}{\partial s}\right|_{s=0}=0;\ \left.\frac{\partial T_2}{\partial s}\right|_{s=L}=0$$

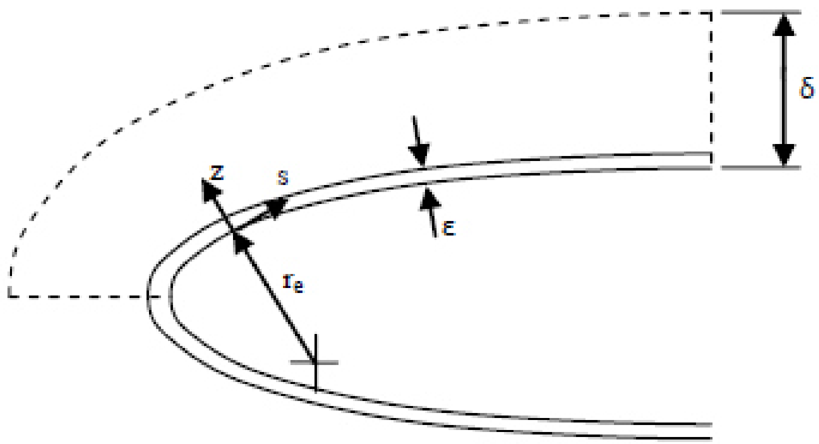
$$T_1(s,\varepsilon)=T_2(s,\varepsilon);T(s,\delta)=T_\infty$$

Single Domain Formulation

$$w_f u(s,z)\frac{\partial T}{\partial s} + w_f v(s,z)\frac{\partial T}{\partial z} = k(z)\frac{\partial^2 T}{\partial s^2} + \frac{\partial}{\partial z}\left[k(z)\frac{\partial T}{\partial z}\right] + w_f\frac{\partial}{\partial z}\left[\frac{l^2}{Pr_t}\frac{\partial u}{\partial z}\frac{\partial T}{\partial z}\right] + G(s,z)$$

$$\left.\frac{\partial T}{\partial s}\right|_{s=0} = 0;\ \left.\frac{\partial T}{\partial s}\right|_{s=L} = 0$$

$$-k_{sol}\left.\frac{\partial T}{\partial z}\right|_{z=0} = q_w; T(s,\delta) = T_\infty$$



Single domain representation for conjugated problem analysis of wing sections

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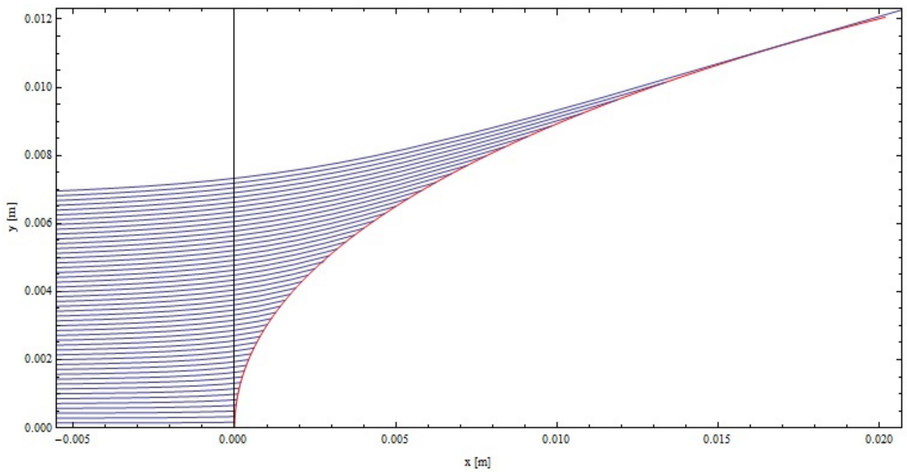
Icing Model:

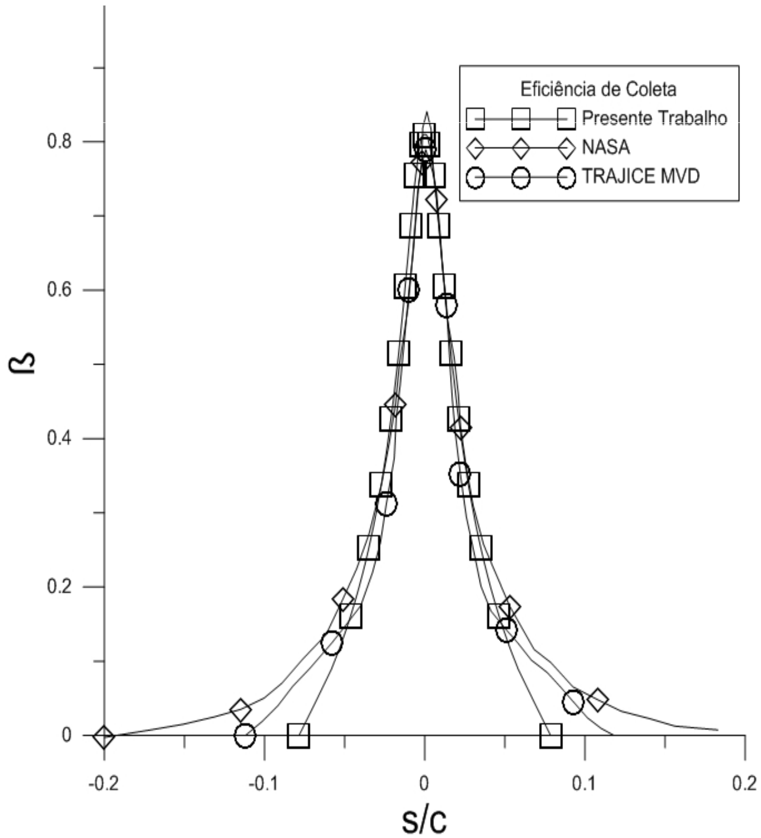
Droplets trajectory – Lagrangean approach

$$m_d \ddot{\boldsymbol{x}}_{\boldsymbol{d}} = -\frac{1}{2}\rho|\dot{\boldsymbol{x}}_{\boldsymbol{d}} - \boldsymbol{u}_{\boldsymbol{e}}|A_d C_D(\dot{\boldsymbol{x}}_{\boldsymbol{d}} - \boldsymbol{u}_{\boldsymbol{e}})$$

$$C_D = \begin{cases} 1 + 0.197{Re_d}^{0.63} + 2.6 \times 10^{-4}{Re_d}^{1.38}, Re_d \leq 3500 \\ 1.699 \times 10^{-5}{Re_d}^{1.92}, Re_d > 3500 \end{cases}$$

$$\beta = \frac{m_i''}{\rho(LWC)U_\infty} \qquad \beta = \frac{\partial y_0}{\partial s} \approx \frac{\Delta y_0}{\Delta s}$$





Icing Model:

Modified Messinger Model

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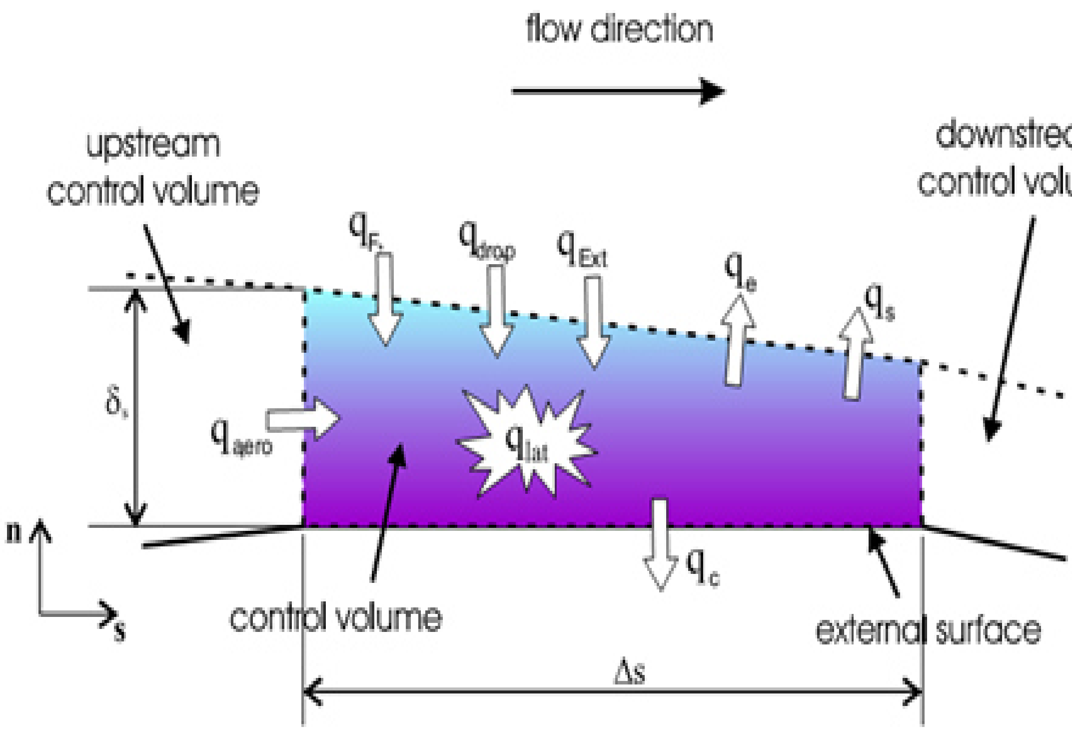
- The energy added comprises terms which are due to freezing, aerodynamic heating, droplet kinetic energy, and external sources (such as the de-icing heater).

-The energy removed includes terms which are due to convection, evaporation, sublimation, droplet warming, and aft conduction.



Icing Model:
Modified Messinger Model

- *Glaze ice*
- Energy balance

$$\rho_g L_F \frac{db^{(p)}}{dt} = \dot{q}''_c + \dot{q}''_e + \dot{q}''_d - \dot{q}''_k - \dot{q}''_a - \dot{q}''_{sw}$$

$$\dot{q}''_c = h_c (T_{sur} - T_\infty)$$

$$\dot{q}''_e = \frac{16.813 L_E}{c_p P_t Le^{2/3}} h_c (T_{sur} - T_\infty)$$

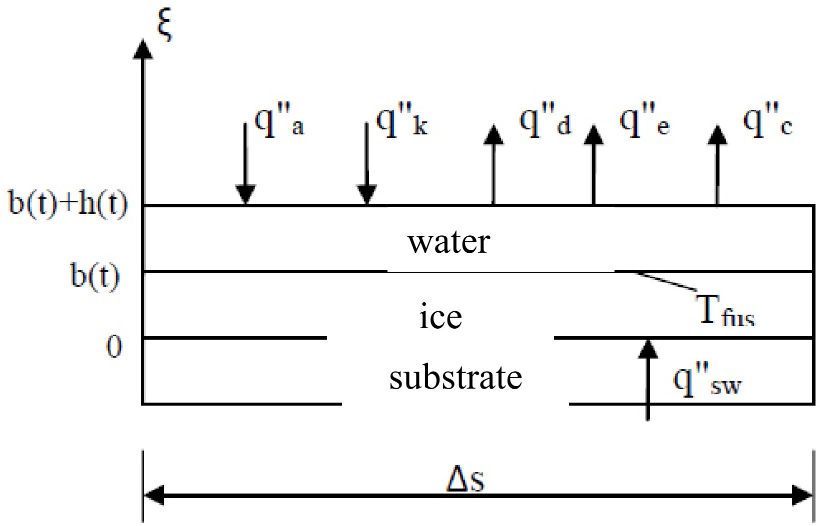
$$\dot{q}''_d = \beta (LWC) U_\infty c_p (T_{sur} - T_\infty)$$

$$\dot{q}''_a = h_c \frac{{U_\infty}^2}{2 c_p} \begin{cases} Pr^{1/2}, & \text{If laminar} \\ Pr^{1/3}, & \text{If turbulent} \end{cases}$$

$$\dot{q}''_k = \beta (LWC) \frac{{U_\infty}^3}{2}$$

$$\dot{q}''_{sw} = -\frac{k_s \Delta T_c}{\delta} \frac{\partial \theta}{\partial \eta}\bigg|_{\eta = \eta_I^-}$$

$$h_c = -\frac{k_f}{\delta} \frac{1}{\theta(S, {\eta_I}^+, \tau)} \frac{\partial \theta}{\partial \eta}\bigg|_{\eta = {\eta_I}^+}$$



A4 Skyhawk – Brazilian Navy

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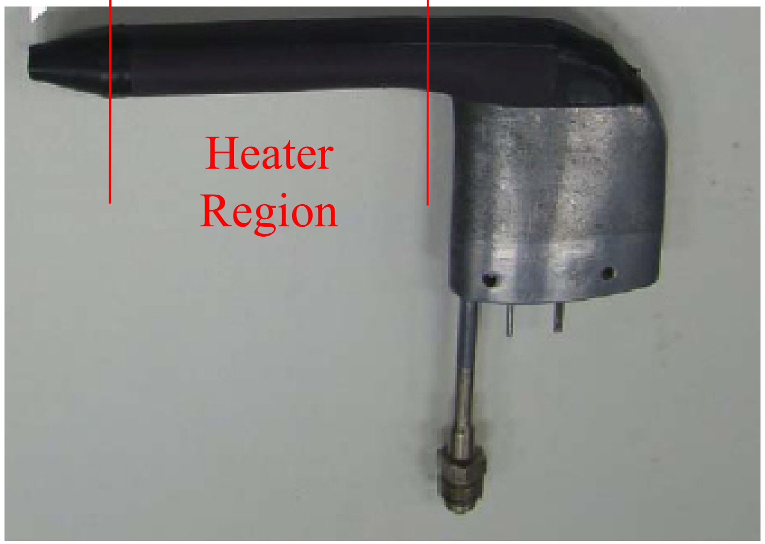
- Experimental analysis: Flight Testing and Wind Tunnel
- Theoretical analysis: Improved Lumped Model and Conjugated Problem
- Validation of Model & Simulation (GITT and Mathematica system)



- A4 Skyhawk – Brazilian Navy (São Pedro d´Aldeia Base)
- Imposed transients by successive heating system switching
- Compressible flow range (Ma=0.5, up to altitude 15,000 ft)

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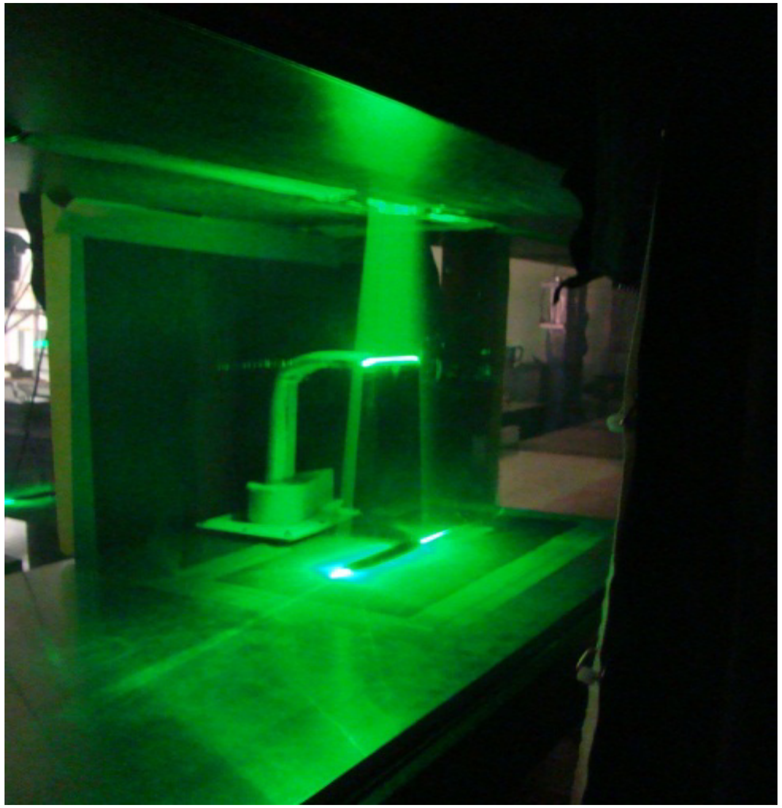
Wind Tunnel Tests

- INMETRO Wind Tunnel Facility
- PIV and Infrared camera measurements
- Incompressible flow range



Model Validation

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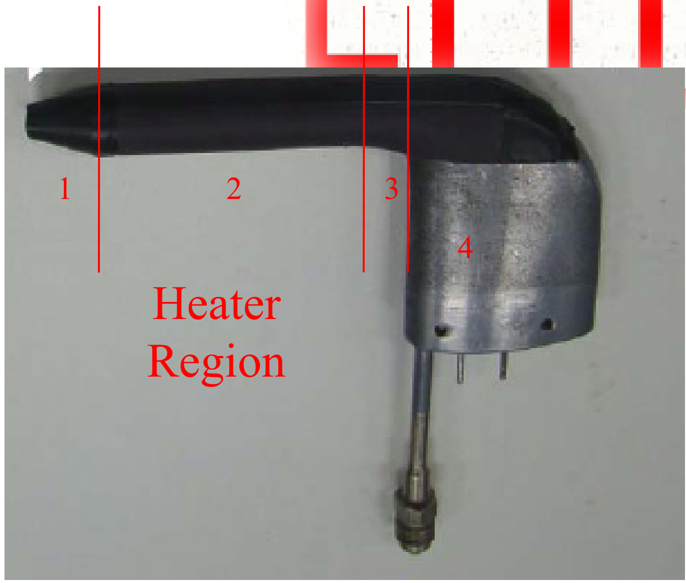
- Infrared camera measurements from wind tunnel experiments versus conjugated problem analysis for Pitot probe surface temperatures



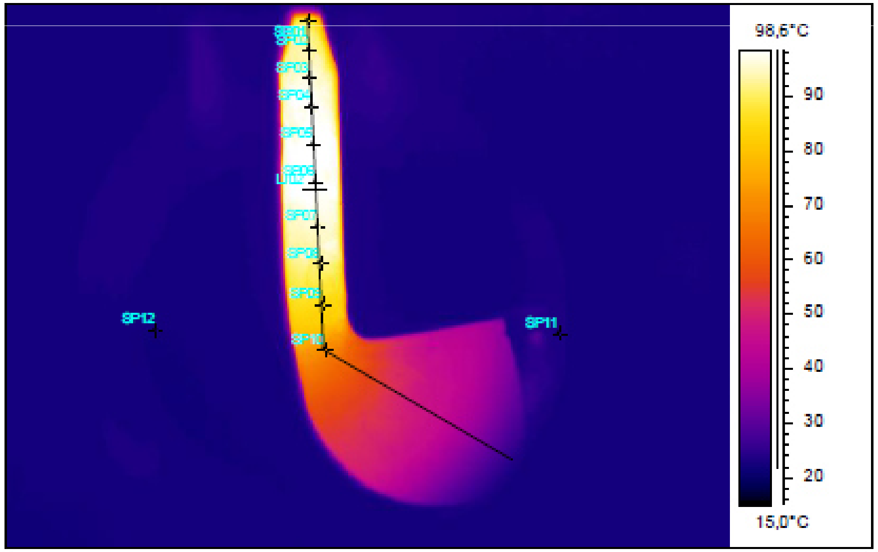


Figure 3. Illustration of the infrared thermography results from the wind tunnel experiments (9.98 m/s, 68 V and 0.69 A in the resistor)

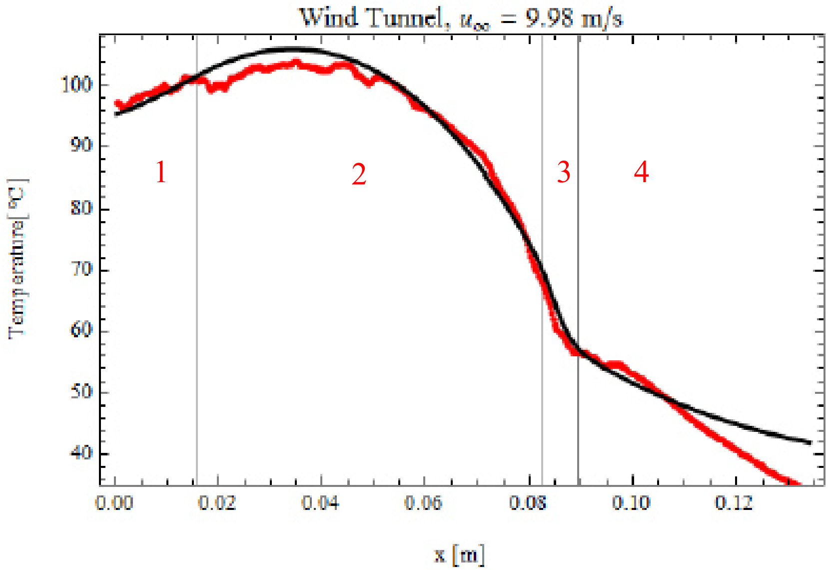


Figure 4. Comparison between the steady state temperature fields: experimental, in red, and simulated, in black.

Model Validation – Steady

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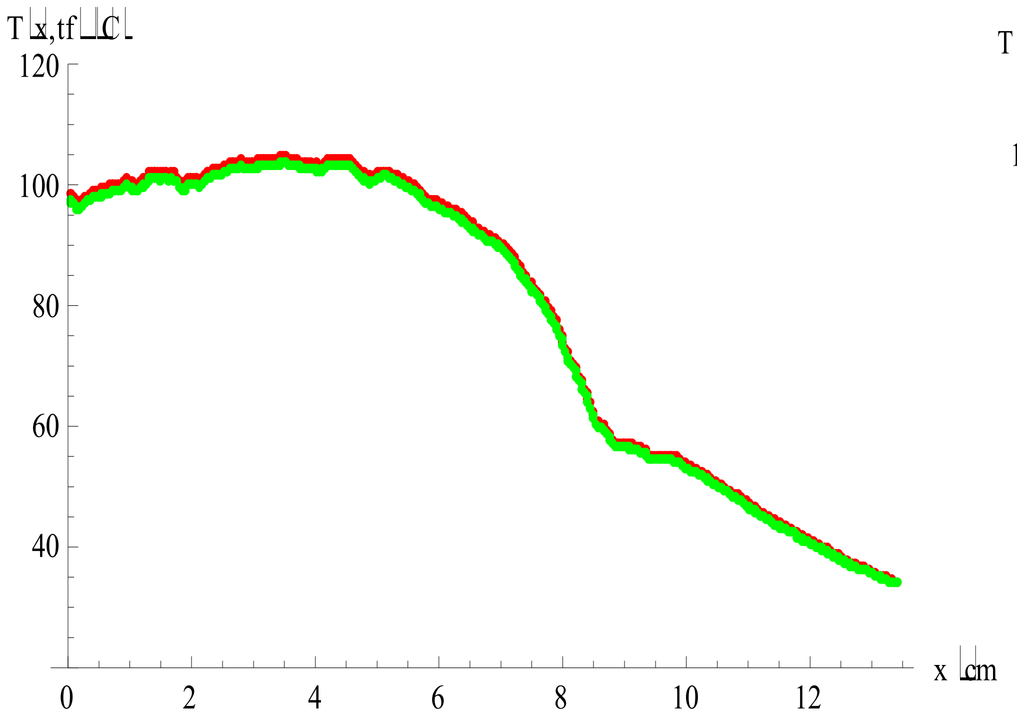
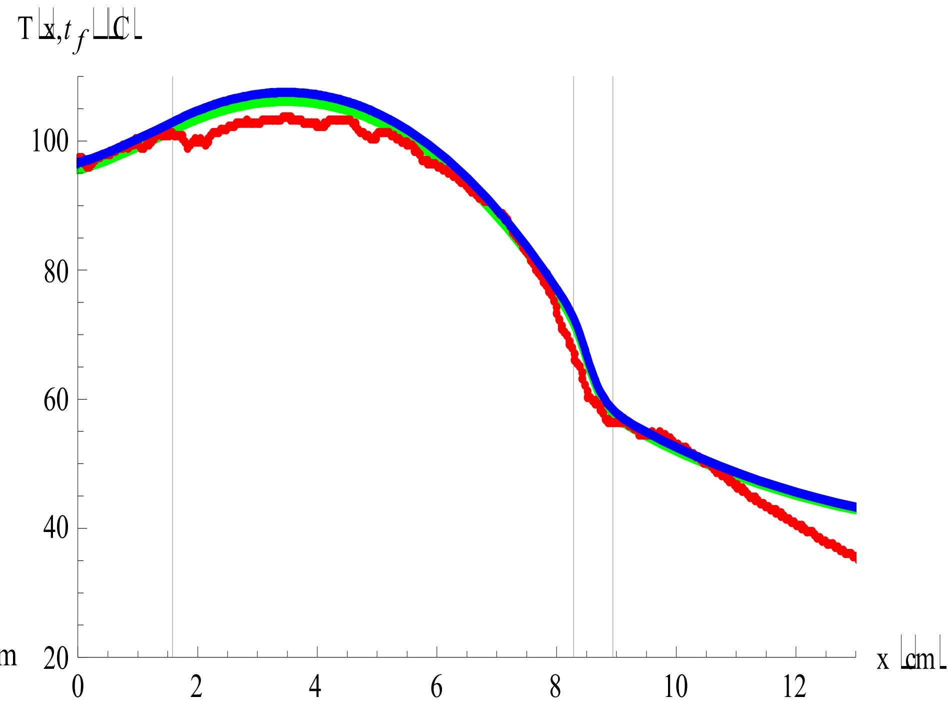


Repetition - experiments (nos. 6 e 7), with 10 m/s, 68 V and 0,69 A.

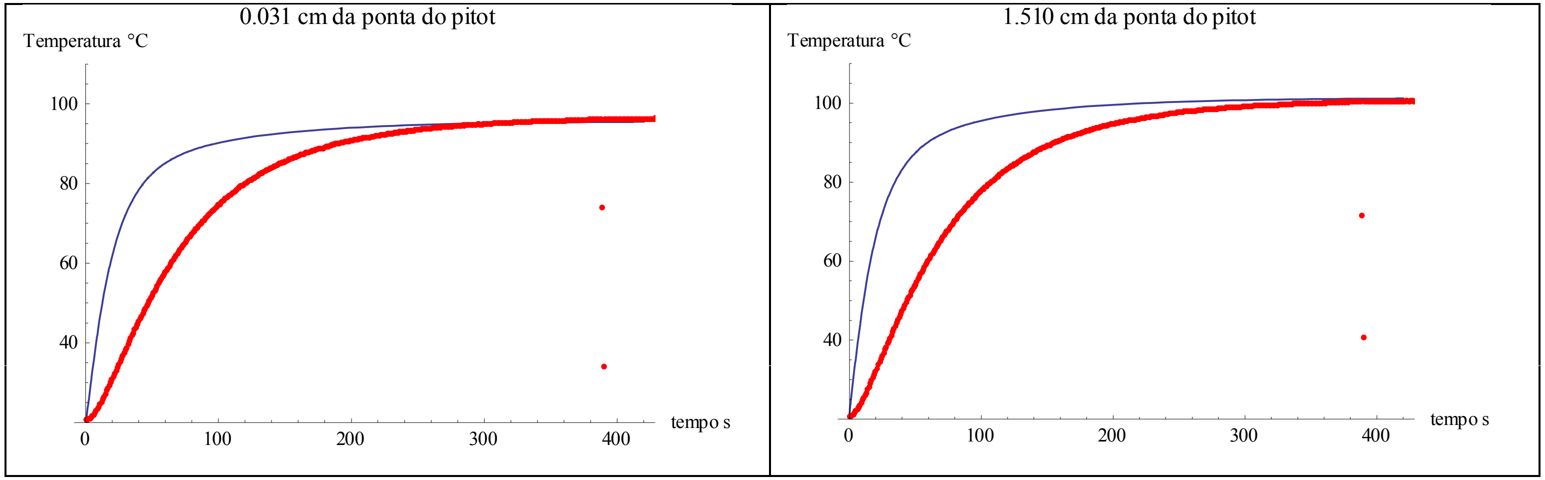


Comparison of experiment with linear and nonlinear models, with 10 m/s, 68 V and 0,69 A.

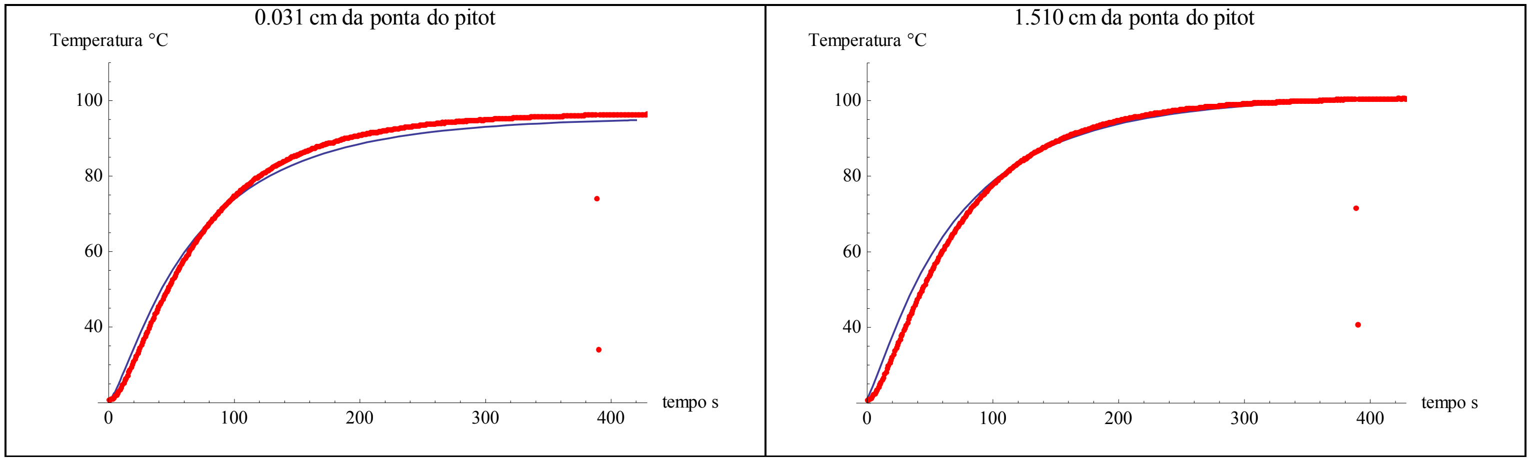
Model Validation - Transient

Transient temperature behavior

Neglecting ceramic capacitance (experim. in red, theory in blue)



Accounting for ceramic capacitance (experim. in red, theory in blue)



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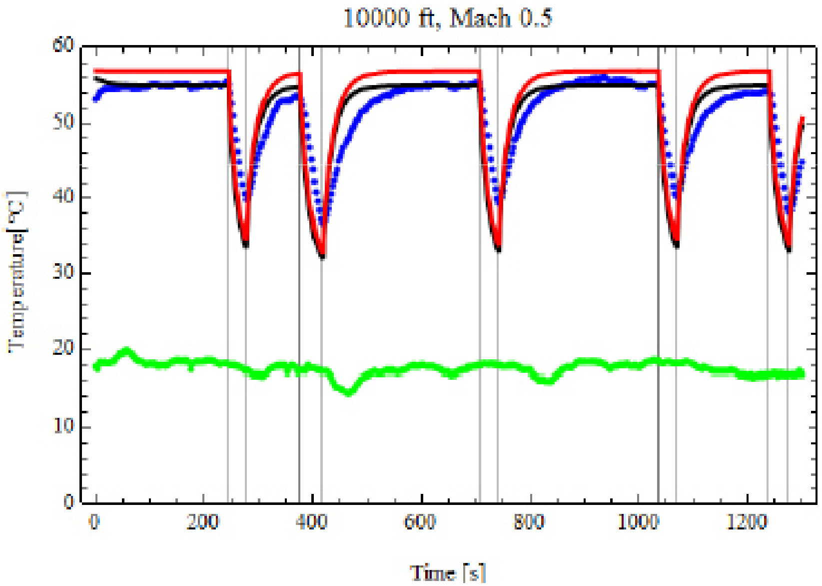
Model Validation

Thermocouple measurements from A4 flight tests

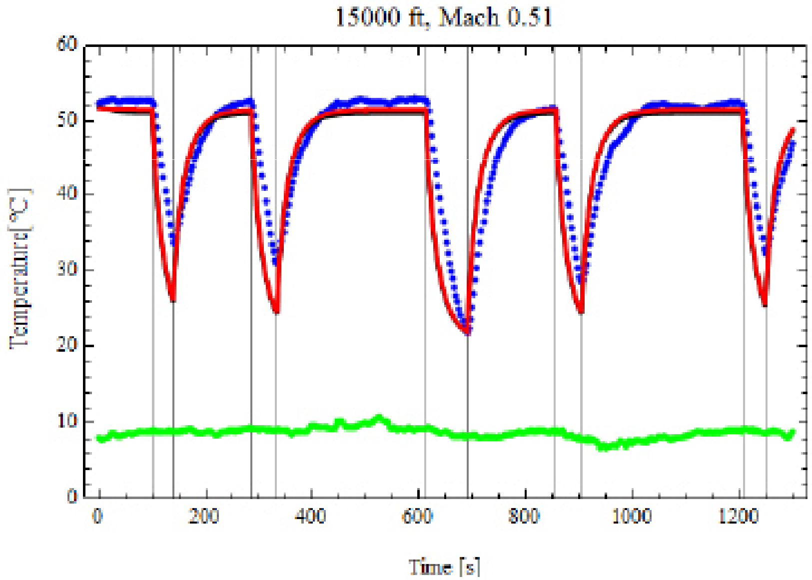
X

conjugated problem analysis for Pitot probe surface temperatures

Improved Lumped and with **Classical Lumped**



(a)

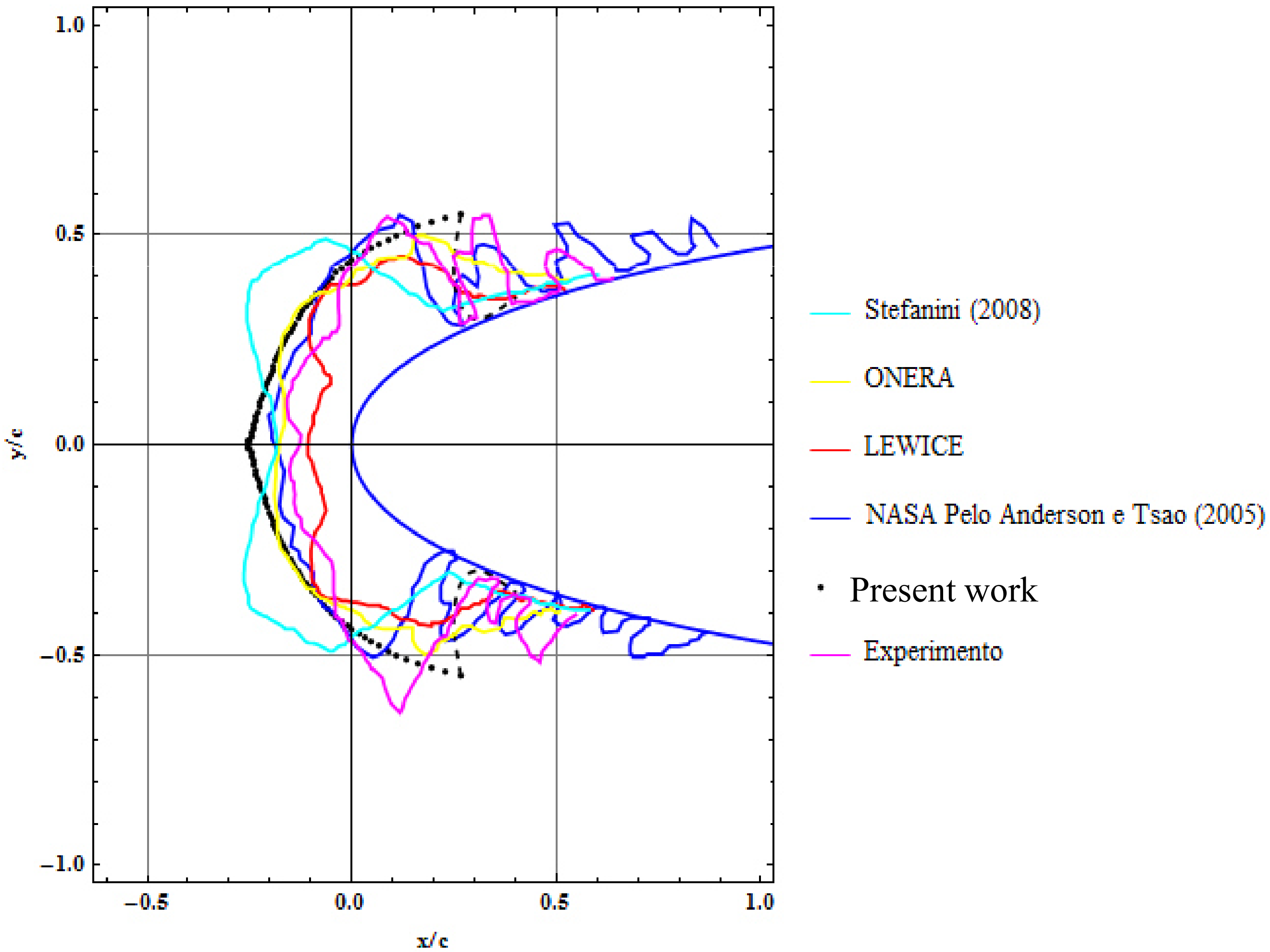


(b)

Figure 6. Time evolution of the temperature at the surface of the Pitot distant 80 mm from the stagnation point, considering the porcelain influence, with improve lumped, in black, with classical lumped, in red, and with experimental results, in blue: (a) flight at 10000 ft and Mach 0.5; (b) flight at 15000 ft and Mach 0.51.

Ice Formation - Airfoil

- Validation and verification against experimental and previous theoretical results from different icing codes for airfoils: - NACA0012



Ice Formation - Pitot

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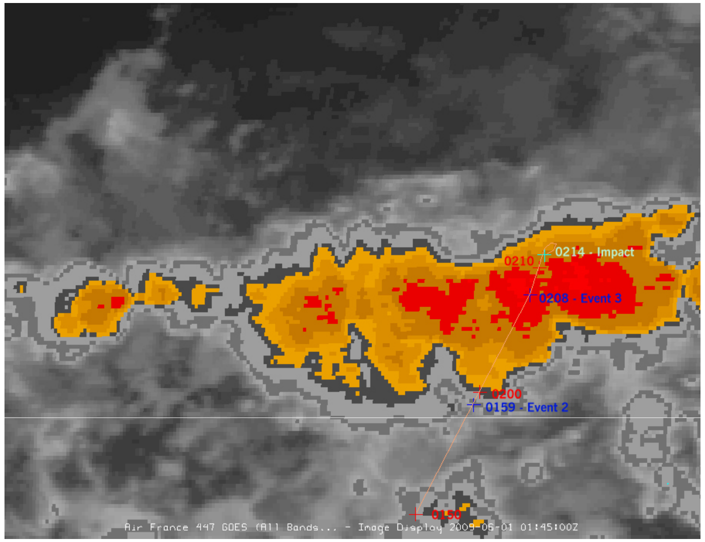
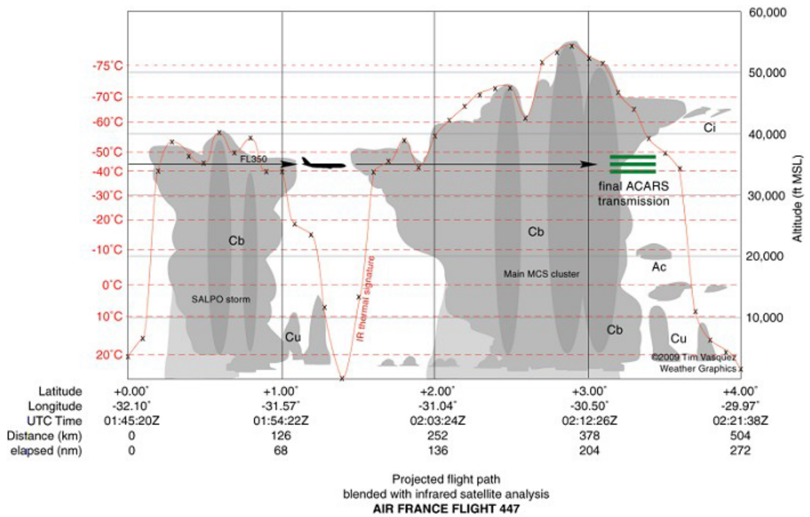
Trajectory of AF447 - flight conditions : T_∞ = -54 ^{0} C, LWC= 1.115 g/m^3, MVD=45 μm, V_∞=800 km/h

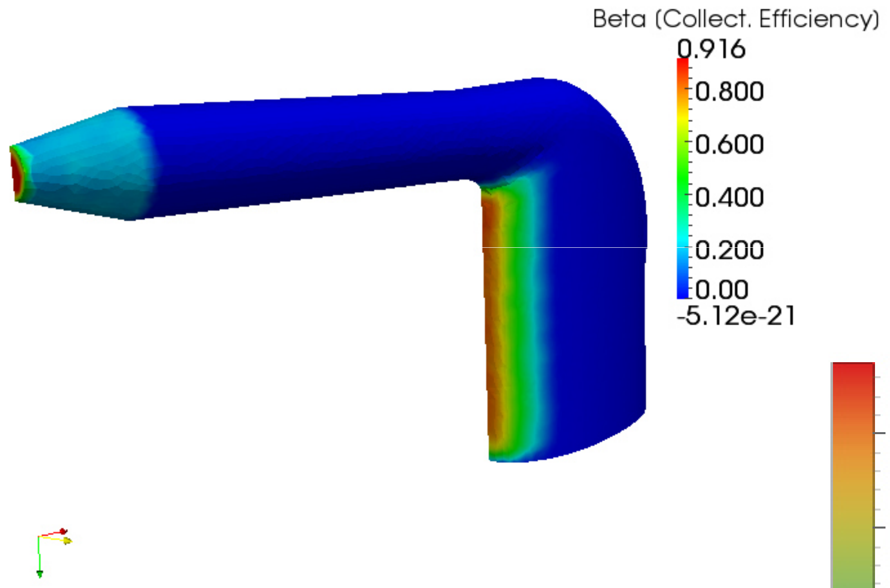




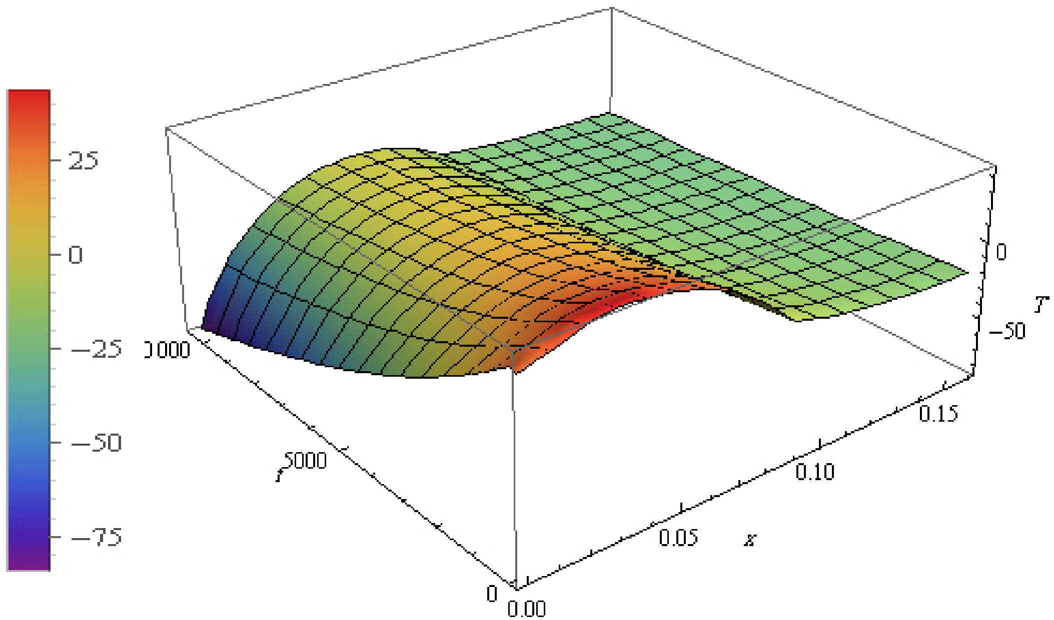
Ice Formation - Pitot

COPPE/UFRJ

- Pitot probe thermal analysis model including ice formation: - AF447 flight conditions : T_∞ = -54 ^{0} C, LWC= 1.115 g/m^3, MVD=45 µm, V_∞=800 km/h



Collection efficiency for Pitot probe

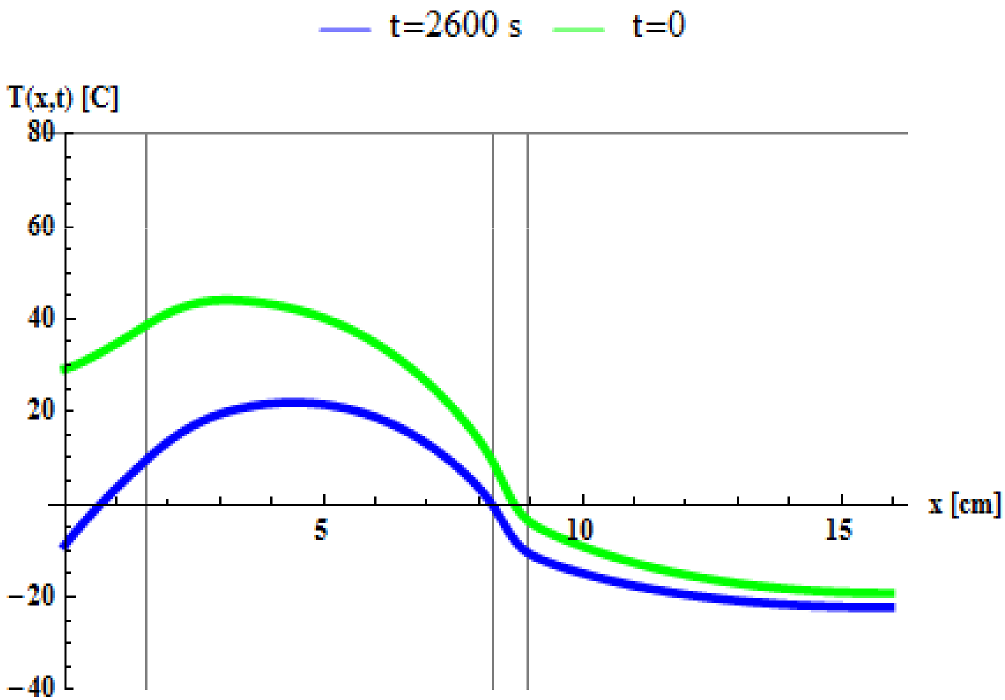


Transient temperature distribution along Pitot length at the external surface

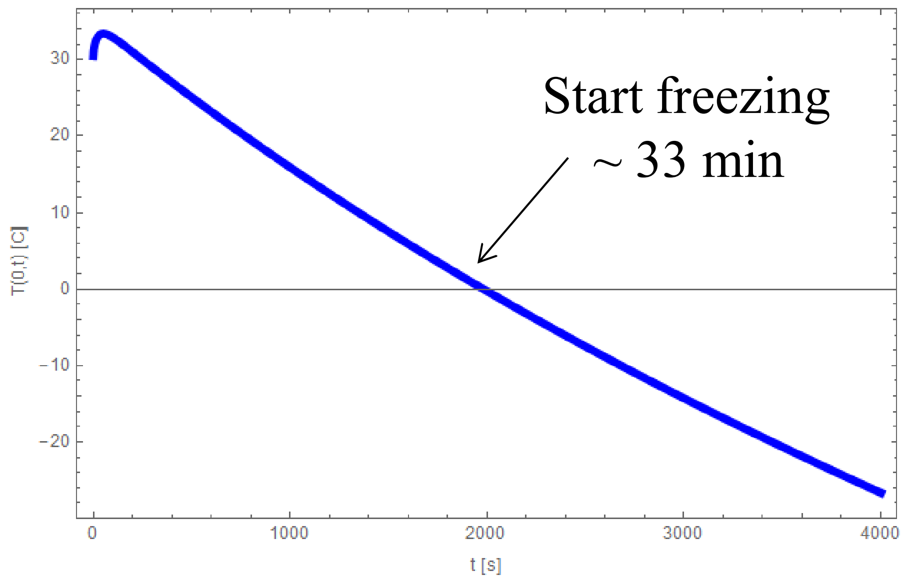
Ice Formation - Pitot

COPPE/UFRJ

- Ice accretion: - present model -AF447 flight conditions: T_∞ = -54 ^{0} C, LWC= 1.115 g/m^3, MVD=45 µm, V_∞=800 km/h



Theoretical prediction of temperatures along the probe PH510 before and after starting icing



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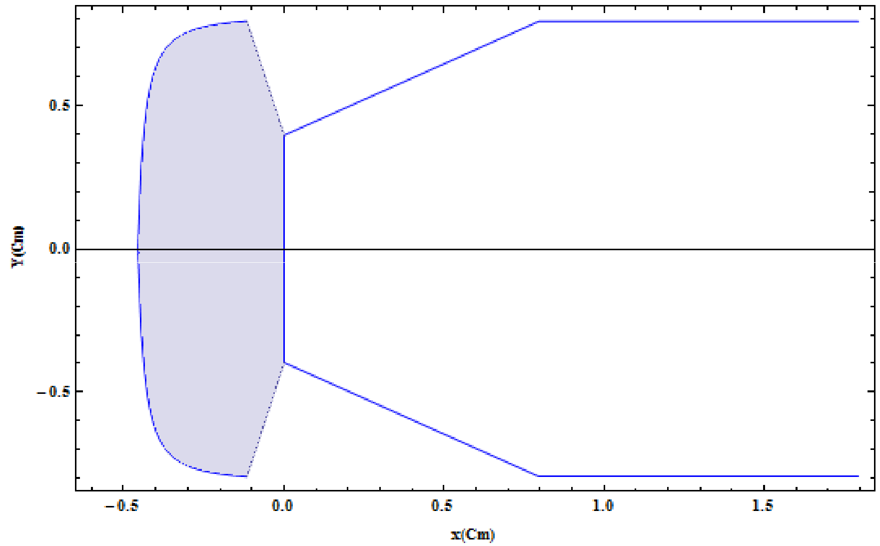
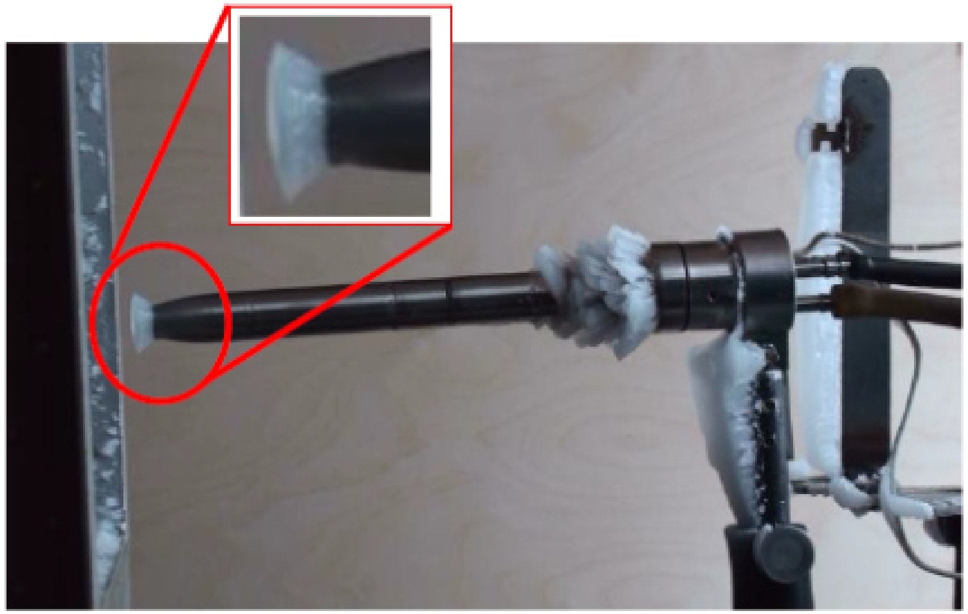
Ice Formation - Pitot

- Ice accretion shape: qualitative comparison (theoretical+experimental)



Ice shape over Pitot probe as obtained in the TsAIG Climatic Icing Wind Tunnel, Russia (Prof. Ivan Egorov, Head of TsAGI research department)



Ice shape in Pitot probe PH-510 under conditions:
u_{∞}= 56 m/s, LWC=0.59 g/m^3,
T_{sur}= -10 ^{0}C, MVD=26.2μm.

State Estimation Problem

(i) Estimating the heat transfer coefficient at the leading edge of Pitot tubes, in order to detect ice accretion;
(ii) Estimating the relative air speed in lack of reliable dynamic pressure readings.

State estimation problem: -available measured data used together with prior knowledge on physical phenomena (direct problem) and measuring devices → sequentially produce estimates of desired dynamic variables.

State Estimation Problem

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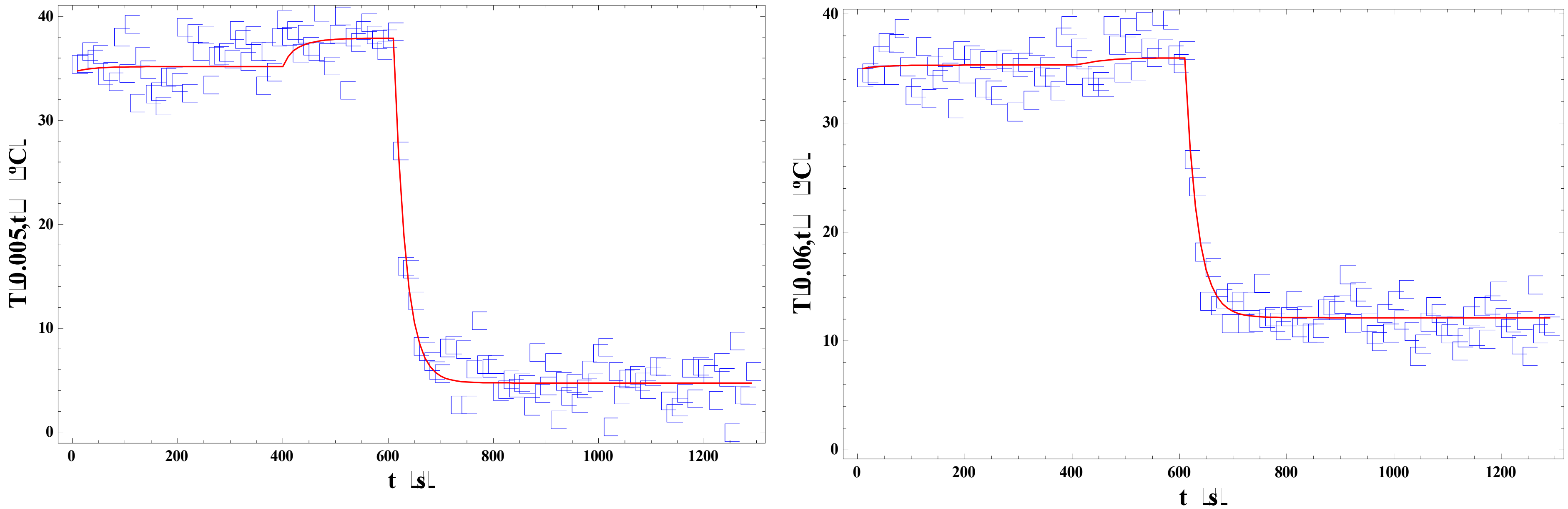

(a) $x = 0.005$ m (b)) $x = 0.06$ m

Simulated measurements (blue stars) and exact temperatures (red lines) at different positions

State Estimation Problem

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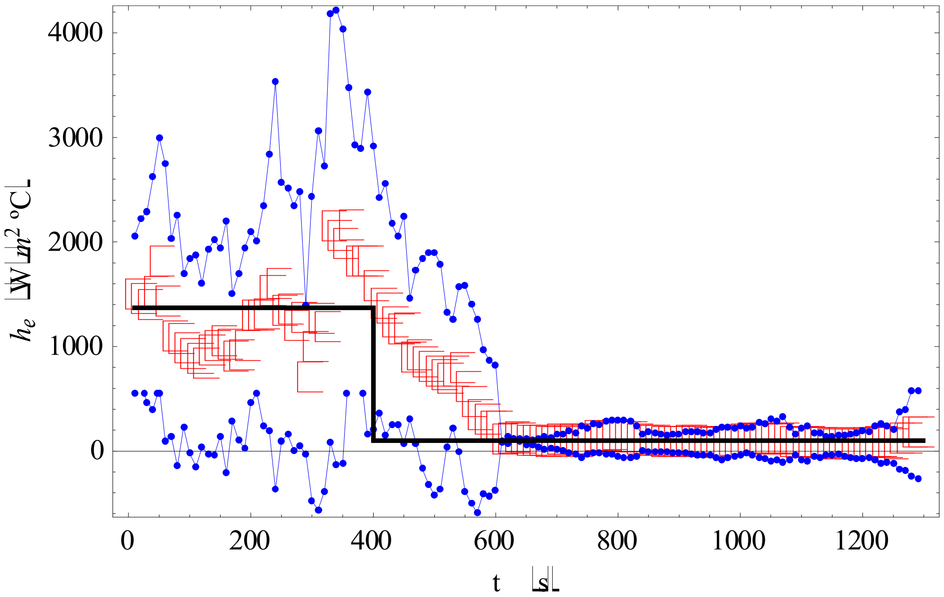
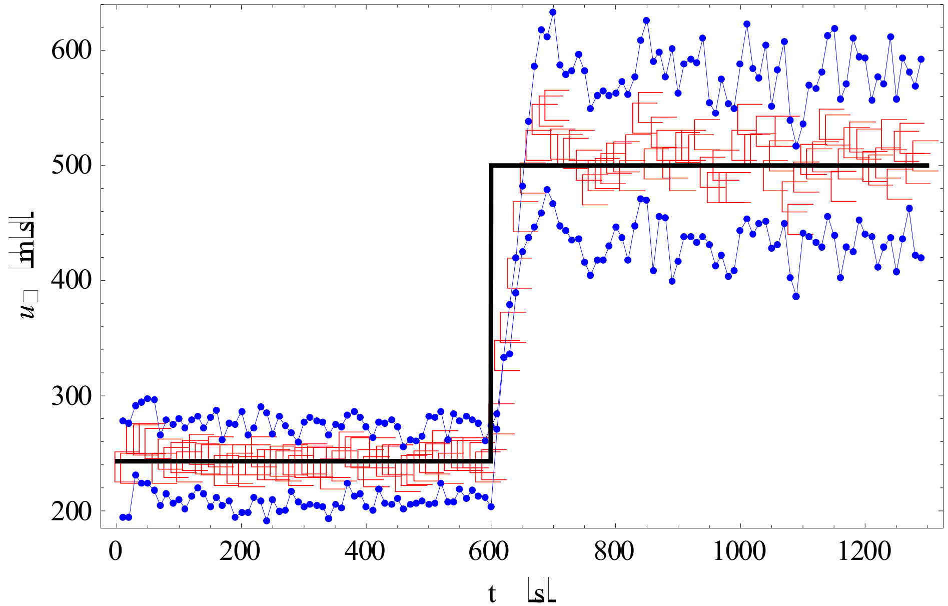
a) heat transfer coefficient - h_e



b) free stream velocity - u_∞



Results for the test-case estimation obtained with 50 particles - means of the state variables (red stars), 99% confidence intervals (blue lines) and exact functional forms (black lines)

Icing Wind Tunnel COPPE

COPPE/UFRJ

- NIDF: Interdisciplinary Nucleus of Fluid Dynamics – CT2 – COPPE-UFRJ

Closed circuit
Total length: 7.5m; Total height: 3.5m
Test section: 0.3m X 0.3 m X 2 m
Velocity: 5 to 58 m/s
Ma=0.3 and T∞= -20C

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Icing Wind Tunnel COPPE

Test section of wind tunnel during experiment

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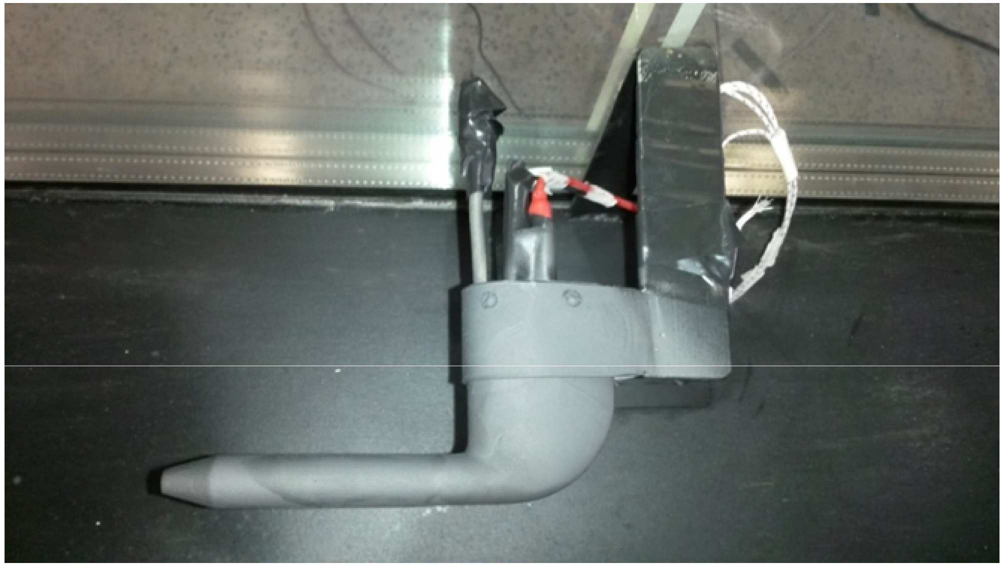
Icing Wind Tunnel COPPE



Pitot tube of A4 Syhawk – graphite ink for termography

Icing experiment in tunnel test section

Icing Wind Tunnel COPPE

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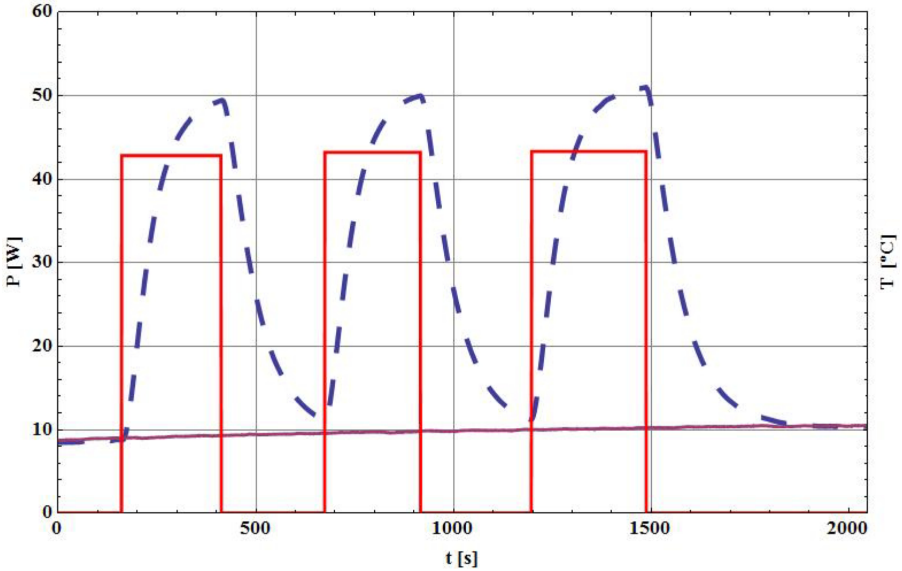
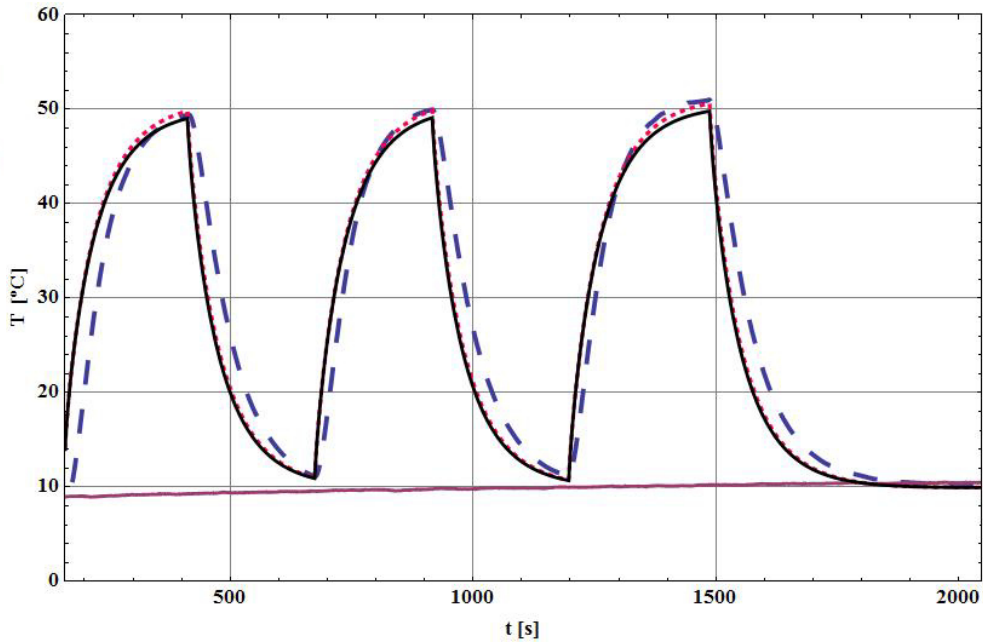


Comparison between the theoretical and experimental transient temperature fields: experimental in blue, and theories in black and red.

Transient power generation in probe and achieved measured temperature transient at 80mm.Temperature of air in the tunnel, lower purple line. (800 rpm - 22,87 m/s)



Results for promoted step transients in power

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Recent and Future Research

Recent & Future Research

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- Multiscale domains and properties
- Convective eigenvalue problems
- Nonlinear eigenvalue problems
- Single domain multiphase formulation

Convective eigenvalue problem

$$w_k(x)\frac{\partial T_k(x,t)}{\partial t}+u_k(x)\frac{\partial T_k(x,t)}{\partial x}=\frac{\partial}{\partial x}[k_k(x)\frac{\partial T_k(x,t)}{\partial x}]-d_k(x)T_k(x,t)+P_k(x,t,\mathbf{T}),\quad x_0<x<x_1,t>0$$

$$\hat{w}_k(x)\frac{\partial T_k(x,t)}{\partial t}=\frac{\partial}{\partial x}[\hat{k}_k(x)\frac{\partial T_k(x,t)}{\partial x}]-\hat{d}_k(x)T_k(x,t)+\hat{P}_k(x,t,T),\quad x_0<x<x_1,t>0$$

$$\hat{w}_k(x)=w_k(x)\hat{k}_k(x)/k_k(x);\ \hat{d}_k(x)=d_k(x)\hat{k}_k(x)/k_k(x);\hat{P}_k(x,t,T)=P_k(x,t,\mathbf{T})\hat{k}_k(x)/k_k(x);$$

$$u_k^*(x)=\frac{1}{k_k(x)}\left[u_k(x)-\frac{dk_k(x)}{dx}\right];\ and\ \hat{k}_k(x)=e^{-\int u_k^*(x)dx}$$

Diffusion formulation incorporates convective effects

$$\frac{d}{dx}[\hat{k}(x)\frac{d\psi(x)}{dx}]+[\mu^2\hat{w}(x)-\hat{d}(x)]\psi(x),\quad x_0<x<x_1$$

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Convective eigenvalue problem

Convergence analysis of eigenfunction expansions with convective and diffusive eigenvalue problems for one-dimensional Burgers equation in both linear and nonlinear formulations.

N	u_0=10, b=0 (linear problem)				u_0=10, b=5 (nonlinear problem)			
	T(0.5,0.05) Conv.	T(0.5,0.05) Dif.	T(0.9,0.01) Conv.	T(0.9,0.01) Dif.	T(0.5,0.05) Conv.	T(0.5,0.05) Dif.	T(0.9,0.01) Conv.	T(0.9,0.01) Dif.
2	0.386565	0.462380	-1.81418	0.430194	0.286084	0.360552	-1.43571	0.450501
4	0.379714	0.356180	0.200342	0.653885	0.276093	0.249478	0.409794	0.688207
6	0.379716	0.388818	0.704696	0.726450	0.276946	0.287617	0.762698	0.781085
8	0.379716	0.375350	0.738297	0.748564	0.276724	0.271688	0.791899	0.812893
10	0.379716	0.382043	0.738805	0.750988	0.276796	0.279440	0.794770	0.816583
12	0.379716	0.378270	0.738798	0.746579	0.276769	0.275180	0.794446	0.810758
14	0.379716	0.380596	0.738798	0.741232	0.276781	0.277753	0.793992	0.803719
16	0.379716	0.379064	0.738798	0.737306	0.276774	0.276080	0.793768	0.798590
18	0.379716	0.380126	0.738798	0.735361	0.276778	0.277236	0.793735	0.796028
20	0.379716	0.379359	0.738798	0.735074	0.276775	0.276386	0.793817	0.795626

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Convective eigenvalue problem

Convergence of eigenfunction expansions with convective and diffusive eigenvalue problems in the solution of the two-dimensional Burgers equation in linear formulation.

N	u_0=10, v_0=1, b=0 (linear problem)					
	T(0.5,0.1,0.01) Conv.	T(0.5,0.1,0.01) Diff.	T(0.1,0.1,0.01) Conv.	T(0.1,0.1,0.01) Diff.	T(0.9,0.1,0.05) Conv.	T(0.9,0.1,0.05) Diff.
10	0.516515	0.485608	0.110953	0.133233	0.108255	0.097938
20	0.491783	0.501325	0.135094	0.155057	0.108368	0.102400
30	0.493395	0.497816	0.140490	0.137969	0.108368	0.106297
40	0.493082	0.492352	0.141373	0.145203	0.108368	0.108567
50	0.493041	0.491333	0.141533	0.139425	0.108368	0.109549
60	0.493100	0.494026	0.141569	0.140748	0.108368	0.109946
70	0.493122	0.495103	0.141574	0.140942	0.108368	0.109924
80	0.493123	0.495309	0.141574	0.140974	0.108368	0.109922

Nonlinear Eigenvalue Problem

Problem rewritten keeping **nonlinear boundary coefficients** (step 1)

$$w(\mathbf{x})\frac{\partial T(\mathbf{x},t)}{\partial t} = \nabla\cdot[k(\mathbf{x})\nabla T(\mathbf{x},t)] - d(\mathbf{x})T(\mathbf{x},t) + P(\mathbf{x},t,T), \quad \mathbf{x}\in V,\ t>0$$

$$T(\mathbf{x},0) = f(\mathbf{x}), \quad \mathbf{x}\in V$$

$$\alpha(\mathbf{x},t,T)\,T + \beta(\mathbf{x},t,T)\,k(\mathbf{x})\,\frac{\partial T}{\partial \mathbf{n}} = \phi(\mathbf{x},t,T),$$

Characteristic linear equation coefficients (w, k, d)

and original nonlinear boundary condition (α, β) coefficients

Analytical Filtering

$$T(\mathbf{x},t) = T^*(\mathbf{x},t) + T_F(\mathbf{x};t)$$

Filtered Problem Formulation (step 1)

$$w(\mathbf{x})\frac{\partial T^*(\mathbf{x},t)}{\partial t} = \nabla\cdot[k(\mathbf{x})\nabla T^*(\mathbf{x},t)] - d(\mathbf{x})T^*(\mathbf{x},t) + P^*(\mathbf{x},t,T^*), \quad \mathbf{x}\in V,\ t>0$$

$$T^*(\mathbf{x},0) = f^*(\mathbf{x}) \equiv f(\mathbf{x}) - T_F(\mathbf{x};0), \quad \mathbf{x}\in V$$

$$\alpha(\mathbf{x},t,T)\ T^* + \beta(\mathbf{x},t,T)\ k(\mathbf{x})\frac{\partial T^*}{\partial \mathbf{n}} = \phi^*(\mathbf{x},t,T^*),$$

$$P^*(\mathbf{x},t,T^*) = P(\mathbf{x},t,T) - w(\mathbf{x})\frac{\partial T_F(\mathbf{x};t)}{\partial t} + \nabla\cdot[k(\mathbf{x})\nabla T_F(\mathbf{x};t)] - d(\mathbf{x})T_F(\mathbf{x};t)$$

$$\phi^*(\mathbf{x},t,T^*) = \phi(\mathbf{x},t,T) - \left[\alpha(\mathbf{x},t,T)\ \ T_F + \beta(\mathbf{x},t,T)\ k(\mathbf{x})\frac{\partial\ T_F}{\partial \mathbf{n}}\right]$$

Generalized Integral Transform Technique
Eigenfunction Expansion

Eigenvalue Problem (step 2)

$$\nabla \cdot [k(\mathbf{x}) \nabla \psi_i(\mathbf{x})] + [\mu_i^2 w(\mathbf{x}) - d(\mathbf{x})]\psi_i(\mathbf{x}) = 0, \quad \mathbf{x} \in V$$

$$\alpha(\mathbf{x},t,T)\psi_i(\mathbf{x};t) + \beta(\mathbf{x},t,T)\mathrm{k}(\mathbf{x})\frac{\partial \psi_i(\mathbf{x};t)}{\partial \mathbf{n}} = 0,$$

Integral Transform Pair (step 3)

$$\bar{T}_i(t) = \int_V w(\mathbf{x})\ \psi_i(\mathbf{x};t) T^*(\mathbf{x},t)\ dv\ , \qquad \text{transform}$$

$$T^*(\mathbf{x},t) = \sum_{i=1}^{\infty} \frac{1}{N_i(t)} \psi_i(\mathbf{x};t)\bar{T}_i(t)\ , \qquad \text{inverse}$$

norm

$$N_i(t) = \int_V w(\mathbf{x})\psi_i^2(\mathbf{x};t)\ dv$$

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Transformed System

Integral Transformation (step 3) $\int_v \psi_i(\mathbf{x};t)\,(.)\,dV$

Transformed ODE System (step 4)

$$\frac{d\overline{T}_i(t)}{dt}+\sum_{j=1}^{\infty}A_{i,j}(t,\overline{\mathbf{T}})\overline{T}_j(t)=\overline{g}_i(t,\overline{\mathbf{T}}),\qquad t>0,\quad \text{i j}=1,2\ldots$$

$$A_{i,j}(t,\overline{\mathbf{T}})=\delta_{ij}\mu_i^2(t)+A_{i,j}^*(t,\overline{\mathbf{T}}),\qquad A_{i,j}^*(t,\overline{\mathbf{T}})=-\frac{1}{N_j(t)}\int_V w(\mathbf{x})\,\frac{\partial}{\partial t}[\psi_i(\mathbf{x};t)]\psi_j(\mathbf{x};t)\,dv$$

Transformed initial conditions and sources (step 4)

$$\overline{T}_i(0)=\overline{f}_i=\int_v w(\mathbf{x})\,\tilde{\psi}_i(\mathbf{x};0)\,f^*(\mathbf{x})\,\mathrm{d}V$$

$$\overline{g}_i(t,\overline{\mathbf{T}})=\int_v \psi_i(\mathbf{x};t)P^*(\mathbf{x},t,T)\,dV+\int_S \phi^*(\mathbf{x},t,T)\left(\frac{\psi_i(\mathbf{x};t)\;-k(\mathbf{x})\dfrac{\partial\psi_i}{\partial\mathbf{n}}}{\alpha(\mathbf{x},t,T)+\beta(\mathbf{x},t,T)}\right)ds$$

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Heat conduction with nonlinear convective-radiative BC´s

$$\frac{\partial\ T(x,t)}{\partial t} = \frac{\partial^2 T(x,t)}{\partial x^2}\ , \qquad 0<x<1, \qquad t>0$$

$$T(x,0)=1\ , \qquad 0 \le x \le 1$$

$$\frac{\partial T(0,t)}{\partial x}=0\ ; \qquad \frac{\partial T(1,t)}{\partial x} + Bi(T(1,t))\,T(1,t)=0\ , \qquad \text{t}>0$$

$$Bi(T(1,t)) = Bi_c T^{1/3}(1,t) + Bi_r \left[(1+\gamma T(1,t) + \frac{\gamma^2}{2} T^2(1,t))\,(1+\frac{\gamma}{2} T(1,t)) \right]$$

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Nonlinear eigenvalue problem

$$\frac{\partial^2 \psi(x;t)}{\partial \mathrm{x}^2} + \mu^2(\mathrm{t})\,\psi(x;t) = 0 \,, \qquad 0<x<1$$

$$\frac{\partial \psi(0;t)}{\partial x} = 0 \;; \qquad \frac{\partial \psi(1;t)}{\partial x} + Bi(T(1,t))\,\psi(1;t) = 0$$

$$\psi(x;t) = \cos[\mu(t)x]$$

$$-\mu(t)\sin[\mu(t)] + Bi(T(1,t))\cos[\mu(t)] = 0 \qquad T(1,t) = \sum_{i=1}^{\infty} \frac{1}{N_i(t)} \cos \mu_i(t) \bar{T}_i(t)$$

Eigenvalues ODE system

$$\frac{d\mu_i(t)}{dt}\{-\sin[\mu_i(t)] - \mu_i(t)\cos[\mu_i(t)] - Bi(T(1,t))\sin[\mu_i(t)]\} +$$

$$+ \frac{dBi(T(1,t))}{dT(1,t)} \cos[\mu_i(t)] \frac{\partial T(1,t)}{\partial t} = 0$$

$$-\mu_i(0)\sin[\mu_i(0)] + Bi(T(1,0))\cos[\mu_i(0)] = 0, \quad \mathrm{i}=1,2,3,\ldots$$

Nonlinear eigenvalue problem

Potential convergence behavior as obtained from the nonlinear eigenvalue problem and the traditional GITT approaches

$Bi_c = 1;\ Bi_r = 1;\ \gamma = 1/3$

N	*T*(*x*=0.2,*t*=0.1)		*T*(*x*=0.8,*t*=0.1)		*T*(*x*=1.0,*t*=0.1)	
	Nonlinear Eig.Probl.	**Linear Eig.Probl.**	**Nonlinear Eig.Probl.**	**Linear Eig.Probl.**	**Nonlinear Eig.Probl.**	**Linear Eig.Probl.**
1	1.01966	1.03587	0.670233	0.640969	0.480618	0.429363
5	0.976123	0.974836	0.727676	0.730316	0.531293	0.519378
10	0.976108	0.975809	0.727709	0.727208	0.531246	0.525529
15	0.976107	0.975981	0.727707	0.727967	0.531242	0.527504
20	0.976107	0.97604	0.727708	0.727645	0.531241	0.528473
25	0.976107	0.976067	0.727708	0.727819	0.531240	0.529048
30	0.976107	0.976081	0.727708	0.727711	0.531240	0.529429
Num.*	**0.976107**		**0.727710**		**0.531245**	

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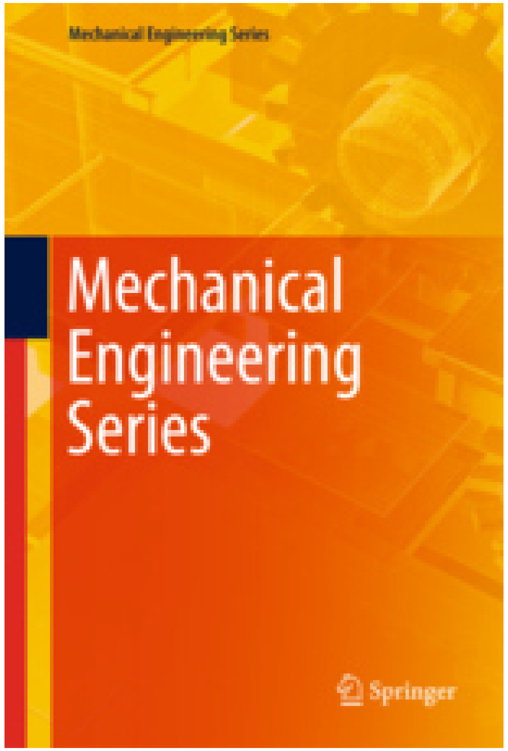

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Cotta, Knupp, Naveira-Cotta

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Dedication

•This work is dedicated to the 228 victims of the AF447 flight and their families (31/05/2009).

This hard lesson will hopefully affect somehow technology development protocols, in a progressively more competitive world, reminding us all that there is no acceptable, sustainable and safe technological development without a supporting extensive scientific analysis.