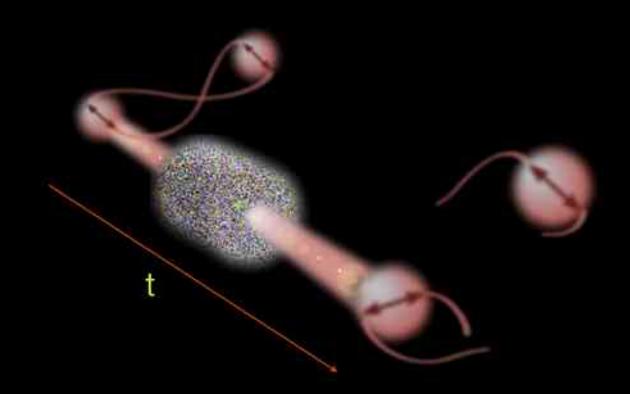
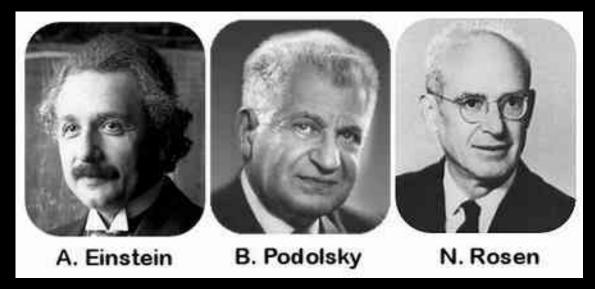
Open system dynamics of entanglement





Fernando de Melo qig@CBPF



Can quantum-mechanical description of physical reality be considered complete? Phys. Rev. 47, 777 (1935)

... the way of "paradoxes"

Best possible knowledge of a whole does not include best possible knowledge of its parts — and that is what keeps coming back to haunt us

Erwin Schrödinger (1935)

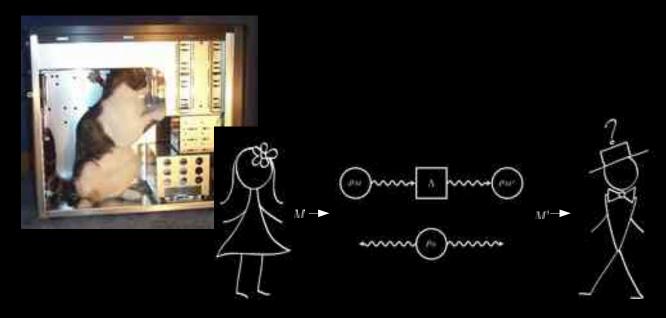


"On the Einstein-Podolsky-Rosen Paradox" Physics 1, 195 (1964)

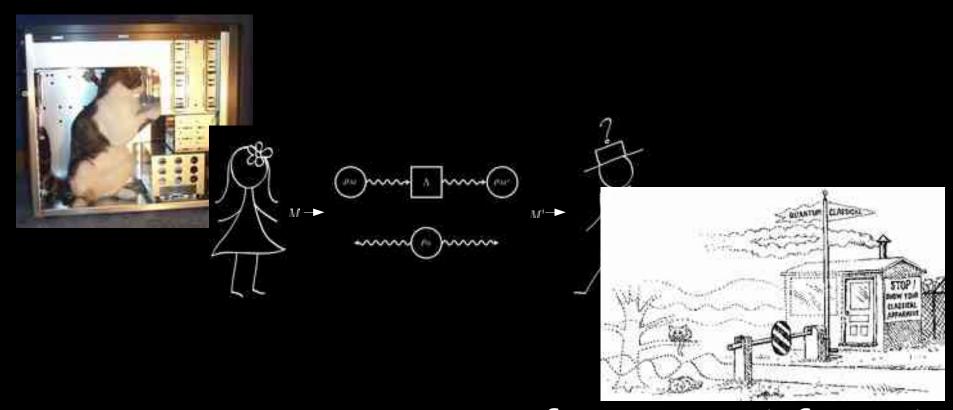
... correlation with physical consequences



... a resource for quantum information



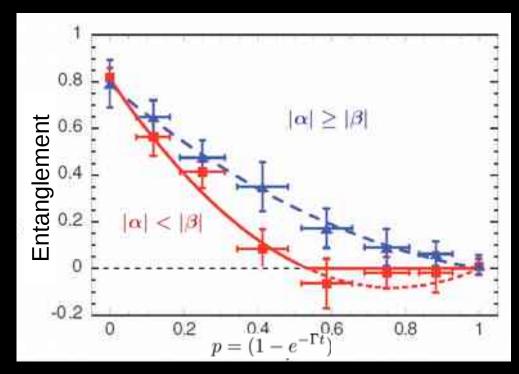
... a resource for quantum information



... a resource for quantum information

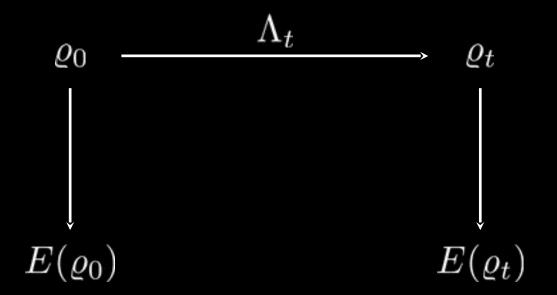
Caveat: Entanglement is fragile



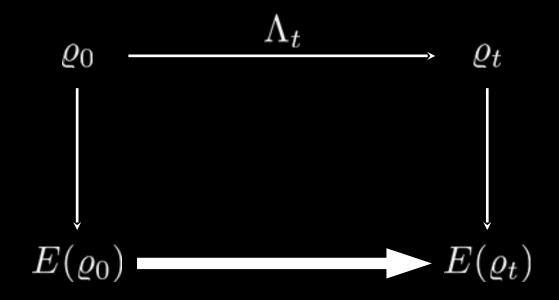


M.P Almeida, FdM, M. Hor-Meyll, A. Salles, S.P. Walborn, P.H. Ribeiro, L. Davidovich Science 316, 579 (2007)

Entanglement open system dynamics

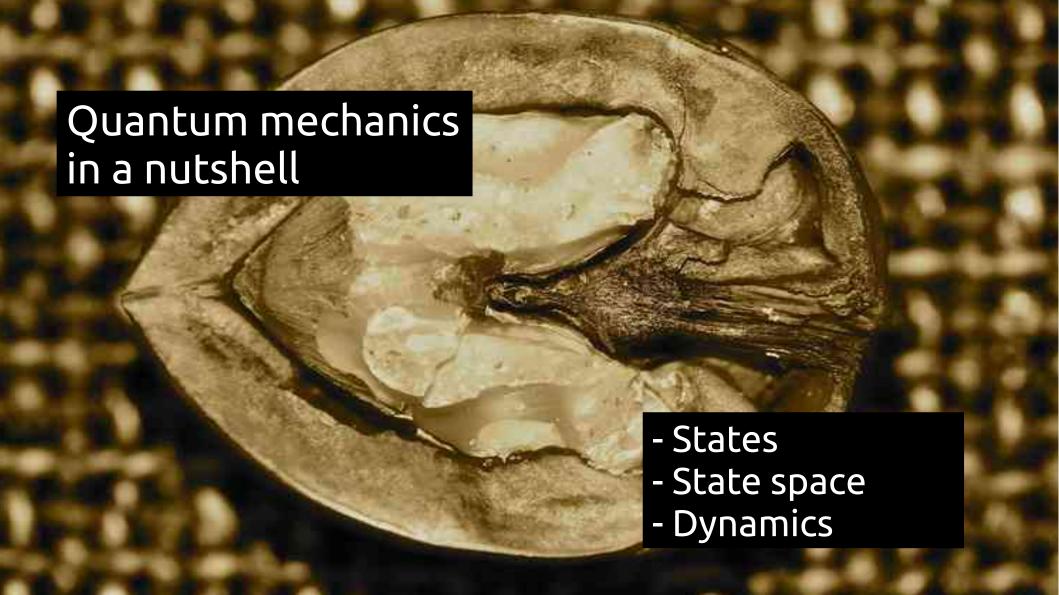


Entanglement open system dynamics



Outline

- Quantum mechanics in a nutshell
- Entanglement: definition and quantification
- Entanglement dynamics
 - Deterministic equation of motion
 - Statistical approach: a universal behavior
- Conclusions and open questions



1st postulate: To every physical system is associated a Hilbert space \mathcal{H} . The state of a quantum system is described by a unit vector $|\psi\rangle \in \mathcal{H}$.

Given an orthonormal basis $\{|e_i
angle\}_{i=0}^{d-1}\in\mathbb{C}^d$ then

$$|\psi\rangle = \sum_{i=0}^{d-1} c_i |e_i\rangle$$

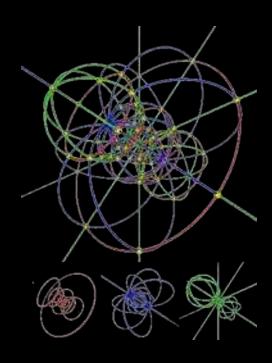
Constraints:

Normalization:

$$\sum_{i=0}^{\infty} |c_i|^2 = 1$$

Modulo global phase: $c_0 \in \mathbb{R}$

State space of $|\psi\rangle$'s : 2d-2 sphere



Consider the case where the state of the system is not exactly know, but it is in the state $|\psi_i\rangle$ with probability p_i .

The expectation value of an observable A (Hermitian linear operator), is given by:

$$\langle \mathcal{A} \rangle = \sum_{i} p_{i} \langle \psi_{i} | \left(\mathcal{A} | \psi_{i} \rangle \right)$$

The expectation value of an observable \mathcal{A} (Hermitian linear operator), is given by:

$$\langle \mathcal{A} \rangle = \sum_{i} p_{i} \langle \psi_{i} | \left(\mathcal{A} | \psi_{i} \rangle \right)$$

$$= \operatorname{Tr} \left[\left(\sum_{i} p_{i} | \psi_{i} \rangle \langle \psi_{i} | \right) \mathcal{A} \right]$$

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$$= \operatorname{Tr} \left[\left(\sum_{i} p_{i} | \psi_{i} \rangle \langle \psi_{i} | \right) \mathcal{A} \right]$$

$$= \operatorname{Tr} \rho \mathcal{A}$$

Properties of the density matrix:

- linear operator

$$\varrho = \sum_{i} p_i |\psi_i\rangle\langle\psi_i|$$

- trace one

$$\operatorname{Tr} \varrho = \sum_{i} p_{i} \operatorname{Tr} |\psi_{i}\rangle\langle\psi_{i}| = \sum_{i} p_{i} = 1$$

- Hermitian

$$\varrho^{\dagger} = \sum_{i} p_{i}^{*}(|\psi\rangle\langle\psi|)^{\dagger} = \varrho$$

- positive semi-definite $\forall |\chi\rangle \in \mathcal{H} \langle \chi|\varrho|\chi\rangle \geq 0$

1st postulate reloaded: To every physical system is associated a Hilbert space \mathcal{H} . The state of a quantum system is described by a density matrix $\varrho \in \mathcal{D}(\mathcal{H})$

The previous postulate is then a special case with the identification $\varrho_{\psi} = |\psi\rangle\langle\psi| \rightarrow |\psi\rangle$

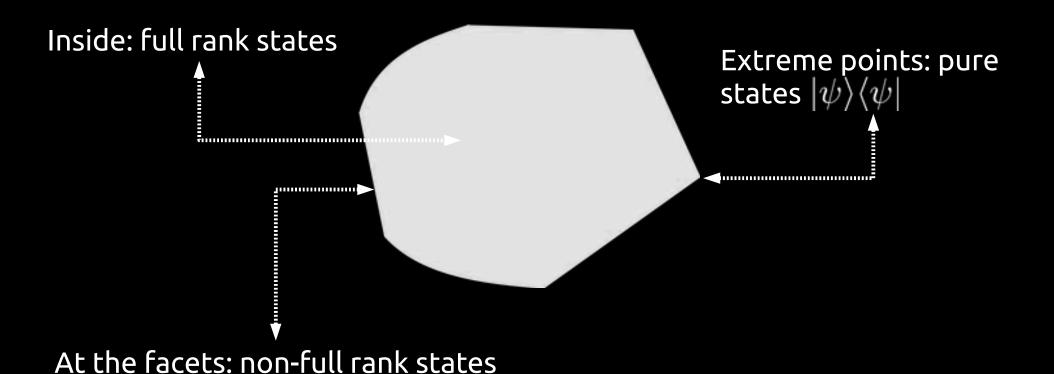
State space of ϱ 's : convex set

If ϱ_1 and ϱ_2 are density matrices, then

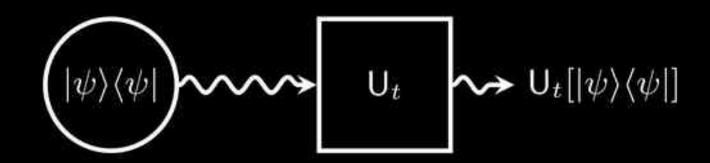
$$\lambda \varrho_1 + (1-\lambda)\varrho_2$$

with $\lambda \in [0, 1]$, is also a density matrix.

State space of ϱ 's: convex set



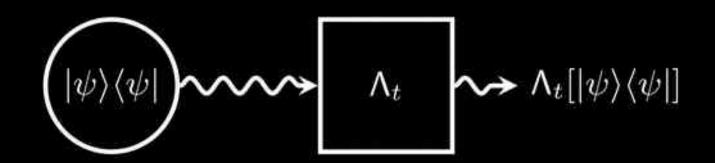
Dynamics: Closed system / Noiseless



Where
$$\mathsf{U}_t[|\psi
angle\langle\psi|]:=U_t|\psi
angle\langle\psi|U_t^\dagger$$
 , and $U_t^\dagger U_t=\mathbb{1}$

Pure states remain pure – no information is lost

Dynamics: Open system / Noisy



Where
$$\Lambda_t[|\psi\rangle\langle\psi|]:=\sum_i K_t^{(i)}|\psi\rangle\langle\psi|{K_t^{(i)}}^\dagger$$
 and $\sum_i {K_t^{(i)}}^\dagger K_t^{(i)}=\mathbb{1}$

Pure states might become mixed (not rank 1) – information might be lost

Dynamics: Open system / Noisy

The linear map $\Lambda_t : \mathcal{D}(\mathcal{H}) \mapsto \mathcal{D}(\mathcal{H})$

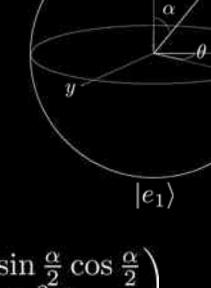
- preserves the trace
- preserves Hermiticity
- preserves positive semi-definiteness (more than that, it is completely positive)

Example: 1 qubit dephasing

Consider the state,

$$|\psi\rangle = \cos\frac{\alpha}{2}|e_0\rangle + e^{i\theta}\sin\frac{\alpha}{2}|e_1\rangle$$

Its density matrix reads



 $|e_0
angle$

$$\varrho_{\psi} = |\psi\rangle\langle\psi| = \begin{pmatrix} \cos^2\frac{\alpha}{2} & e^{-i\theta}\sin\frac{\alpha}{2}\cos\frac{\alpha}{2} \\ e^{i\theta}\sin\frac{\alpha}{2}\cos\frac{\alpha}{2} & \sin^2\frac{\alpha}{2} \end{pmatrix}$$

Example: 1 qubit dephasing

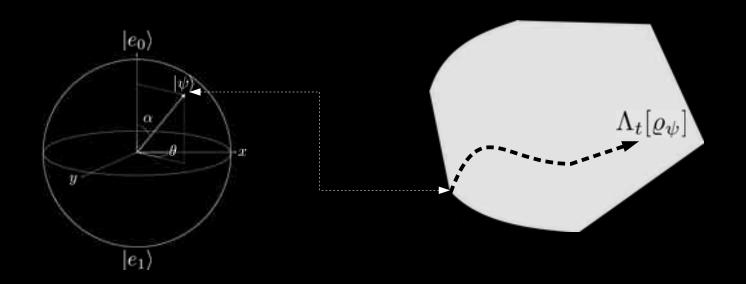
Now consider the map Λ_t with

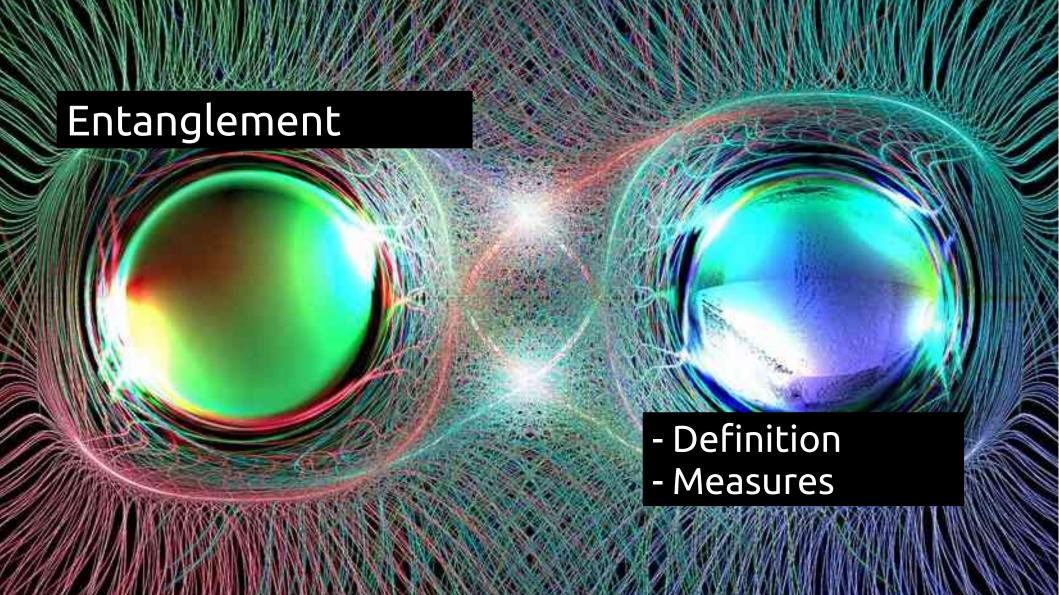
$$K_t^{(0)} = \begin{pmatrix} 1 & 0 \\ 0 & e^{-\Gamma t/2} \end{pmatrix}$$
 $K_t^{(1)} = \begin{pmatrix} 0 & 0 \\ 0 & \sqrt{1 - e^{-\Gamma t/2}} \end{pmatrix}$

with $\Gamma > 0$ is a decay rate

The dynamics is then

$$\Lambda_t[\varrho_{\psi}] = \sum_i K_t^{(i)} |\psi\rangle\langle\psi| K_t^{(i)\dagger} = \left(e^{-\Gamma t/2} e^{\mathrm{i}\theta} \sin\frac{\alpha}{2}\cos\frac{\alpha}{2} \right) \left(e^{-\Gamma t/2} e^{-\mathrm{i}\theta} \sin\frac{\alpha}{2}\cos\frac{\alpha}{2} \right)$$





Consider now a system composed of two subsystems, say A and B. It is assigned to it a composed Hilbert space $\mathcal{H} = \mathcal{H}_A \otimes \mathcal{H}_B$

But there exist states

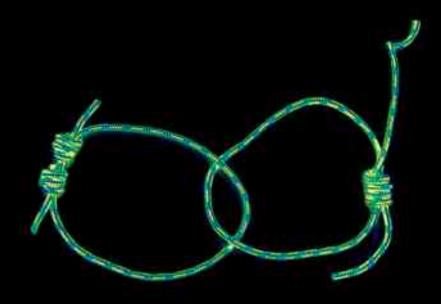
$$|\psi\rangle \in \mathscr{H} = \mathscr{H}_A \otimes \mathscr{H}_B$$

for which there exist **NO** $|\phi\rangle \in \mathcal{H}_A$ and $|\chi\rangle \in \mathcal{H}_B$, such that

$$|\psi\rangle \stackrel{!}{=} |\phi\rangle \otimes |\chi\rangle$$

These are the entangled (pure) states

Bipartite entangled state



$$|\psi\rangle = \frac{1}{\sqrt{2}}(|e_0\rangle \otimes |e_0\rangle + |e_1\rangle \otimes |e_1\rangle)$$

Tripartite entangled states

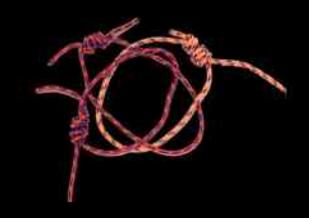


Cluster state

$$\frac{1}{2} (|e_0 e_0 e_0\rangle + |e_0 e_0 e_1\rangle + |e_0 e_1 e_0\rangle - |e_0 e_1 e_1\rangle + \\ + |e_1 e_0 e_0\rangle + |e_1 e_0 e_1\rangle - |e_1 e_1 e_0\rangle + |e_1 e_1 e_1\rangle)$$

GHZ state

$$\frac{1}{\sqrt{2}}(|e_0e_0e_0\rangle + |e_1e_1e_1\rangle)$$



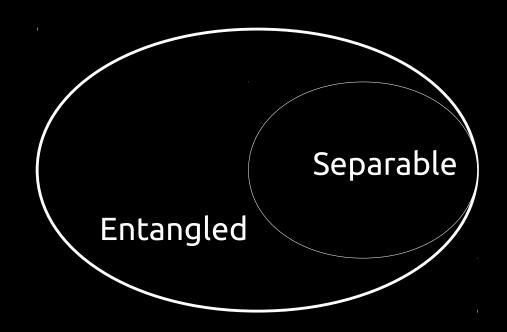
Entanglement definition

A multipartite state $\varrho \in \mathcal{D}(\mathcal{H}_A) \otimes \mathcal{D}(\mathcal{H}_B) \otimes \cdots \otimes \mathcal{D}(\mathcal{H}_N)$ is **separable** if it can be written as

$$\varrho = \sum_i p_i |\psi_A^{(i)}\rangle\!\langle\psi_A^{(i)}|\otimes|\psi_B^{(i)}\rangle\!\langle\psi_B^{(i)}|\otimes\cdots\otimes|\psi_N^{(i)}\rangle\!\langle\psi_N^{(i)}|$$
 with $p_i\geq 0$ and $\sum_i p_i=1$

The state is **entangled** otherwise

Entanglement in the space of states



Watch out: This is not a Venn diagram!

Let $E: \mathcal{D}(\mathcal{H}_A) \otimes \cdots \otimes \mathcal{D}(\mathcal{H}_N) \mapsto [0,1]$ be a tentative entanglement measure. It then must satisfy:

- $E(\varrho) = 0$ iff ϱ is separable
- $E(\lambda \varrho + (1 \lambda)\varrho') \le \lambda E(\varrho) + (1 \lambda)E(\varrho'), \ \forall \lambda \in [0, 1]$
- $|E(\varrho) E(\varrho')| \xrightarrow{||\varrho \varrho'|| \to 0} 0$
- $E(\Lambda_{LOCC}[\varrho]) \leq E(\varrho)$

Recipe to construct an entanglement measure:

- 1. Take a valid measure over pure states
- 2. Extend it over density matrices. How?

Watch out: Entanglement measures cannot be linear!

Consider a purported measure E and the entangled states:

$$|\Phi^{+}\rangle = \frac{1}{\sqrt{2}}(|e_{0}e_{0}\rangle + |e_{1}e_{1}\rangle) \qquad |\Phi^{-}\rangle = \frac{1}{\sqrt{2}}(|e_{0}e_{0}\rangle - |e_{1}e_{1}\rangle)$$

s.t.
$$E(|\Phi^+\rangle\langle\Phi^+|)=E_+>0$$
 and $E(|\Phi^-\rangle\langle\Phi^-|)=E_->0$

Watch out: Entanglement measures cannot be linear!

If linear, then

$$E\left(\frac{1}{2}|\Phi^{+}\rangle\langle\Phi^{+}| + \frac{1}{2}|\Phi^{-}\rangle\langle\Phi^{-}|\right) = \frac{1}{2}(E_{+} + E_{-}) > 0,$$

Despite of the fact that

$$\frac{1}{2}|\Phi^{+}\rangle\langle\Phi^{+}| + \frac{1}{2}|\Phi^{-}\rangle\langle\Phi^{-}| = \frac{1}{2}\left(|e_{0}\rangle\langle e_{0}| \otimes |e_{0}\rangle\langle e_{0}| + |e_{1}\rangle\langle e_{1}| \otimes |e_{1}\rangle\langle e_{1}|\right)$$

is separable!



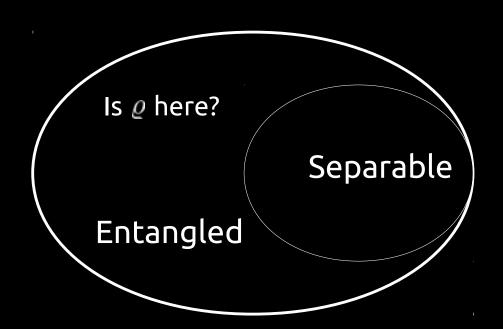
Given the state

$$\varrho = \sum_{i} p_{i} |\psi_{i}\rangle\langle\psi_{i}|$$

then

$$E(\varrho) = \min_{\{p_i, |\psi_i\rangle\}} \sum_{i} p_i E(|\psi_i\rangle)$$

Finally: given ϱ , is it entangled and how strongly?



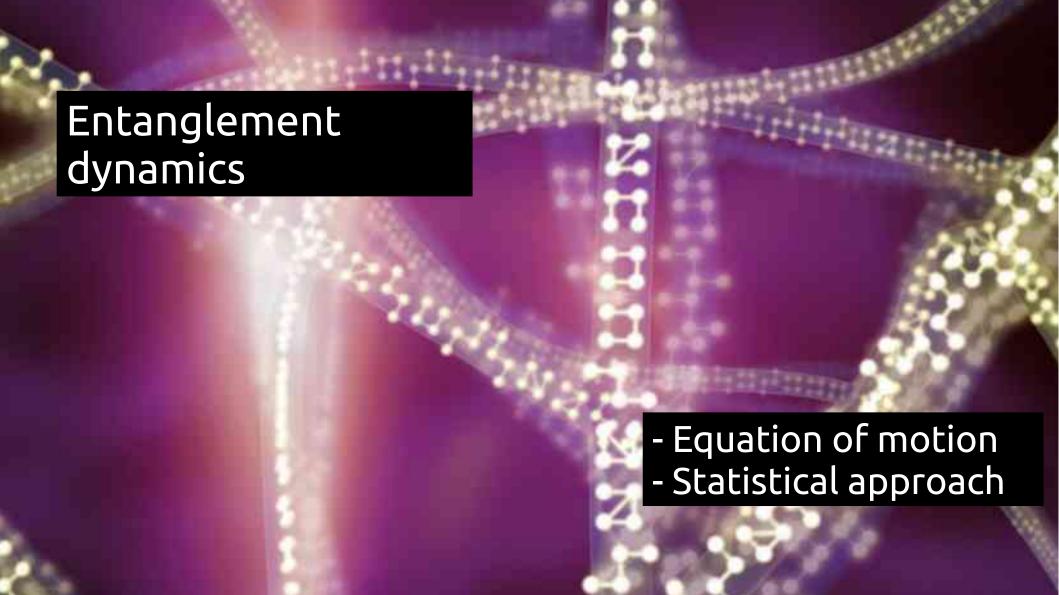
Finally: given ϱ , is it entangled and how strongly?

Is *Q* here? Separable Entangled

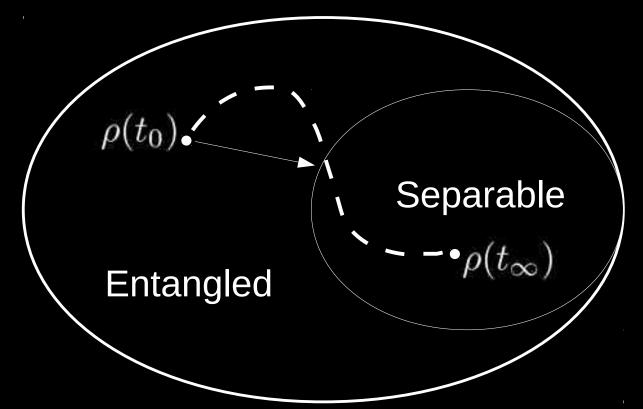
It's NP-Hard to discover that!

Only known cases: 2x2 and 2x3

L. Gurvits, In Proceedings of the thirty-fifth ACM symposium on Theory of computing, pages 10 –19, (2003), ACM Press

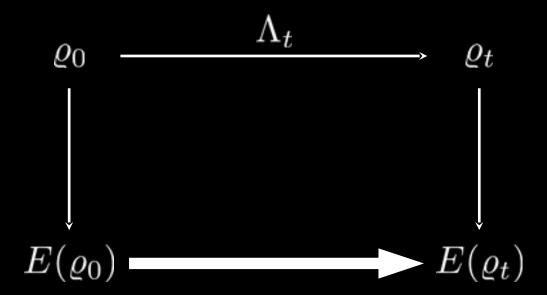


Where are we?



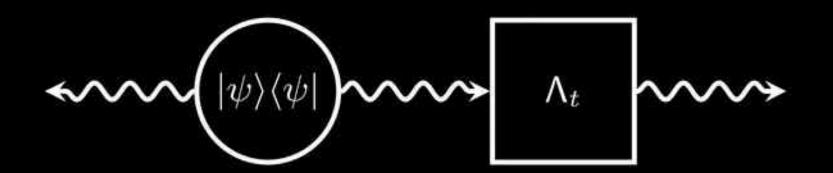
In trouble, definitely.

Dynamical approach:



Deterministic equation of motion

Bipartite states (dxd)



$$C_d\Big(\mathbb{1}\otimes \Lambda_t[|\psi\rangle\langle\psi|]\Big) = C_d\Big(|\psi\rangle\langle\psi|\Big) \ C_d\Big(\mathbb{1}\otimes \Lambda_t[|\Phi_d^+\rangle\langle\Phi_d^+|]\Big)$$

where
$$\left|\Phi_{d}^{+}\right\rangle = \frac{1}{\sqrt{d}}\sum_{i=0}^{d-1}\left|e_{i}\;e_{i}\right\rangle$$

T. Konrad, FdM, M. Tiersch, C. Kasztelan, A. Aragão, and A. Buchleitner, Nat. Phys. 4, 99 (2008)
M. Tiersch, FdM, and A. Buchleitner,

Phys. Rev. Lett. 101, 170502, (2008)

Proof ingredients

• (G-)Concurrence: determinant structure

$$|\psi\rangle = \sum_{i,j=0}^{d-1} \psi_{ij} |e_i e_j\rangle \Rightarrow C_d(|\psi\rangle) = d|\det \psi|^{2/d}$$

Jamiołkowsky isomorphism

$$|\psi\rangle = M_{\psi} \otimes \mathbb{1} |\Phi_d^+\rangle \text{ with } M_{\psi} = \sqrt{2} \sum_{i,j} \psi_{i,j} |e_i\rangle\langle e_j|$$

A. Jamiołkowsky, Rep. Math. Phys. **3**, 275 (1972)

Further results:

$$C_d\Big(\mathbb{1}\otimes \Lambda_t[\varrho_0]\Big) \leq C_d\Big(\varrho_0\Big) C_d\Big(\mathbb{1}\otimes \Lambda_t[|\Phi_d^+\rangle\langle \Phi_d^+|]\Big)$$

$$C_d\Big(\Phi_t \otimes \Lambda_t[\varrho_0]\Big) \leq C_d\Big(\varrho_0\Big) \ C_d\Big(\mathbb{1} \otimes \Lambda_t[|\Phi_d^+\rangle\langle \Phi_d^+|]\Big) C_d\Big(\Phi_t \otimes \mathbb{1}[|\Phi_d^+\rangle\langle \Phi_d^+|]\Big)$$

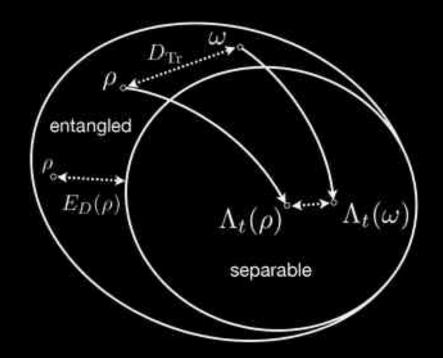
Statistical approach: a universal behavior

Multipartite states

Concentration of measure

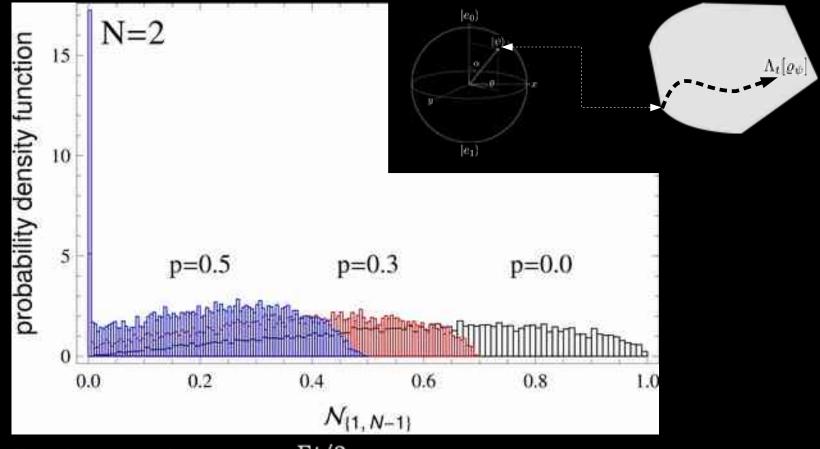
Consider a system in $\mathscr{H} = \mathscr{H}_A \otimes \cdots \otimes \mathscr{H}_N$. If one uniformly samples pure states $|\psi\rangle \in \mathscr{H}$, the chance that it is maximally entangled approaches 1 exponentially fast with the total dimension $d = d_A.d_B \ldots d_N$

P. Hayden, D. Leung, and A. Winter, Comm. Math. Phys. 265, 95 (2006)

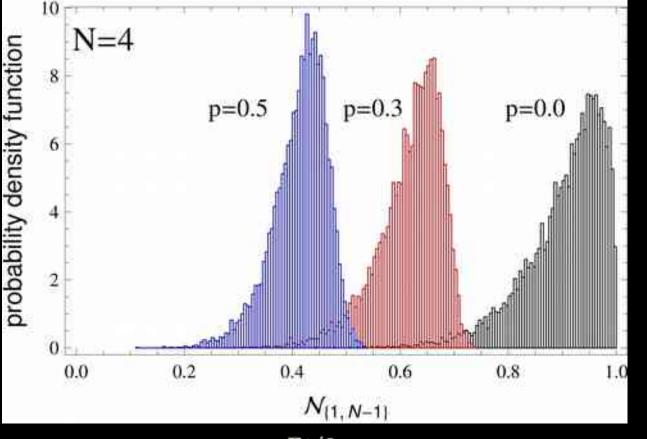


$$||\Lambda_t(\varrho) - \Lambda_t(\omega)||_1 \le \eta_{\Lambda_t} ||\varrho - \omega||_1 ; \eta_{\Lambda_t} \le 1$$

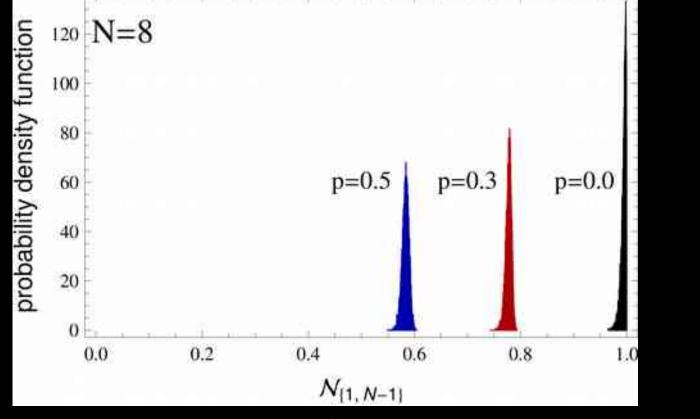
M. B. Ruskai, Rev. Math. Phys. 6, 1147 (1994).



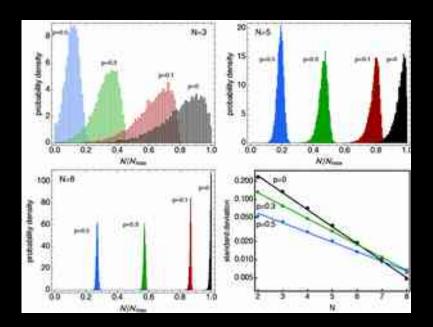
Dephasing $(p = 1 - e^{\Gamma t/2})$



Dephasing $(p = 1 - e^{\Gamma t/2})$



Dephasing $(p = 1 - e^{\Gamma t/2})$



$$\Pr\left(|E[\Lambda_t(|\psi\rangle\langle\psi|)] - E(t)| > \epsilon\right) \le 4\exp\left(-C\frac{2a-1}{4\eta_E^2\eta_{\Lambda_t}^2}\epsilon^2\right)$$

with
$$C = (24 \pi^2)^{-1}$$
 and $E(t) := \int d\chi \ E[\Lambda_t(|\chi\rangle \langle \chi|)]$

M. Tiersch, FdM, and A. Buchleitner, J. Phys. A 46, 085301 (2013)

Proof ingredients

Lipschitz continuous entanglement measure

$$|E(\varrho) - E(\omega)| \le \eta_E ||\varrho - \omega||_1$$

Levy's lemma:

Let
$$f: \mathbb{S}^n \to \mathbb{R}$$
, with $|f(X) - f(Y)| \le \eta ||X - Y|| \ \forall X, Y \in \mathbb{S}^n$

$$\Pr(|f(X) - \langle f \rangle| > \epsilon) \le 4 \exp\left(-C\frac{n+1}{\eta^2}\epsilon^2\right)$$

for a random $X \in \mathbb{S}^n$, and C > 0.

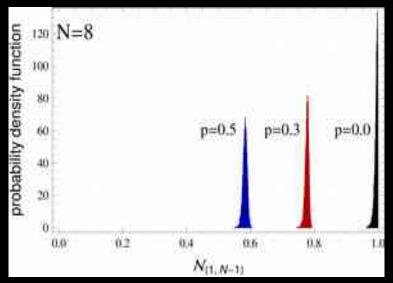


Conclusions

- Entanglement theory is rich and interesting
- Determining entanglement is a hard task

 The complexity seems to be reduced when introducing dynamical aspects

Open questions



- Finding an equation of motion for the mean entanglement
- Use an efficient sampling distribution

Interested?



Open-system dynamics of entanglement: a key issues review

Launules Autitia", Ferroandis dy Moto and Luiz Devidovichi

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1. Introduction

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To appear in Rep. Prog. Phys.

78 pages 560 references 33 figures

Also available at arXiv:1402.3713



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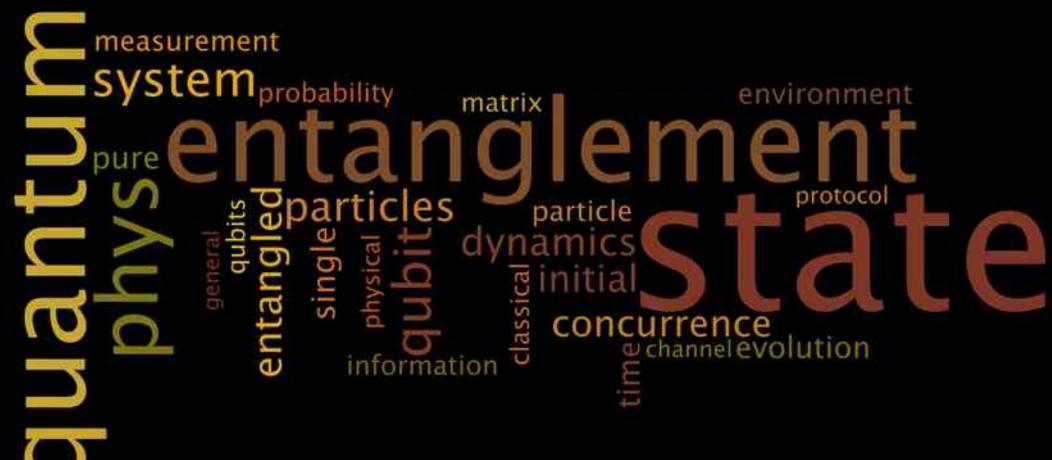


Coordenadas: 05.12, 13h CBPF

Inscrições e informações:

http://qigcbpf.wordpress.com/bell50anos/

Grátis!



Obrigado!