

Modelos gráficos dinâmicos para tomada de decisão: sistemas de especialistas e segurança alimentar no Brasil

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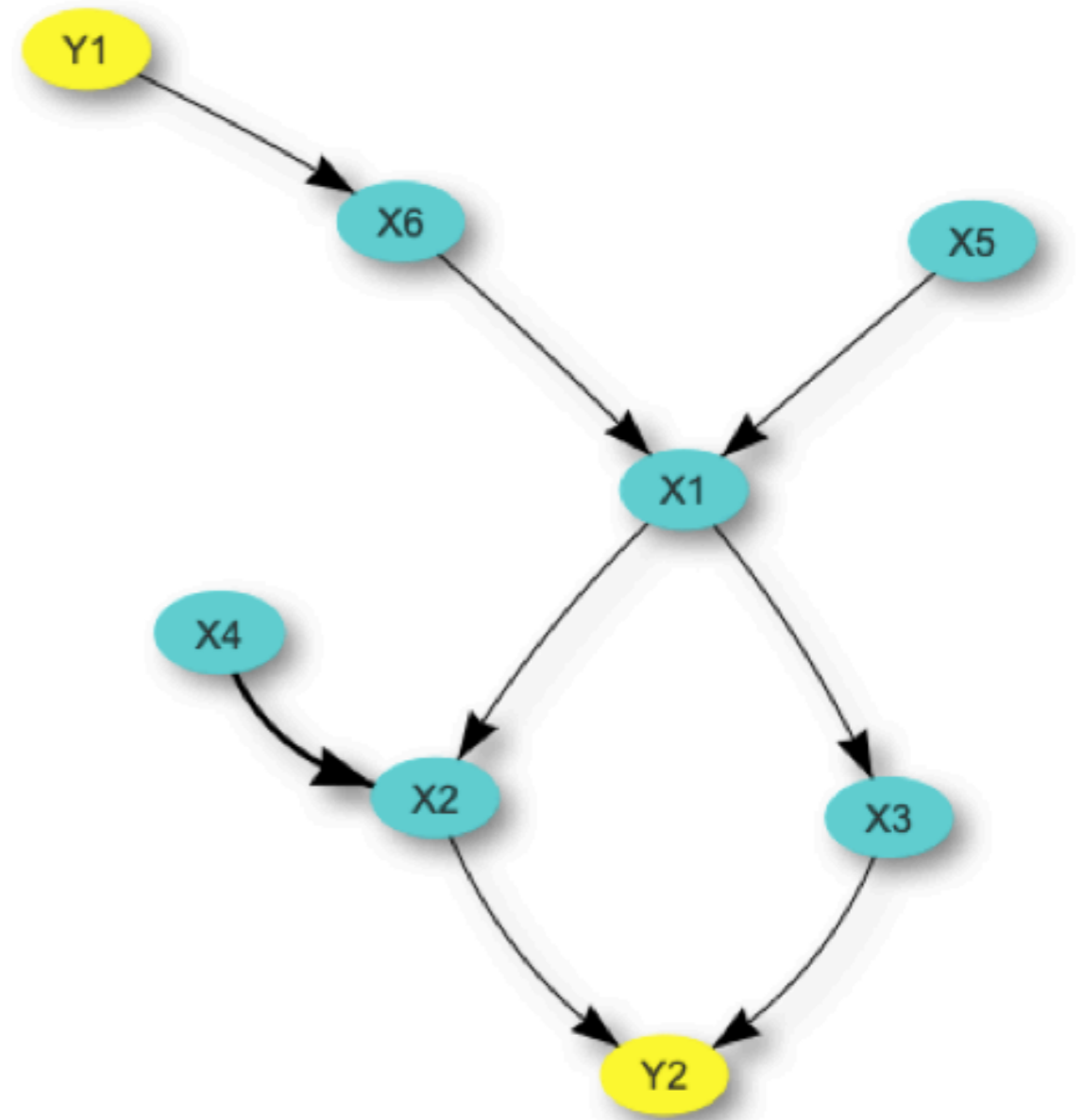
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Outline

- Expert systems and decisions
- Brief introduction to decision support systems based on graphical models
- Probabilistic graphical modeling: divide and conquer
- The food security problem
- The food network construction and policy comparisons
- Some future steps (the Brazilian case and Dirichlet evolution)



Expert system and AI

Expert systems

- A system that performs intellectually demanding tasks is often said to exhibit **artificial intelligence (AI)**.
- If the system depend on an ability which is restricted to a particular area of expertise it will be called **expert system**.
- By formulating the expert's knowledge in an appropriate **formal (computer) language**, the reasoning conducted by the expert can be carried out by a computer.
- Thus, AI can be used to automate tasks to support belief updating aiding **decision making under uncertainty**;
- Note that here we are not interested in replacing the experts with AI, we want to build **a support decision tool** to aid the decision making in complex problems.

Source: Bayesian Networks and Influence Diagrams: A Guide to Construction and Analysis, Springer.

Simple example

- Consider a physician who is consulted by a patient with stomach pain.
- The physician considers an interview with the patient, past knowledge about cause–effect mechanisms involving stomach pain, the patient’s medical records, to **make a diagnosis and a treatment plan**.
- The physician’s knowledge can be translated to a **computer language**, and his reasoning to reach a conclusion can be carried out by a computer.
- A **Probabilistic network** is an example of such a language that has gained popularity over the last couple of decades.

Uncertainty in Expert Systems

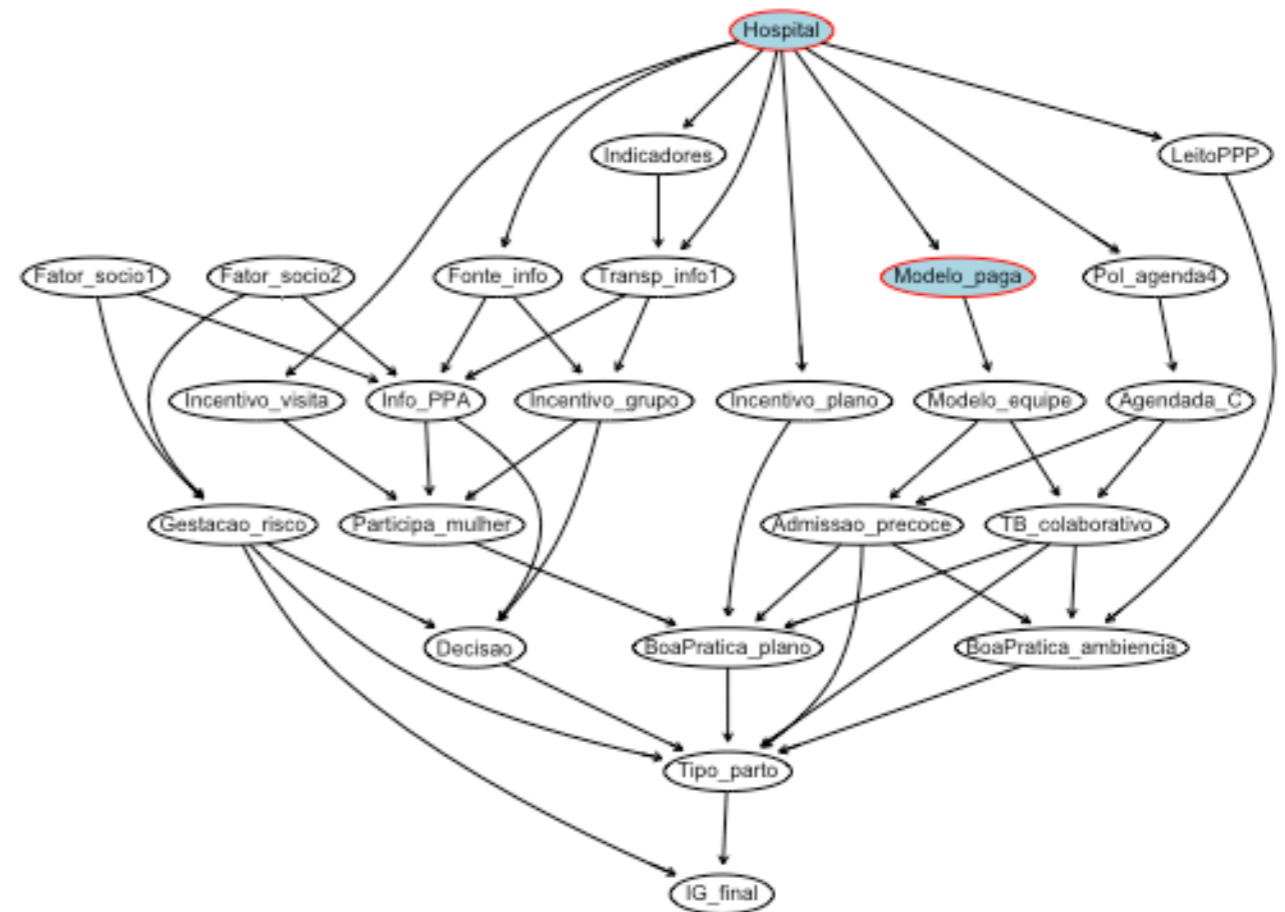
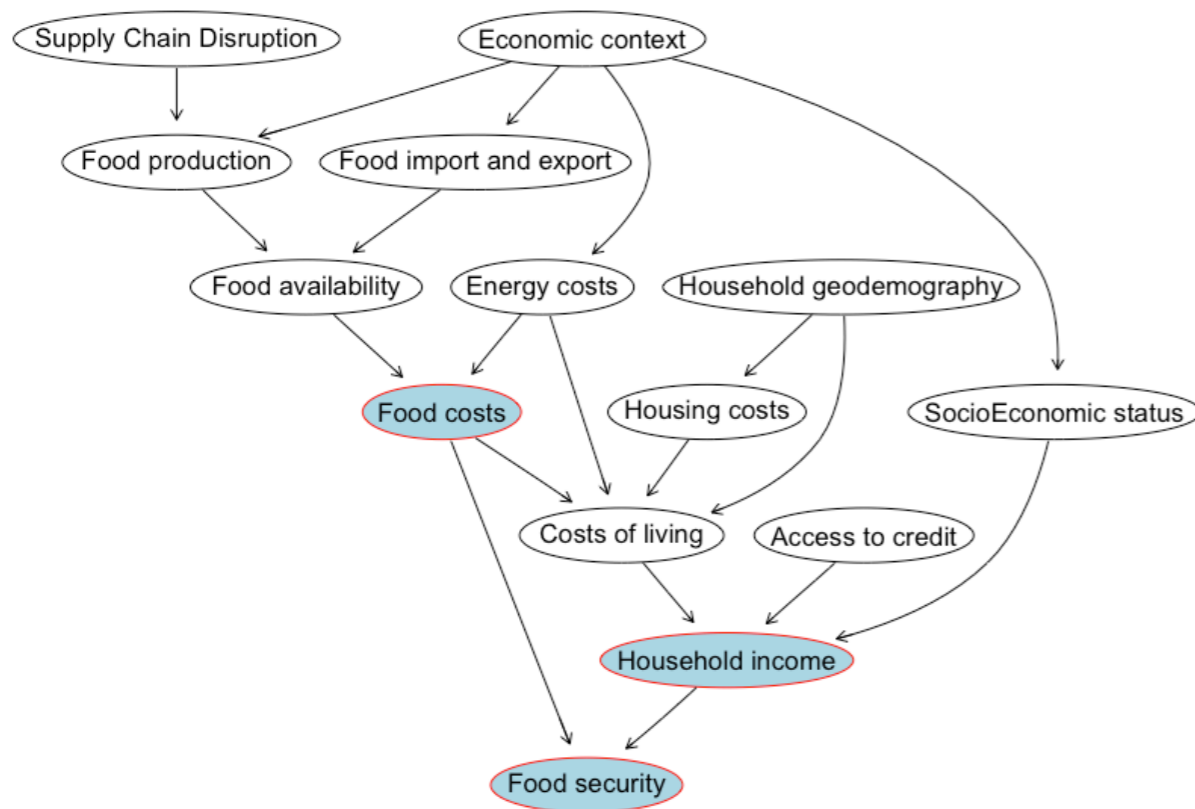
- Consider the medical expert system and cause-effect relations:

Smoking → Bronchitis → Dyspnoea

- These rule-based systems with **certainty factors** have serious limitations!
- Note that only a proportion of the smoking patients suffer from bronchitis. And dyspnoea appears as a symptom only for some of the patients with bronchitis.
- The majority of cause–effect mechanisms of interest in our attempts to model parts of the world in expert (or AI) systems are **uncertain**.
- Thus, we focus our attention on a method based on a **probabilistic interpretation of certainty factors**, leading to the definition of **Probabilistic Bayesian networks** (Kim & Pearl 1983, Pearl 1988).

Many factors and Decision under uncertainty

- In ever-larger dynamic systems, such as the food security, it is increasingly difficult for decision makers to effectively account for all the variables within the system that may influence the outcomes of interest.



Many factors and Decision under uncertainty

- It is well known that the human brain, when faced with too many alternatives, is not able to choose the optimal option.

$$\tilde{A} = \mathit{arg\ max}_{A \in \mathcal{A}} E[U(\tilde{y}; A)]$$

- Tversky & Kahneman (1981) have shown that people usually do not make decisions that maximize their expected utility!
- Thus supporting human decisions by recommendations from **decision support systems** can improve the quality of decisions.
- In this context, **Bayesian Networks** offer a useful approach designed to accommodate uncertainties using both **expert knowledge** and **experimental data**.

Probabilistic Bayesian Networks

Probabilistic Bayesian Networks

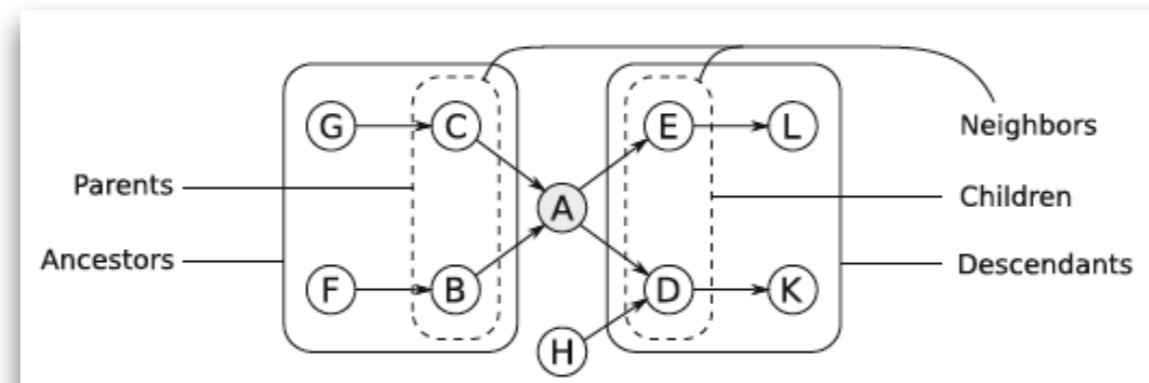
A probabilistic network model is a directed acyclic graph (DAG) with the vertices representing variables and the edges representing relations among the variables.

A graph \mathcal{G} is a mathematical object with

- nodes $V = \{v_1, \dots, v_p\}$;
- arcs A , so that $a_{ij} = (v_i, v_j)$.

A Bayesian network model will assume vertices \mathbf{Y} such that

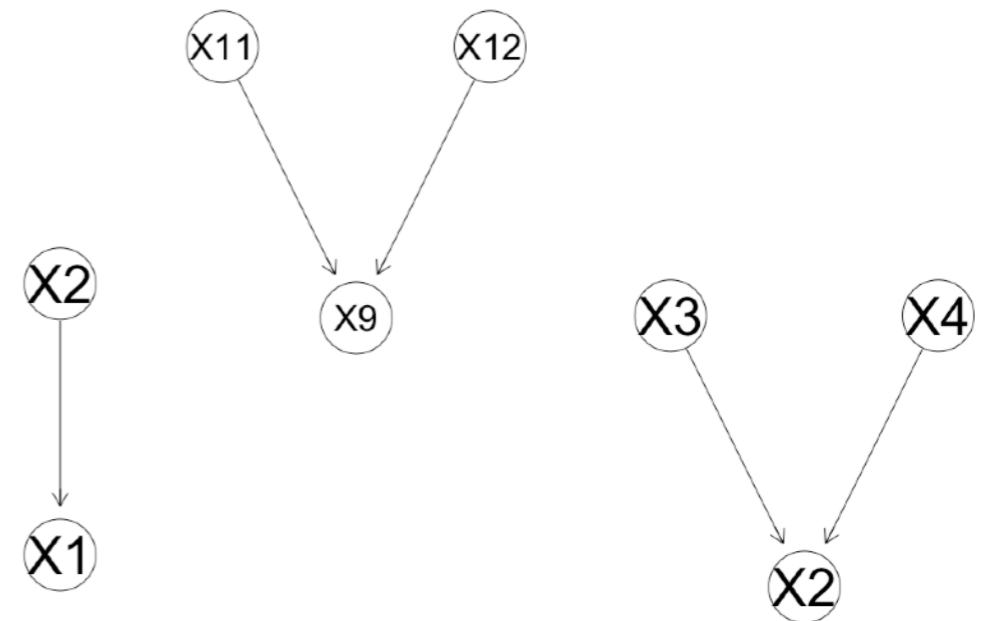
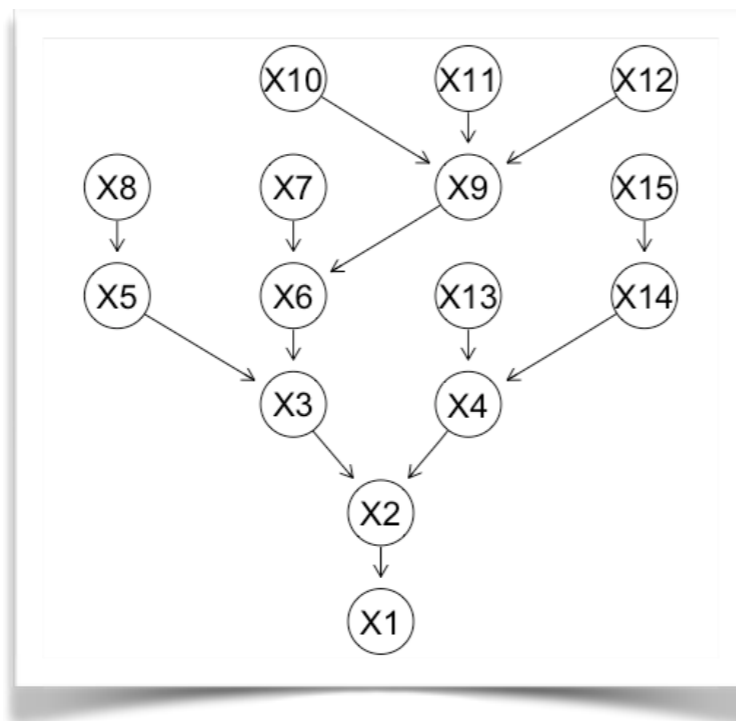
$$\mathbf{Y}_i \perp \mathbf{Y}_j \mid \mathbf{Y}_{\Pi_i}, \quad i, j = 1, \dots, p, \quad j \notin \Pi_i.$$



D is conditionally independent of E given A.

Bayesian network: divide to conquer

- BN are **graphical models** that can represent causal relations among variables.
 - *Causality is not a well-understood concept. Is a causal relation a property of the real world, or rather, is it a concept in our minds helping us to organize our perception of the world?*
 - *Bayesian networks are defined as causal networks with the strength of the causal links represented as conditional probabilities.*
- We consider the idea of conditional probabilities to divide a large multivariate problem in smaller ones based on **conditional independence**.



Bayesian network (BN)

- A Bayesian network (Pearl, 1988) expresses human-oriented qualitative structure translated into a joint probability distribution for the vector $\mathbf{Y} = (Y_1, \dots, Y_p)$.

$$f(y | \mathcal{G}, \theta) = \prod_{i=1}^p f_i(y_i | Y_{\Pi_i}, \mathcal{G}, \theta_i),$$

with Y_{Π_i} the parents of y_i .

Local Markov property

- A BN is defined by two basic elements:

A set of conditional independence statements, represented by the GRAPH.

+

A set of local conditional distributions.

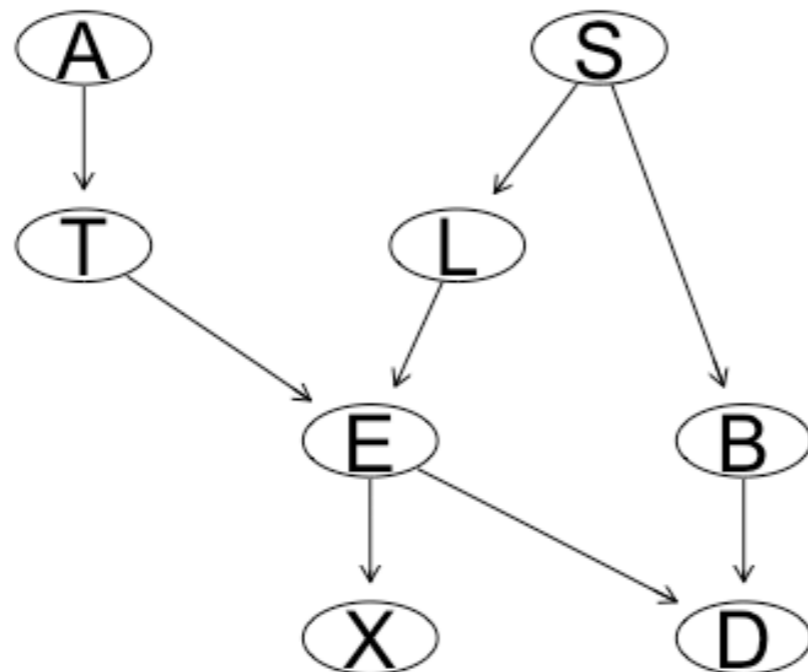
Example: Lung diseases (Lauritzen and Spiegelhalter, 1988)

- Divide to conquer: we can use **parallel computing** for large tasks and we have much less parameters.
- Example: data set from Lauritzen and Spiegelhalter (1988) about lung diseases and visits to Asia.

A: visit to Asia? S: smoking?

T: tuberculosis? L: lung cancer? B=bronchitis?

E: either T or L? X: positive X-ray? D: dyspnoea?



- Parameters in the joint model:
 $2^8 - 1 = 255$.

- Parameters in the BN model: 18

Steps of our BN framework

- (1) Construct the network structure (define all variables and connections) based on **soft elicitation**;
- (2) Estimate the state and static parameters in the Dynamic Network Model based on the observed data (**Divide and conquer strategy**);
- (3) Obtain the expected utility for each policy (**Composition sampling**);
- (4) Compare utilities which will be available to aid the decision maker.

Learning Bayesian Networks

Learning Bayesian Networks

Model selection and estimation of BNs are known as learning, and are usually performed as a two-step process:

- structure learning, estimating the graph from the data (or expert knowledge);
- parameter learning, estimating the local distribution parameters given the graph learned in the previous step.

Structure Learning

Many conditional independence tests are performed.
Computational complexity is super-exponential in the number of nodes (p) in the worst case.

- **Structure learning:** identifying the graph of the BN. It should be the minimal map of the dependence structure of the data.
- **Causal inference:** Learning the structure is useful, because we can use structure to infer causal relationships, and consequently predict the effects of interventions in the outcome of interest.
- **Possible routes:** constraint-based, score-based, hybrid algorithms or elicitation based on expert knowledge.

Soft elicitation

- The *facilitated modelling*: analysts, problem owners and experts meet in workshops to ‘solve’ the problem.
 - What are the processes, inputs, outputs, actors etc
 - How do these entities interact?
 - What are the uncertainties?
 - How might these be modelled?
 - What relevant data and expertise are available?

Parameter learning

- The use of local conditional distributions alleviates the curse of dimensionality.
- The three most common choices for local distributions are
 - Discrete BN: $Y_i \mid Y_{\Pi_i}$ is Multinomial;
 - Continuous BN: $Y_i \mid Y_{\Pi_i}$ is Gaussian;
 - Hybrid BN: $Y_i \mid Y_{\Pi_i}$ is a mixture of Gaussian distributions for each level of a discrete valued parent;

Parameter learning with temporal dynamics

- In DBNs the time slices are connected through temporal links to form the full model which accommodates dependencies within and between time slices.
- Consider the general setting such that

$$\mathbf{Y}_{it} \perp \mathbf{Y}_{Q_i}^t \mid \mathbf{Y}_{\Pi_i}^t, \mathbf{Y}_i^{t-1}, \quad i = 1, \dots, n,$$

with $\{\mathbf{Y}_t : t = 1, \dots, T\}$ a multivariate time series composing a DAG whose vertices are univariate processes and Π_i the index parent set of Y_{it} and $\mathbf{Y}_i^t = (Y_{i1}, \dots, Y_{it})'$ the historical data.

Each variable at time t depends on its own past series, the past series of its parents and the value of its parents at time t .

Parameter learning with temporal dynamics

- The observation and system equations are defined as a **Multiregression Dynamical Model** (Queen and Smith, 1993) and is given by

$$\begin{aligned}Y_{it} &= F_{it}\theta_{it} + \epsilon_{it}, \\ \theta_{it} &= G_{it}\theta_{i,t-1} + \omega_{it},\end{aligned}$$

with $\epsilon_{it} \sim N[0, V_{it}]$ and $\omega_{it} \sim N[0, W_{it}]$.

- Define $V_{it} = \phi_{it}^{-1}$, the variance evolution follows the gamma model given by

$$\phi_{it} \mid D_{t-1} \sim \text{Gamma}(\delta_i^* n_{i,t-1}/2, \delta_i^* d_{i,t-1}/2),$$

with $\delta_i^* \in (0, 1)$ being the discount factors.

Estimation of model parameters is based on the FFBS algorithm (Fruhwirth-Schnatter, 1994). Discount factors are selected by model comparison.

Posterior inference and prediction

- **Global independence**: parameters associated with each variable in the network are independent;

$$\Theta = \bigcup_{i=1}^P \Theta_i$$

- **Local independence**: parameters associated with each state of the parents of a variable are independent;

$$\Theta_i = \bigcup_{j=1}^{q_i} \Theta_{ij}$$

- These two assumptions together make **computation fast and scalable** to large networks.
- Utility of competing policies are compared via predictive distribution computation which is obtained by composition sampling.

Nodes are listed on topological order and simulated from the observation equations conditional on sampled states.

Approximate inference: Logic sampling

- Utility of competing policies are compared computing the predictive distribution which is approximated using the logic sampling.
- The method considers Monte Carlo simulation to obtain the marginal probability distribution of interest.
- **Basic idea**: to sample from a BN we transverse the network in topological order, visiting parents before children and generate a value of each visited node according to the conditional probability of that node.

- 1 – Order the variables in topological order $Y_{(1)} \prec \dots \prec Y_{(p)}$;
- 2 – Set $n_{\mathcal{E}}=0$ and $n_{\mathcal{E},\mathcal{Q}}=0$;
- 3 – Repeat for $m = 1, \dots, M$:
 - 3.1 – Generate $Y_{(i)}$ from $Y_{(i)} \mid Y_{\Pi(i)}$;
 - 3.2 – If \mathbf{y} includes \mathcal{E} , set $n_{\mathcal{E}} = n_{\mathcal{E}} + 1$;
 - 3.3 – If \mathbf{y} includes \mathcal{E} and \mathcal{Q} , set $n_{\mathcal{E},\mathcal{Q}} = n_{\mathcal{E},\mathcal{Q}} + 1$;
- 4 – Estimate $p(\mathbf{y}_q = \mathcal{Q} \mid \mathcal{E}, \mathcal{G}) = n_{\mathcal{E},\mathcal{Q}}/n_{\mathcal{E}}$.

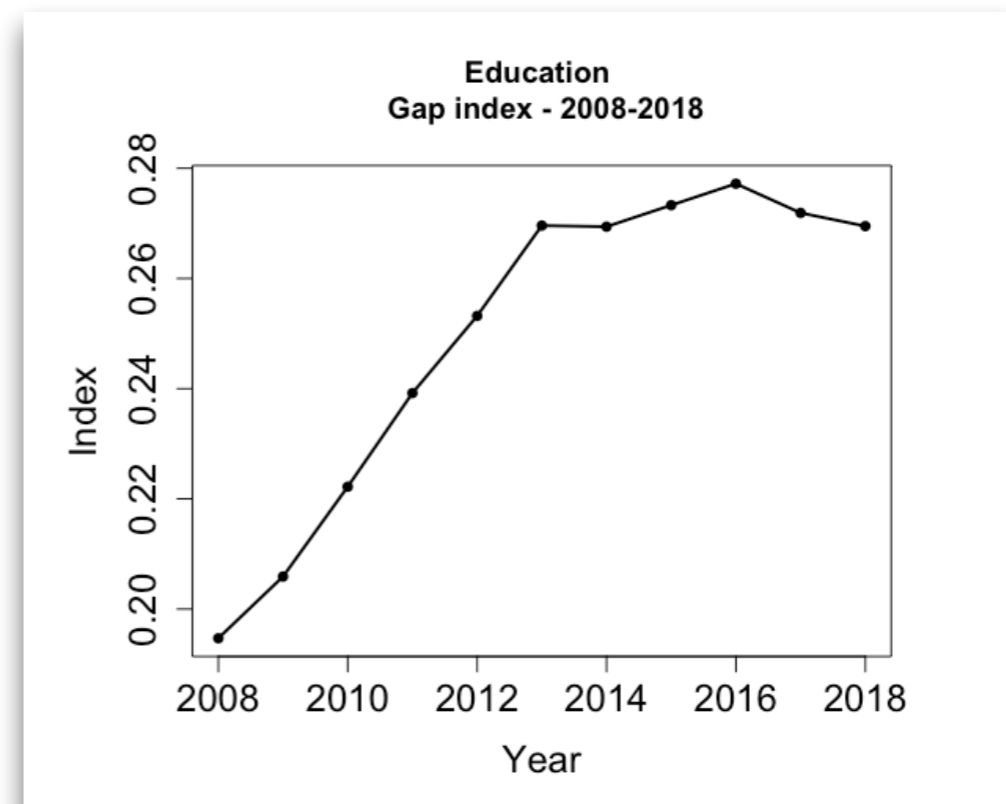
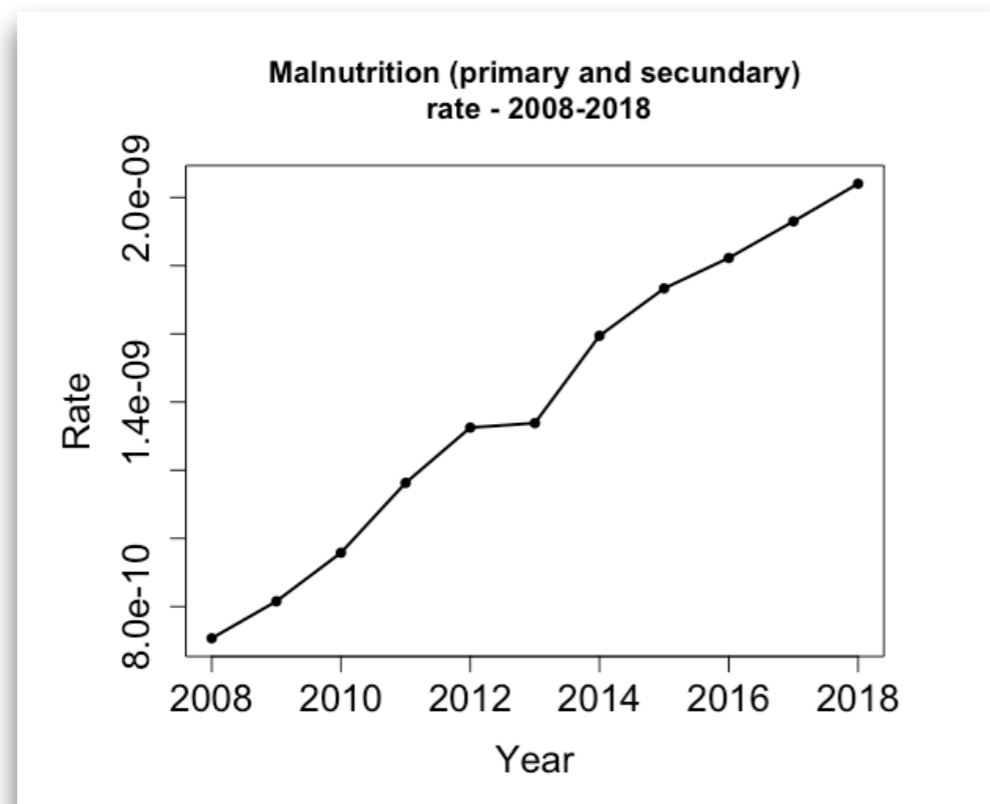
Food security

The food security problem

- Food security exists when all people, at all times, have physical and economic access to sufficient, safe and nutritious food to meet their dietary needs and food preferences for an active and healthy life (FAO, 1996).
- Missing meals and changing diet is a common response to food insecurity (Seligman et al., 2010).
- Rising food insecurity has been associated with **malnutrition**, sustained deterioration of mental health, inability to manage chronic disease, worse child health (Loopstra et al., 2015a; Loopstra,2014) and it has been found to **affect school children's academic performance**, weight gain, and social skills (Faught et al., 2017).
- In this study, we consider as the main outputs of interest **malnutrition** and **school performance** of children receiving free meals.

Food security in the UK

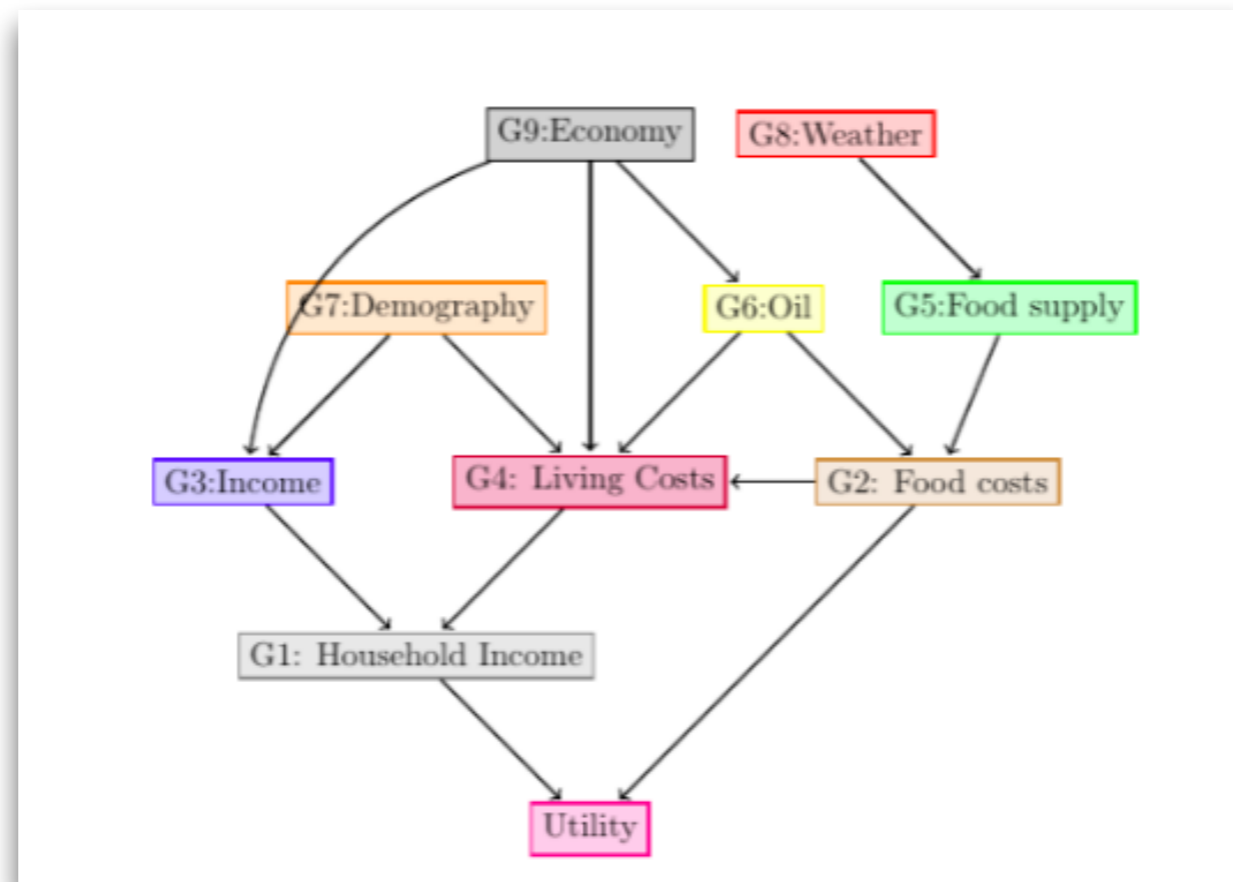
- Health - the count of finished admission episodes with a primary or secondary diagnosis of malnutrition coded ICD-10.
- Education - the gap index measuring the differences between the disadvantaged and non-disadvantaged groups in Key stage 2 and 4 (Hill, 2014).



Factors related to food insecurity

- Policymakers are legally and morally obligated to act, but often require decision support from different areas of expertise.
- In particular, government policies on welfare, farming, the environment, employment, health, etc. all have an impact on food security at various levels and by various routes.
- Each of the influencing variables are **dynamic sub-systems** within domain expertise, many supported by sophisticated probabilistic models.

Expert panels



Within the food system, examples of these are medium to long range weather, which influences food supply forecast using **large numerical models**, and the behaviour of global markets and prices under various plausible scenarios modelled with economic models such as **autoregressive or moving average**.

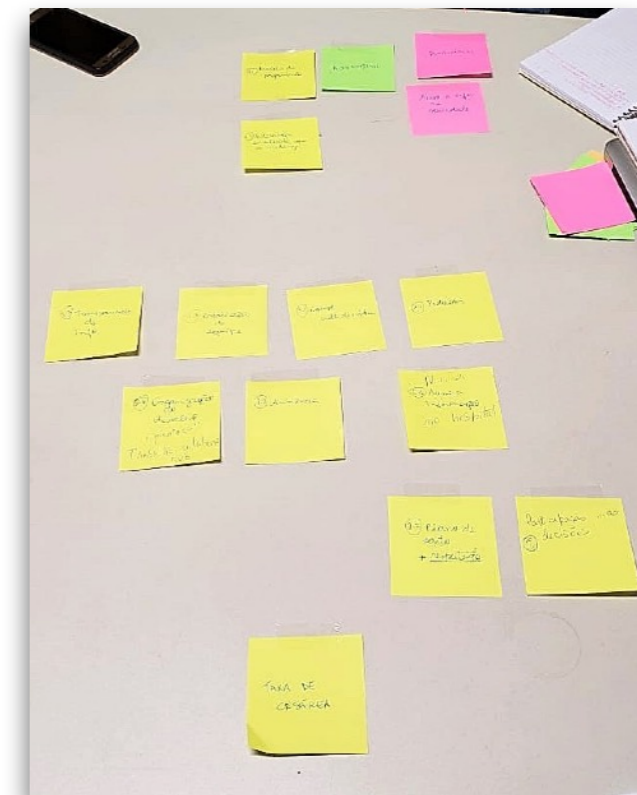
This work proposes an integrating decision support system (IDSS) (Smith et al., 2015; Barons et al., 2018), designed to provide decision support for household food security in the UK.

Expert panel in elicitation workshop and meetings

What is an **expert**? According to O'Hagan et al 2006, it is someone who has great knowledge of the subject matter.

Experts in elicitation workshop: Delegates from the Warwickshire Council (public health, legal & governance, data and statistics, renewable energy, social & financial inclusion, localities & partnerships, child poverty, education, emergency planning, libraries & customer services, and corporate policy departments).

These delegates were engaged in what can be called joint model building or **soft elicitation** (Wilkerson, 2021; French, 2021).

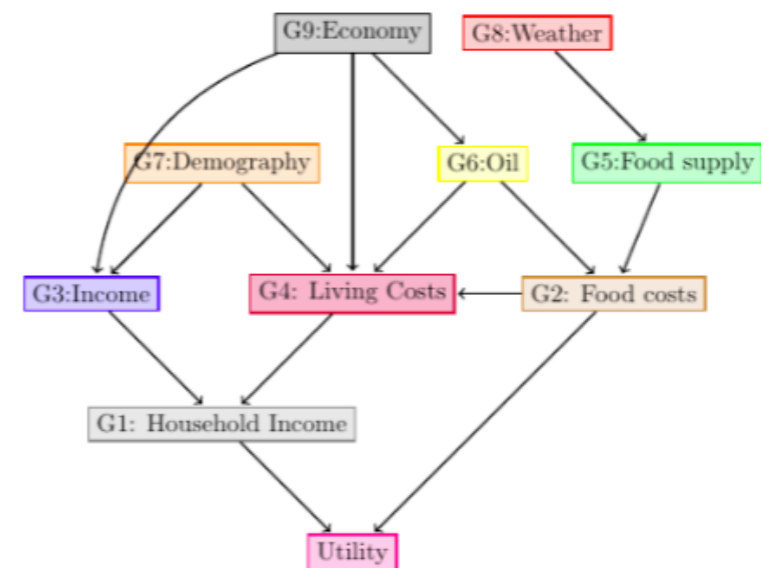


Variables in the expert panels

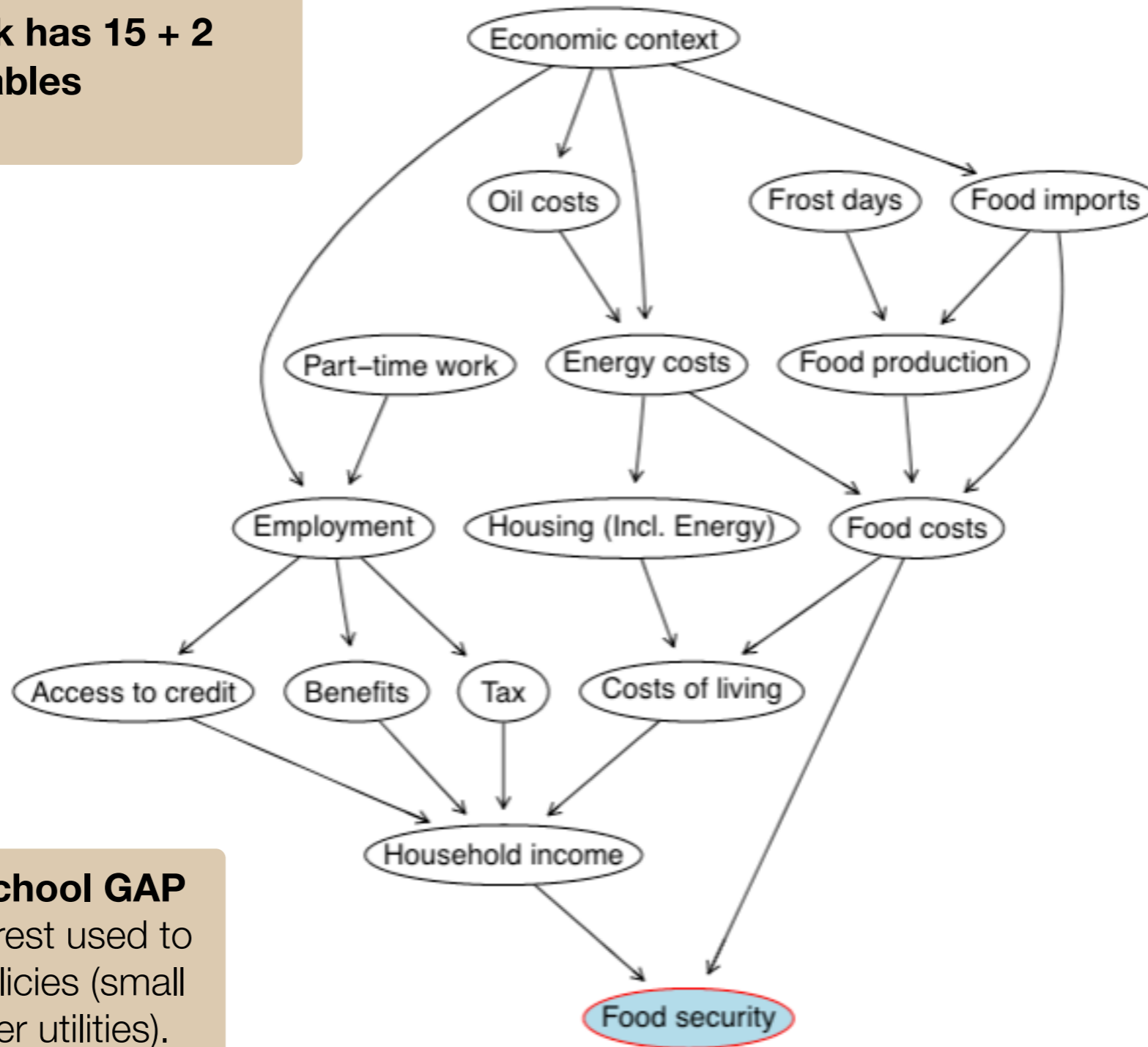
- Panel G1 (household income) - HIncome: Real net households adjusted disposable income per capita less the final consumption expenditure per head.
- Panel G2 (food costs) - CFood: CPI index of 9 food groups (cereals, meat, fish, eggs, milk, oils and fat, fruits, vegetables and beverages).
- Panel G3 (income) - Lending: Net lending (+)/net borrowing (-) by sector as a percentage of GDP - Household and non-profit institution serving households; Tax: Original household income minus post-tax income (deflated to 2018 index). Income has been equalised using the modified-OECD scale; Unemployment: Male unemployment rate, aged 16 and over, seasonally adjusted; Benefits: Social assistance benefits in cash as a percentage of GDP.
- Panel G4 (costs of living) - CLiving: Consumer price indices of the main variables composing the expenditures of a household including energy (CHousing), food (CFood), recreation (CRecreation), and transport (CTransport).
- Panel G5 (food supply) - FProduction: Producer price inflation (Output of food products); FImports: Food imports from European Union countries plus imports from other countries.
- Panel G6 (Oil costs- COil: Liquid fuels, vehicle fuels and lubricants; CEnergy: CPI of energy, 2015=100.
- Panel G7 (Demography) - PartTime: Part-time workers (Ill or disabled);
- Panel G8 (Weather) - Frost: Number of days of air frost.
- Panel G9 (Economy) - GDP: Gross Domestic Product at market prices

Food groups necessary for dietary diversity (G Kennedy et al., 2010)

9 panels and 15 variables



The network has 15 + 2 variables



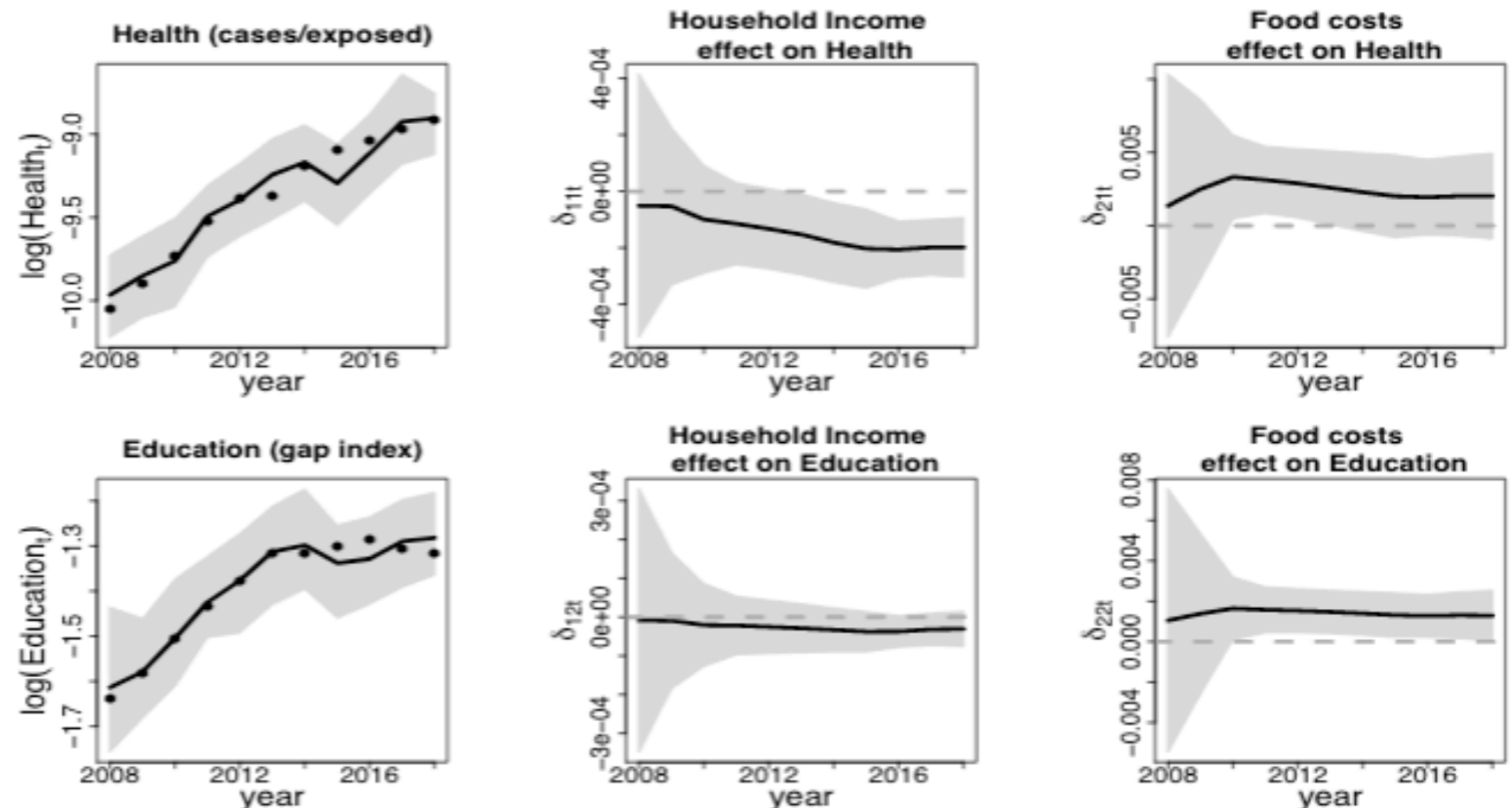
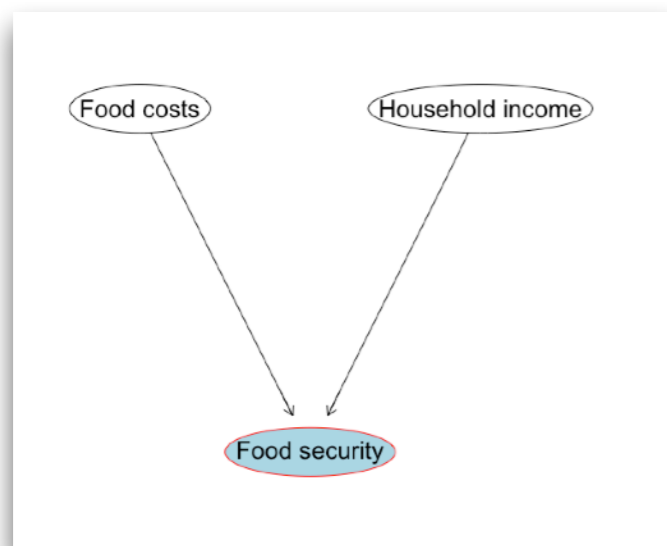
Malnutrition and **School GAP** are the nodes of interest used to compute utility of policies (small values result in better utilities).

Food security network

Qualitative system representation

Sub-network analysis

Consider the Food security variable and its parents (Household income and Food costs).



The dynamical model for this sub-network is

$$FoodSecurity_{1t} = \theta_{01,t} + \theta_{11,t}HIncome_t + \theta_{21,t}CFood_t + \epsilon_{1t}$$

$$FoodSecurity_{2t} = \theta_{01,t} + \theta_{12,t}HIncome_t + \theta_{22,t}CFood_t + \epsilon_{2t}$$

Some examples of sub-networks

- Disposable household income model:

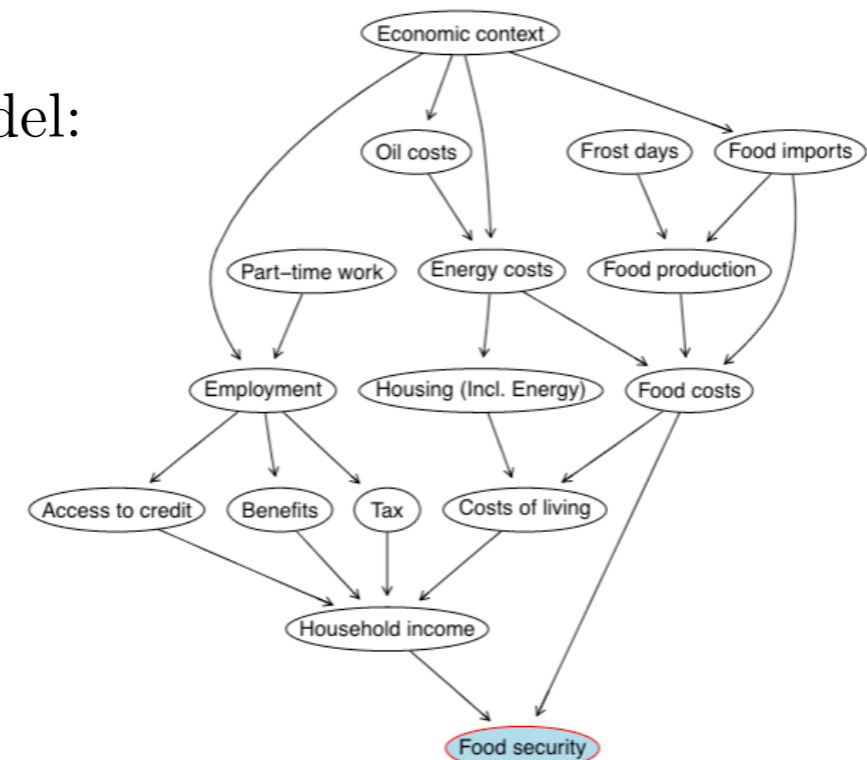
$$HIncome_t = \theta_{01,t} + \theta_{11,t}Lending_t + \theta_{21,t}Tax_t + \theta_{31,t}Benefits_t + \theta_{41,t}CLiving_t + \epsilon_{1t}.$$

- Costs of living model:

$$\begin{aligned} Cliving_t &= \theta_{04,t} + \theta_{14,t}CFood_t + \theta_{24,t}CHousing_t + \epsilon_{4t}, \\ CHousing_t &= \theta_{04,t}^* + \theta_{14,t}^*CEnergy_t + \epsilon_{4t}^*. \end{aligned}$$

- Gross domestic product (economic context) model:

$$GDP_t = \theta_{09,t} + \epsilon_{9t}.$$



MDM fit

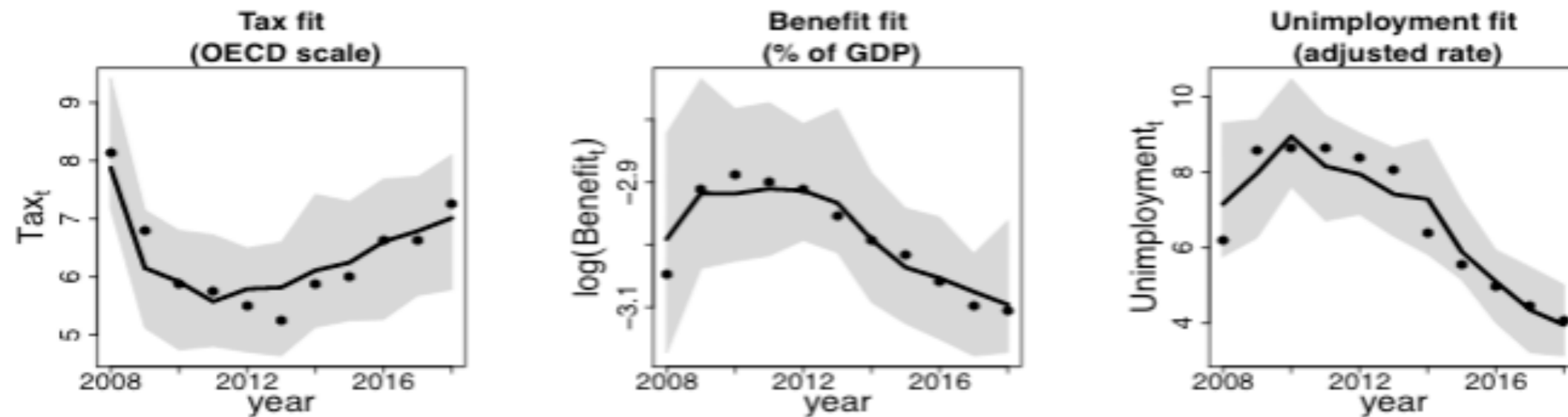


Figure presents the MDM fit for some variables in the food security network.

Note that effects are not constant over time, indicating the need for dynamic means to account for nonstationarities.

Comparing policies

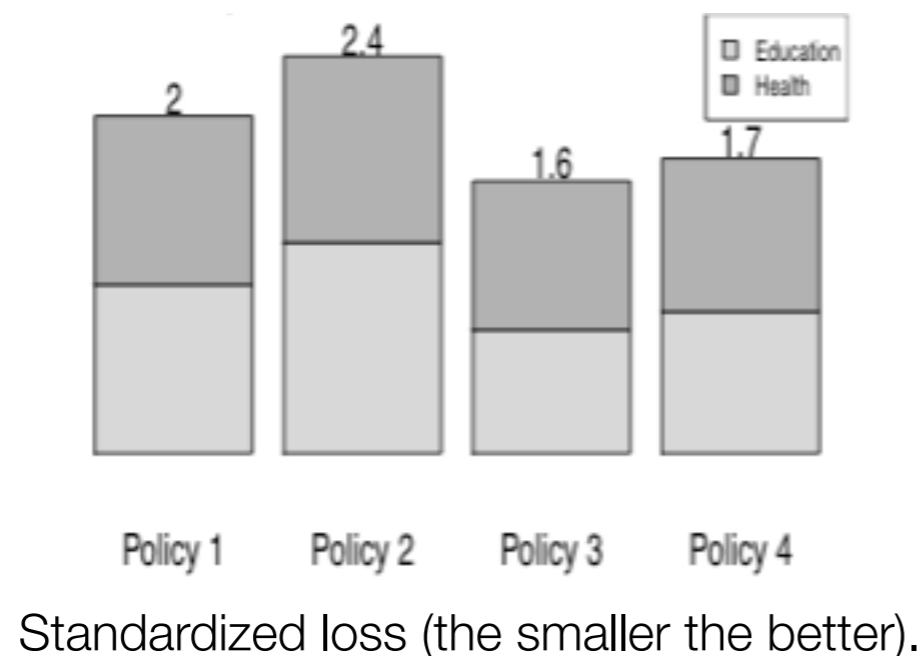
- Next we compute the marginal posterior probability of queries to evaluate the consequences of the intervention E.
- The expected loss was computed for competing policies.
- Let $Z_1 =$ gap in education and $Z_2 =$ malnutrition. It was therefore decided that one family of appropriate loss functions might take the form:

$$L(z) = \sum_{i=1}^2 1 - \exp(-c_i z_i),$$

where (c_1, c_2) were elicited parameters.

Malnutrition and Gap
0 => Loss 0;
Large Malnutrition
and/or Gap => Large
Loss

Comparing policies (Posterior predictive median of Loss function)



We see that Policy 3 gives the lowest loss.

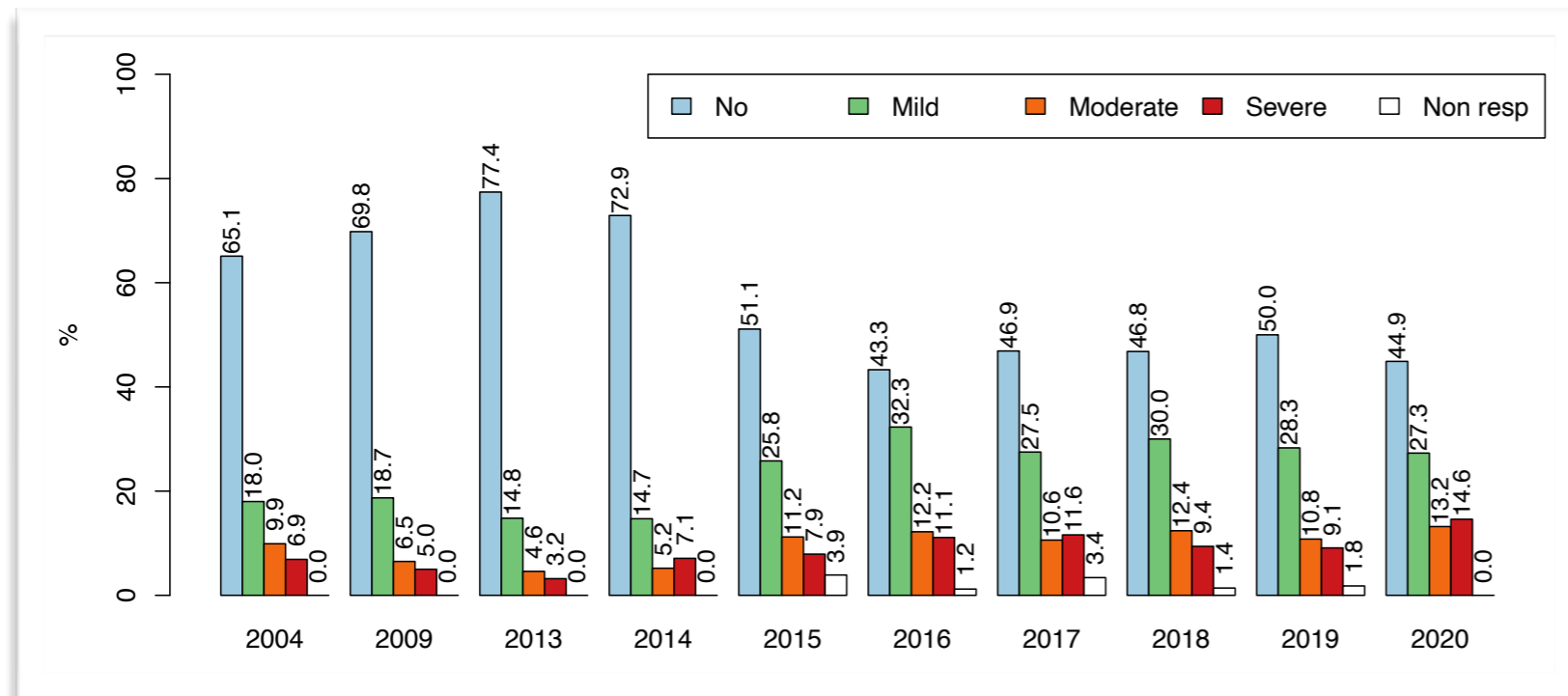
The impact of increases in food cost can be measured by policy 2.

- Policy 1: 'do nothing', i.e. all variables kept at the baseline observed values.
- Policy 2: accounts for an increase of 25% in food costs driven by economic or political policy, such as Brexit (Barons and Aspinall, 2020).
- Policy 3 represents a subsidy policy leading to a decrease of 25% in food costs.
- Policy 4 is a compound economic, welfare and incentive policy leading to a 15% reduction in food prices plus an increase in household income by 15%.

Food security - Brazilian case

Food security in Brazil

- Food security measure (EBIA): it consists of 14 questions related to the direct experience of food insecurity. Score: 0 = food security; 1 to 3 = mild food insecurity; 4 to 5 = moderate food insecurity; and 6 to 8 = severe food insecurity.
- Data: **Brazilian National Household Sample Survey (PNAD, 2004, 2009, 2013), FAO (2014, 2015, 2016, 2017, 2018, 2019), POF (2018), Food for Justice (2020), Vigisan/Penssan (2020).**



PNAD: Pesquisa Nacional por Amostra de Domicílios

POF: Pesquisa de Orçamentos Familiares

FIES: Food Insecurity Experienced Scale

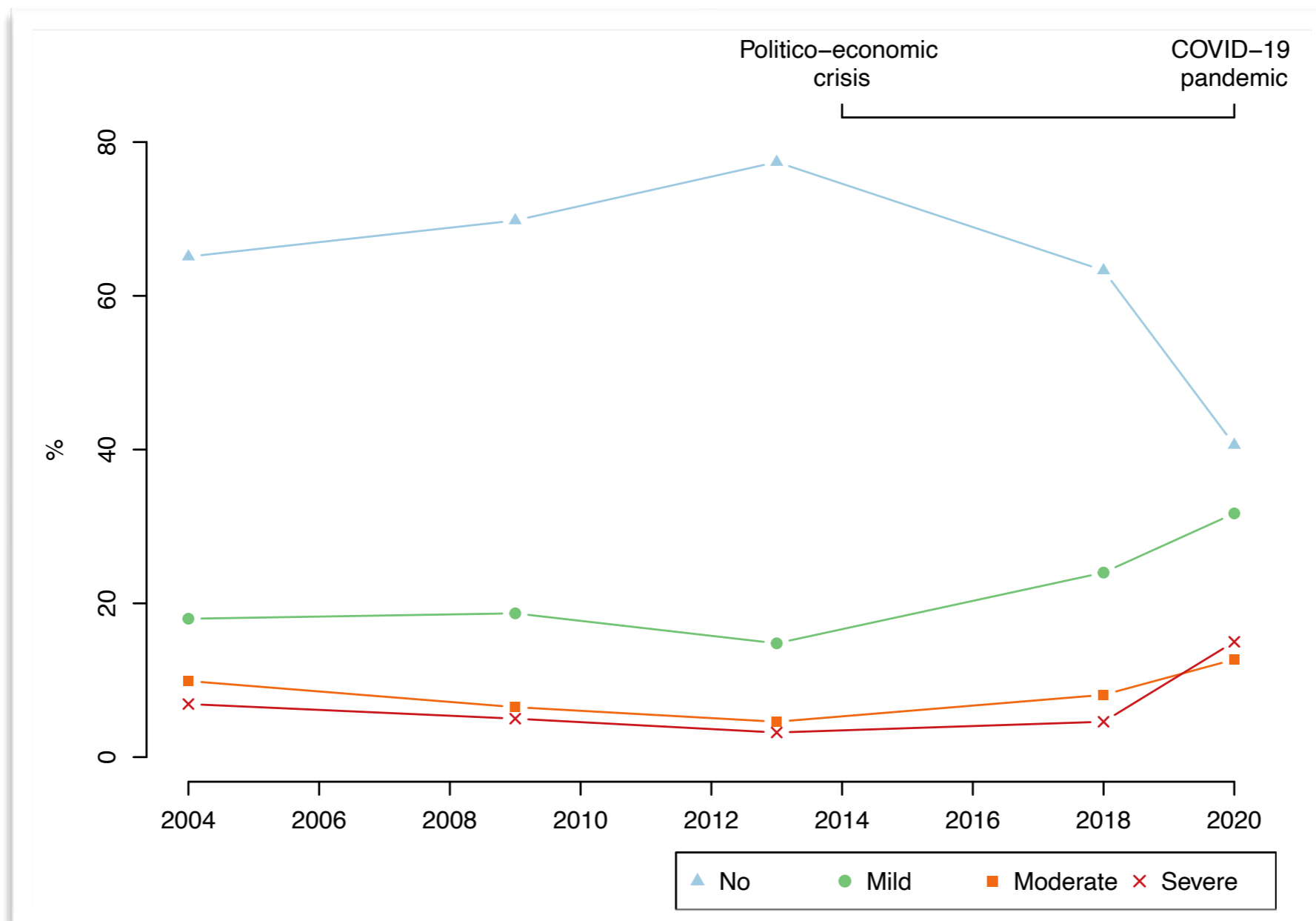
FAO: United Nations' Food and Agriculture Organization

Vigisan/Penssan: Rede Brasileira de Pesquisa em Soberania e Segurança Alimentar (Rede PENSSAN), como parte do projeto VigiSAN (Vigilância da Segurança Alimentar e Nutricional)

Joint work with Luiz Eduardo S. Gomes (PhD student, IM, UFRJ).

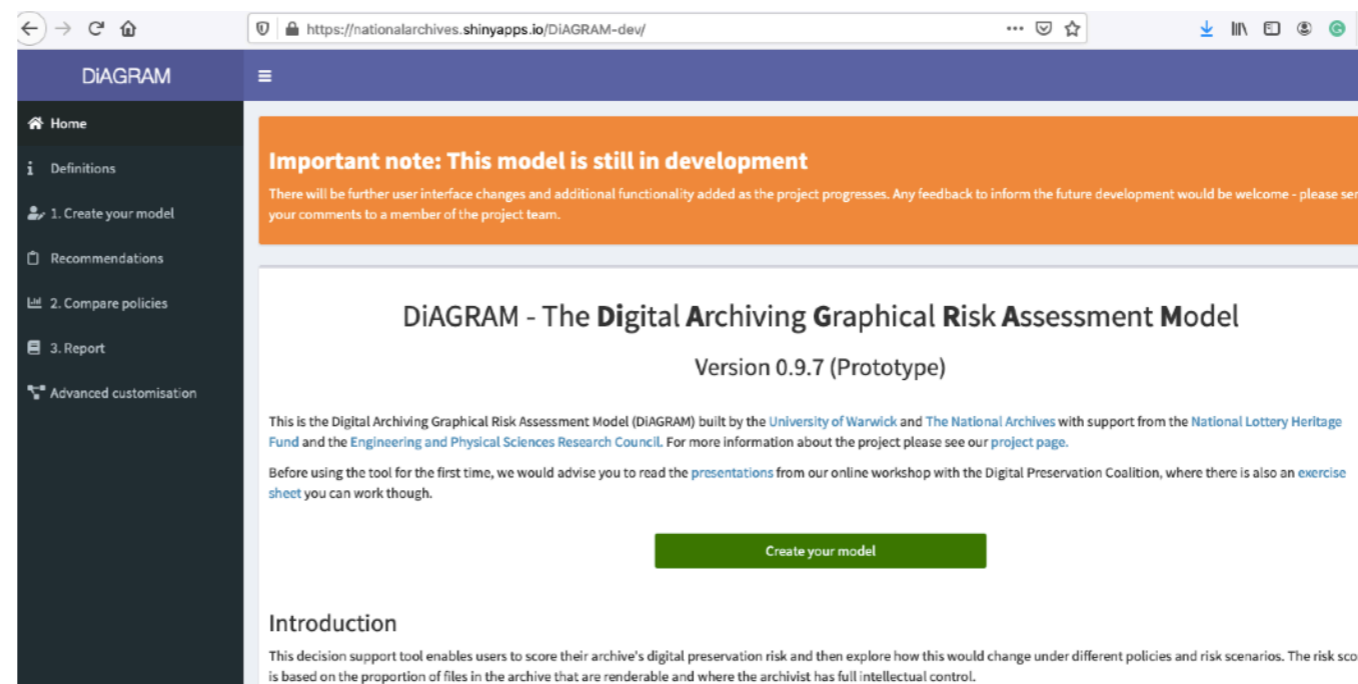
Food security in Brazil

- It has been observed a decrease in the food security levels from 2013.



Food security in Brazil

- Even with a better distribution of income, and consequently lower malnutrition at the bottom, average consumption would not grow by much as it depends on food availability and production, population growth etc. *Source: International Institute of Sustainable Development.*
- The *Guaranteed Price Policy* is the main Brazilian agricultural policy for achieving food security. Other policies: Brazil's Zero Hunger Program ('Programa Fome Zero', 2003), and More Food Program (Programa Mais Alimento, 2010).
- The proposed model aims to measure the impact of these policies to support decisions about future policies.



Challenges

- The time series are irregularly spaced in time from different sources.
- The construction of the BN graph should be adapted for the Brazilian case.
- The variables that need to be measured to compose the network are absent in some of the surveys.
- The time series of food security is a categorical variable.

Dirichlet evolution in time for the probability of food insecurity

- In the case of **discrete data**,

$$Y_{it} \mid Y_{\Pi_i} = j \sim \text{Multinomial}(M_{ij}, \theta_{ijt})$$

- We extend the static model of Heckerman (1995) to account for estimation of probabilities evolving smoothly over time, allowing for detection of **change of regimes, sustainability of policies** etc.

$$\mathbf{Y}_{it} \perp \mathbf{Y}_{Q_i}^t \mid \mathbf{Y}_{\Pi_i}^t, \mathbf{Y}_i^{t-1}, i = 1, \dots, p,$$

$$\theta_{ijt} = \frac{1}{S_{ijt}} \psi_{ijkt} \odot \theta_{ij,t-1}, \psi_{ijkt} \mid \delta_{ij} \sim \text{Beta}(\delta_{ij} a_{ijk,t-1}, (1 - \delta_{ij}) a_{ijk,t-1})$$

Our Dirichlet evolution leads to a conjugate-analysis-based forward filter and backward sampler that is computationally fast and scalable

T.C.O. Fonseca and M.A.R. Ferreira (2017) Dynamic Multiscale Spatiotemporal Models for Poisson Data (2017), Journal of the American Statistical Association 112:517, 215-234

Dirichlet evolution in time: filtering

Assume $\boldsymbol{\theta}_{ij0|\mathcal{D}_0} \sim \text{Dirichlet}(\mathbf{a}_{ij0})$ and consider the Dirichlet evolution coefficient δ_{ij} . Then, for $t = 1, 2, \dots, T$

1. Posterior for $\boldsymbol{\theta}_{ij,t-1} \mid \mathcal{D}_{t-1}, \delta_{ij} \sim \text{Dirichlet}(\mathbf{a}_{ij,t-1})$;
2. Prior for $\boldsymbol{\theta}_{ij,t} \mid \mathcal{D}_{t-1}, \delta_{ij} \sim \text{Dirichlet}(\mathbf{a}_{ij,t|t-1})$ where $\mathbf{a}_{ij,t|t-1} = \delta_{ij}\mathbf{a}_{ij,t-1}$;
3. Posterior for $\boldsymbol{\theta}_{ij,t} \mid \mathcal{D}_t, \delta_{ij} \sim \text{Dirichlet}(\mathbf{a}_{ij,t})$ where $\mathbf{a}_{ij,t} = \delta_{ij}\mathbf{a}_{ij,t-1} + \mathbf{n}_{ij,t}$, $n_{ij,t}$ the counts of \mathbf{Y}_{it} such that $\mathbf{Y}_{\Pi_i} = j$.

Dirichlet evolution in time: smoothing

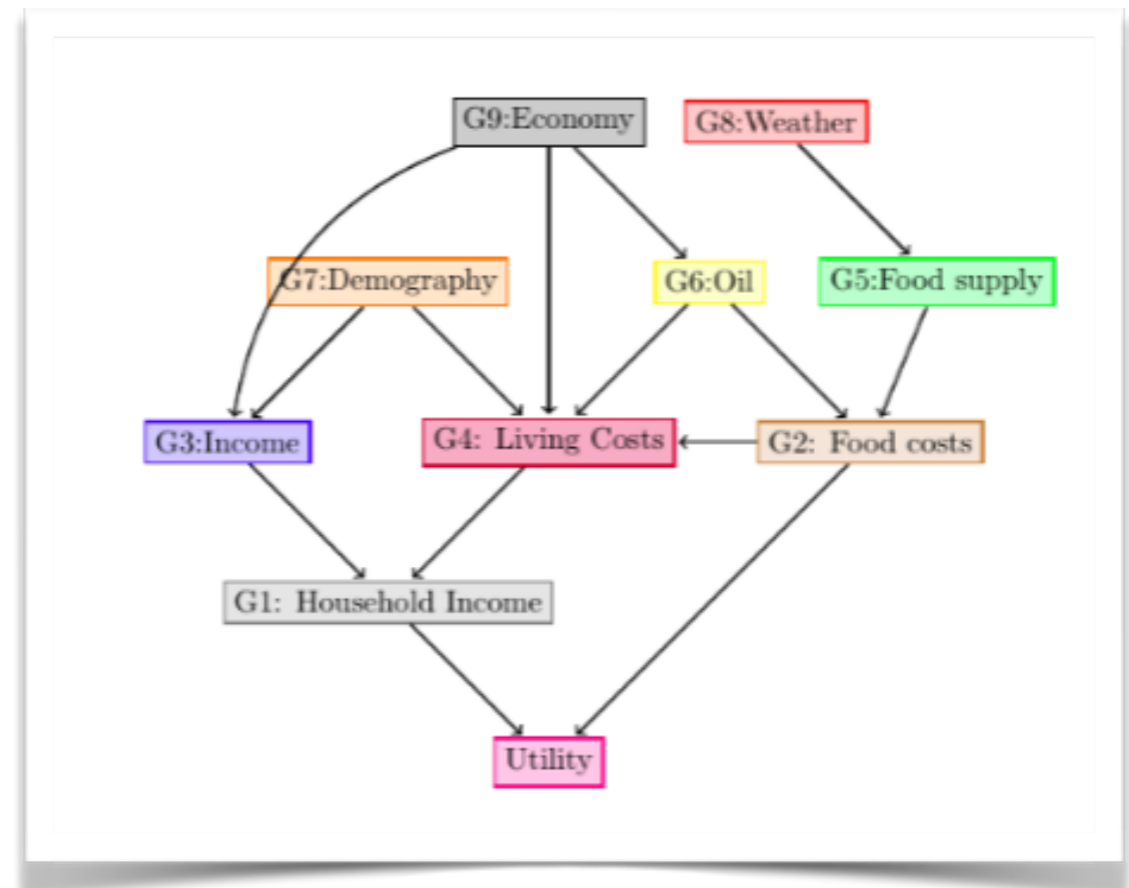
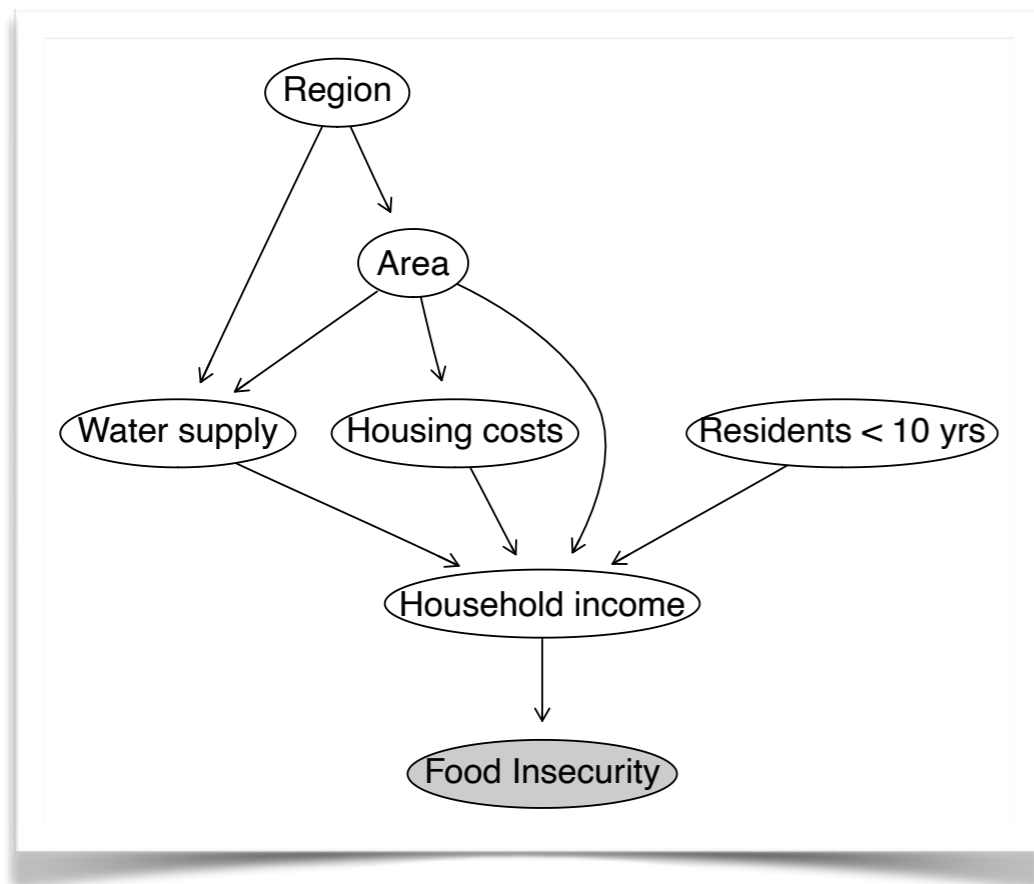
The temporal smoothing (full posterior) for $\boldsymbol{\theta}_{ij,t}$ given the Dirichlet evolution coefficient δ_{ij} is

1. $\boldsymbol{\theta}_{ij,t-1} \mid \mathcal{D}_T, S_{ij,t-1}, \boldsymbol{\theta}_{ij,t}, \delta_{ij} \sim \text{Mod-Dirichlet}((1-\delta_{ij})\mathbf{a}_{ij,t-1}, S_{ij,t-1}\boldsymbol{\theta}_{ij,t});$
2. $S_{ij,t-1} \mid \mathcal{D}_T, \boldsymbol{\theta}_{ij,t}, \delta_{ij} \sim \text{Beta}(\delta_{ij}\tilde{a}_{ij,t-1}, (1-\delta_{ij})\tilde{a}_{ij,t-1}), \tilde{a}_{ij,t-1} = \sum_k a_{ijk,t-1}.$

Full posterior obtained by sampling from Mod-Dirichlet or inference using first analytical moments.

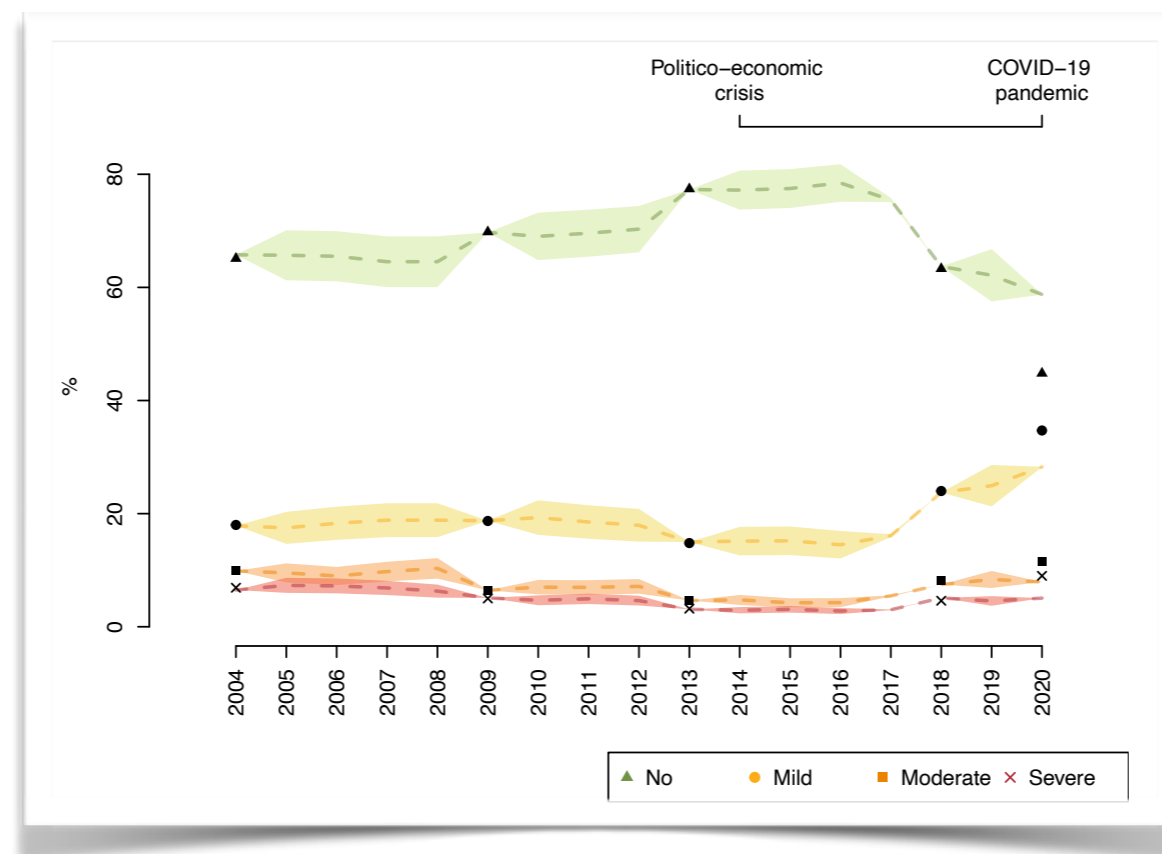
Adapting the graph

- Simple initial model: it has income and water supply representing the household conditions; It includes the region, area (rural and urban) and residents <10 years old to represent demographic features.
- Future work: we will to include economy, food availability, food costs, and weather variables in the model.



Marginal probability of food security over time

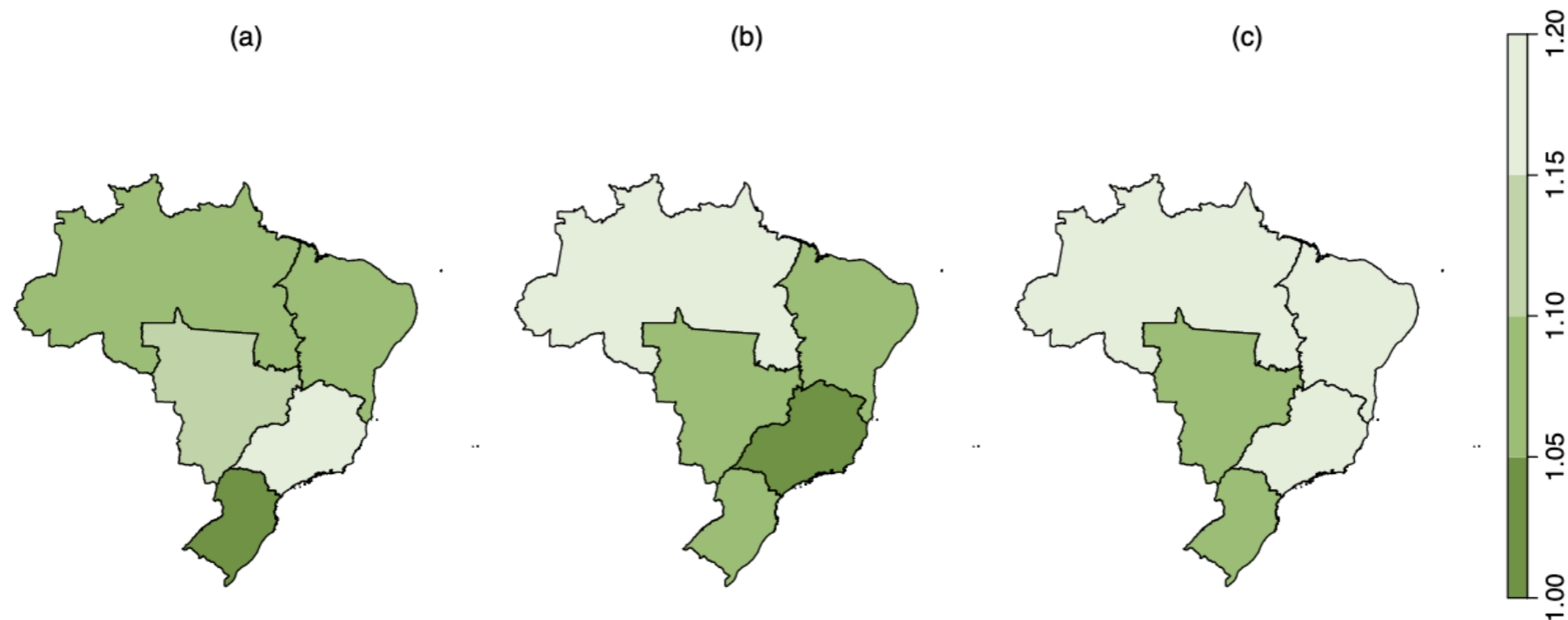
- Here we considered two sets of data for model fitting: PNAD+POF+F4J and PNAD+FIES.



- From 2004 to 2013, there was a progressive increase in food security. There was a decrease in food security after the politico-economic crisis in 2013.
- Note the abrupt change for 2020, which was not captured by our model.

Effect of water supply

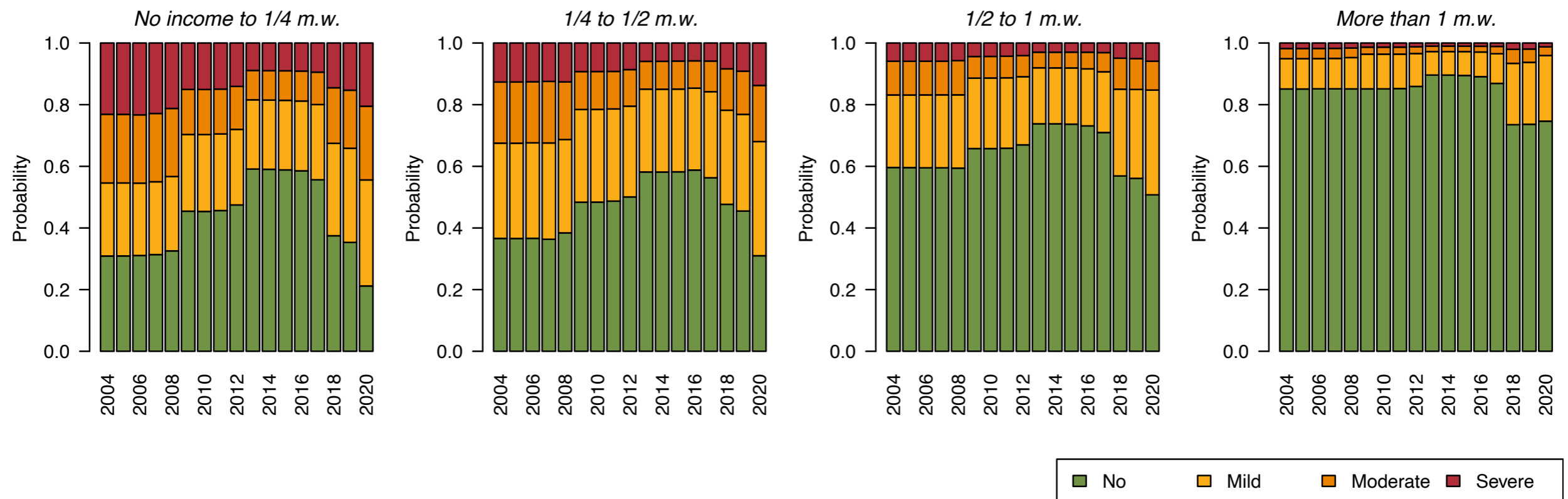
- Relative risk of **food security** in the presence and absence of water supply for (a) 2010, (b) 2015, (c) 2020.



- Lighter colors indicate larger gaps between who has and who does not have water supply. This gap has increased over the years.

Conditional probabilities over time

- For $<1/2$ m.w. the pandemic impact was much larger with increase of severe and mild food insecurity.



Policy evaluation

- Scenario simulation was used to evaluate illustrative policies such as income transfer to household.

Policy	% of food-secure households	
	2019	2020
P1: do nothing	60.8 (60.8, 60.9)	45.8 (45.7, 45.8)
P2: decrease housing costs (income <1/2 m.w.)	61.8 (61.7, 61.8)	46.8 (46.7, 46.9)
P3: Income transfer (600 reais for income <1/2 m.w.)	62.7 (62.6, 62.8)	47.7 (47.6, 47.7)

- Income transfer has larger impact than decrease of housing costs.

Resources

- A decision support system for addressing food security in the UK - <https://arxiv.org/abs/2004.06764>
- Code: <https://github.com/thaiscofonseca/foodnetwork>
- App for the DiAGRAM - The Digital Archiving Graphical Risk Assessment Model (BN analysis in digital preservation) - <https://nationalarchives.shinyapps.io/DiAGRAM> Funder: National Lottery Heritage Fund and the Engineering and Physical Sciences Research Council.
- The Digital Archiving Graphical Risk Assessment Model (UFMG talk, August 2021) - <https://www.youtube.com/watch?v=kmJ70KdioNw>
- Safeguarding the Nation's Digital Memory: Towards a Bayesian Model of Digital Preservation Risk (2021) M J Barons, S Bhatia, T C O Fonseca, A Green, S Krol, H Merwood, A Mulinder, S Ranade, J Q Smith, T Thornhill, D H Underdown Archives and Records, 42:1, 58-78.
- BN analysis for the Brazilian Birth Care System - <https://www.youtube.com/watch?v=xJINfq3q3Fg> - Funder: GCRF Accelerator Account Fund, 2019/2020, UK and the Bill & Melinda Gates Foundation (PPA project).

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