A FÍSICA ESTATÍSTICA DA TURBULÊNCIA

L. Moriconi

IF-UFRJ

I. Introdução

II. A Teoria K41

III. O Fenômeno da Intermitência

IV. Modelagem Lagrangeana

V. Conclusões

Conselho Nacional de Desenvolvimento Científico e Tecnológico

Fundação Carlos Chagas Filho de Amparo à Pesquisa do Estado do Rio de Janeiro

Prelúdio: A Fascinação da Turbulência

H. Lamb (1932):
"I am an old man now, and when I die and go to heaven there are two matters on which I hope for enlightenment. One is quantum electrodynamics and the other is the turbulent motion of fluids. And about the former I am rather optimistic".

R.P. Feynman:
"…the most important, unsolved problem of classical physics".

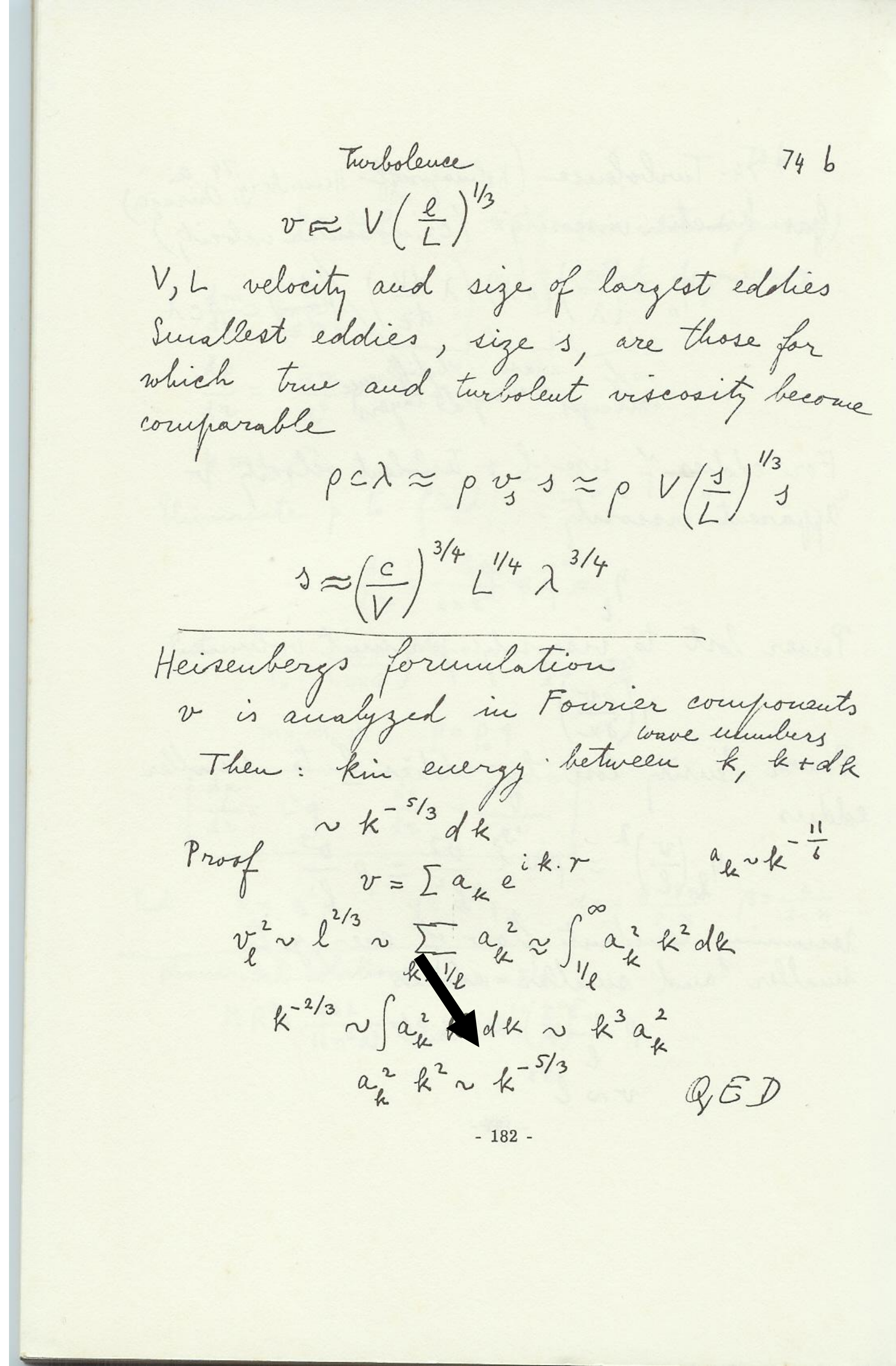
W. Heisenberg:
Depois da segunda guerra mundial, Heisenberg foi detido em Farm Hill, perto de Cambridge, pelos aliados. Impossibilitado de frequentar bibliotecas e grupos científicos, decidiu investigar (apenas em companhia de Weizsacker) o problema da turbulência, caracterizado, àquela época, por poucos resultados consolidados.

E. Fermi:

A última página das suas notas de Termodinâmica e Física Estatística (1951-52).

K. Wilson: Nobel Lecture (1982).

“Theorists have difficulties with this kind of problem because they involve very many degrees of freedom. (…) the entire problem of fully developed turbulence, many problems in critical phenomena and (…) strongly coupled quantum fields have defeated analytic techniques up till now”



Turbolence 74 b

$$v \approx V\left(\frac{l}{L}\right)^{1/3}$$

V, L velocity and size of largest eddies

Smallest eddies, size s, are those for which true and turbolent viscosity become comparable

$$\rho c\lambda \approx \rho v_s s \approx \rho V\left(\frac{s}{L}\right)^{1/3} s$$

$$s \approx \left(\frac{c}{V}\right)^{3/4} L^{1/4} \lambda^{3/4}$$

Heisenbergs formulation

v is analyzed in Fourier components

Then: kin energy between wave numbers k, k+dk

$$\sim k^{-5/3}\, dk$$

Proof $v = \sum a_k e^{ik\cdot r}$ $\quad a_k \sim k^{-11/6}$

$$v_l^2 \sim l^{2/3} \sim \sum_{k>1/l} a_k^2 \approx \int_{1/l}^{\infty} a_k^2 k^2\, dk$$

$$k^{-2/3} \sim \int a_k^2 k^2\, dk \sim k^3 a_k^2$$

$$a_k^2 k^2 \sim k^{-5/3}$$

QED

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I. INTRODUÇÃO

Leonardo da Vinci
1452-1519

Criador do termo
"Turbolenza"
Turba = multidão desordenada

- Estruturas de largas escalas decaem lentamente na turbulência.
- Fluxos turbulentos podem ser representados como a superposição de movimentos principais e flutuantes.

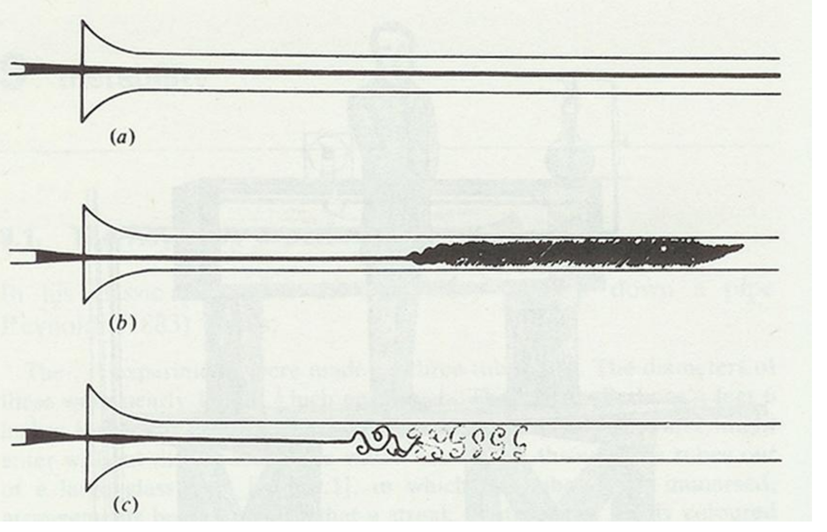
``Observe the motion of the surface of the water, which resembles that of hair, which has two motions, of which one is caused by the weight of the hair, the other by the direction of the curls; thus the water has eddying motions, one part of which is due to the principal current, the other to random and reverse motion." (Leonardo da Vinci)

Experimento de Reynolds (1883)

V_1

V_2

V_2



$V_1 < V_2$

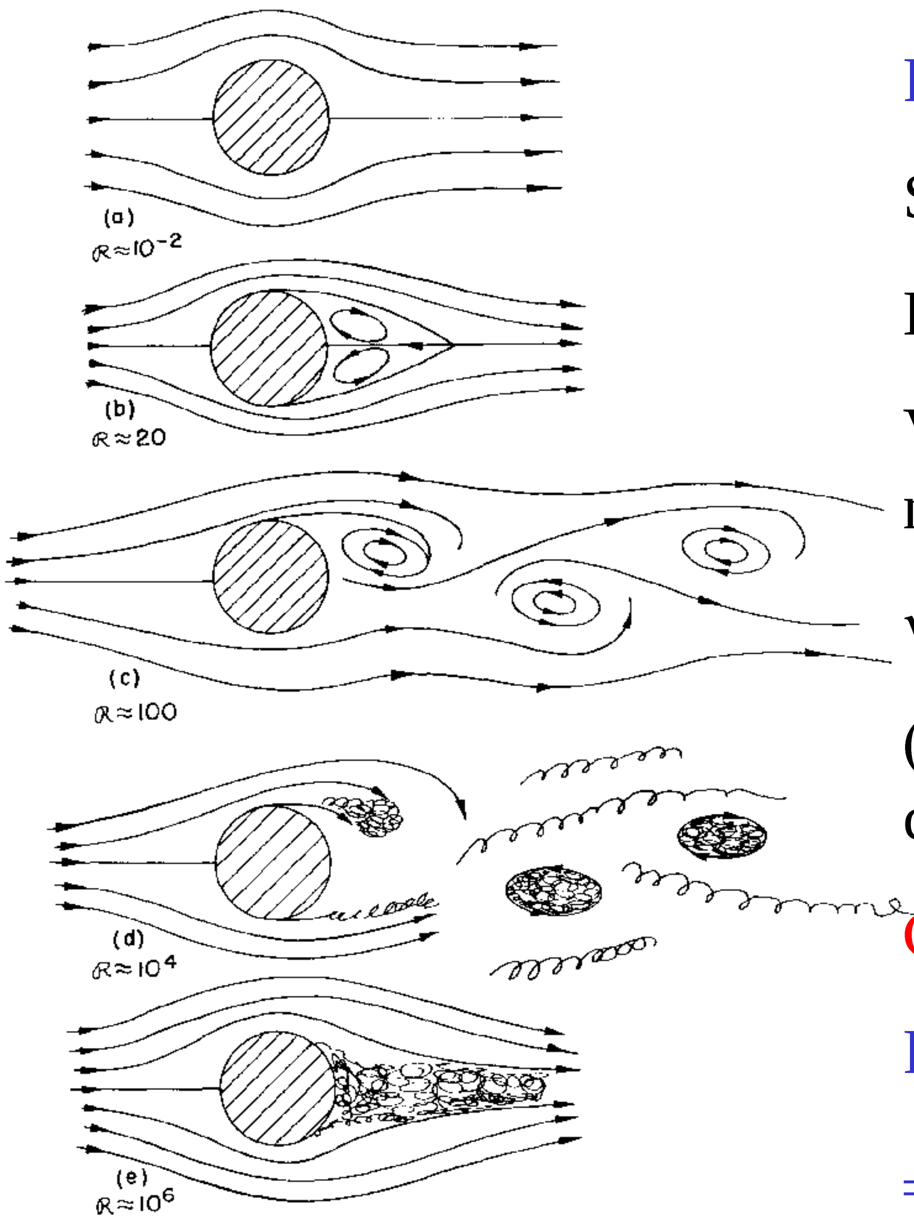


Fig. 41-6. Flow past a cylinder for various Reynolds numbers.

Número de Reynolds:

$R = LV/\nu$

Seja:

L = Diâmetro do Cilindro

V = Velocidade do Escoamento no Infinito

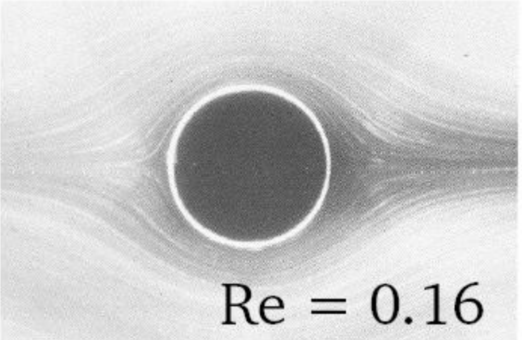
ν = Viscosidade Cinemática

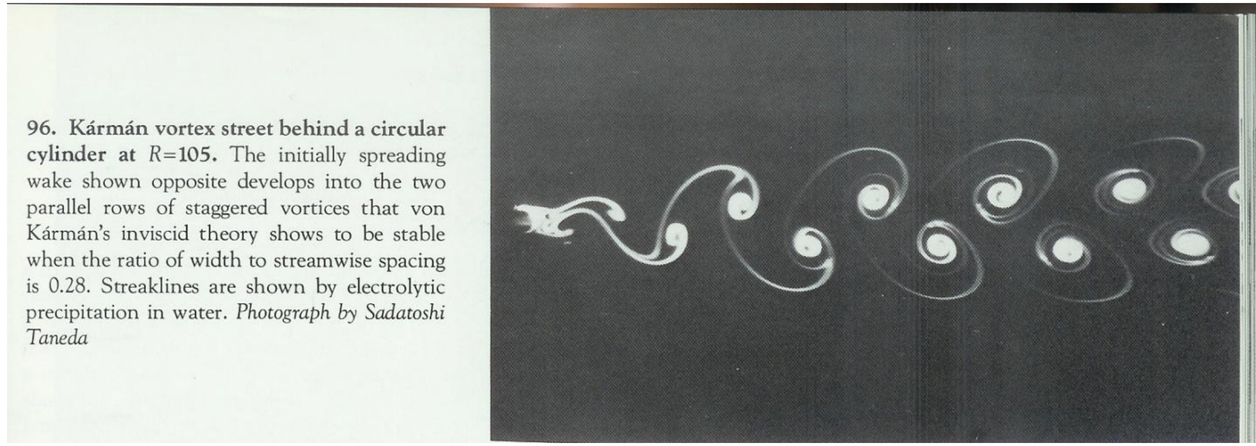
(na água, em unidades cgs, $\nu \sim 1/100$)

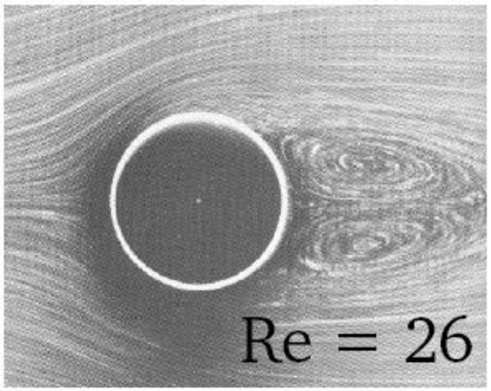
Observe que

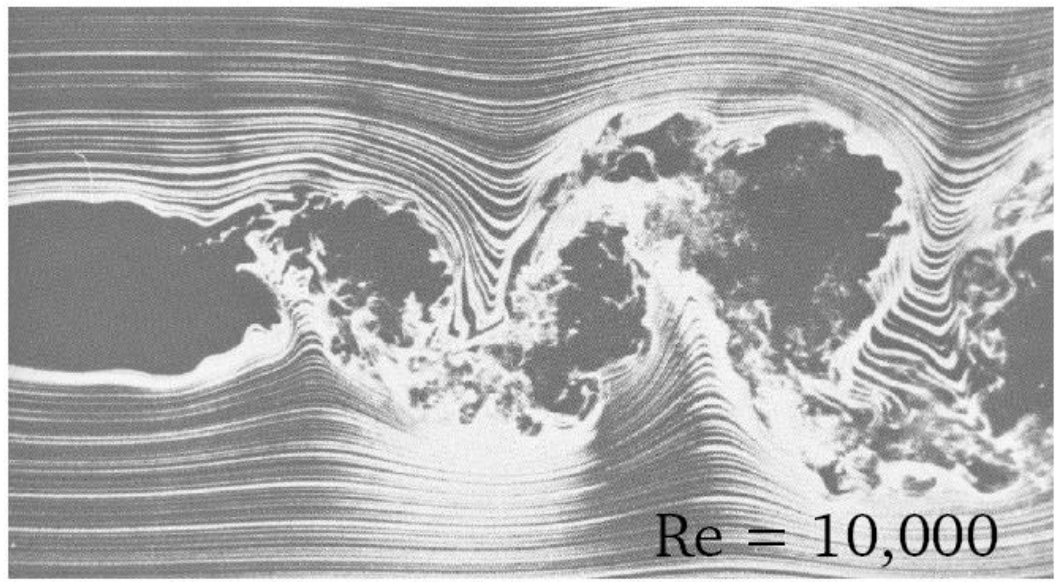
$R = LV/\nu = (L^2/\nu)/(L/V)$

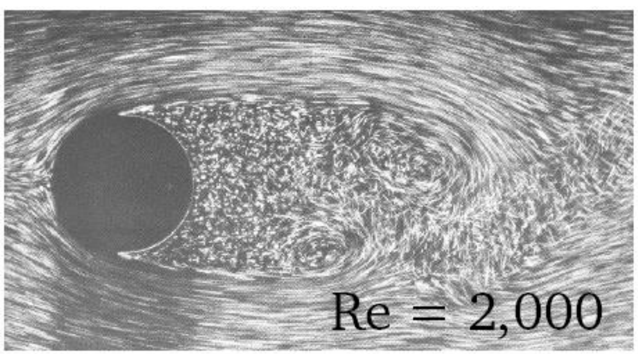
$= T_d/T_c$


Re = 0.16


96. Kármán vortex street behind a circular cylinder at R=105. The initially spreading wake shown opposite develops into the two parallel rows of staggered vortices that von Kármán's inviscid theory shows to be stable when the ratio of width to streamwise spacing is 0.28. Streaklines are shown by electrolytic precipitation in water. Photograph by Sadatoshi Taneda


Re = 26


Re = 10,000


Re = 2,000

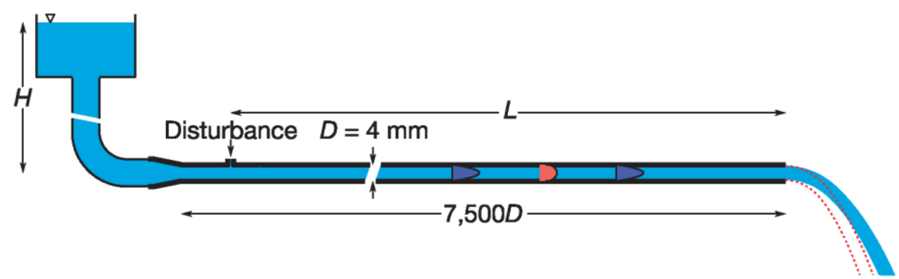
Vol 443|7 September 2006|doi:10.1038/nature05089 nature

LETTERS

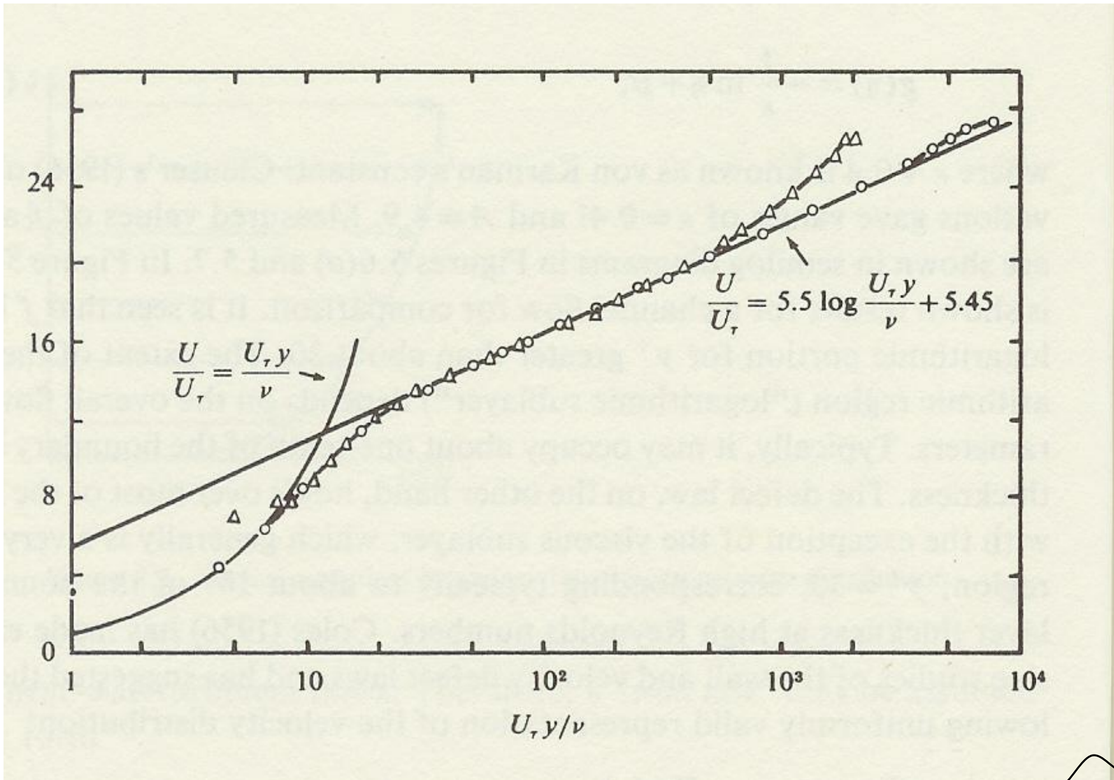
Finite lifetime of turbulence in shear flows

Björn Hof[1,2], Jerry Westerweel[2], Tobias M. Schneider[3] & Bruno Eckhardt[3]

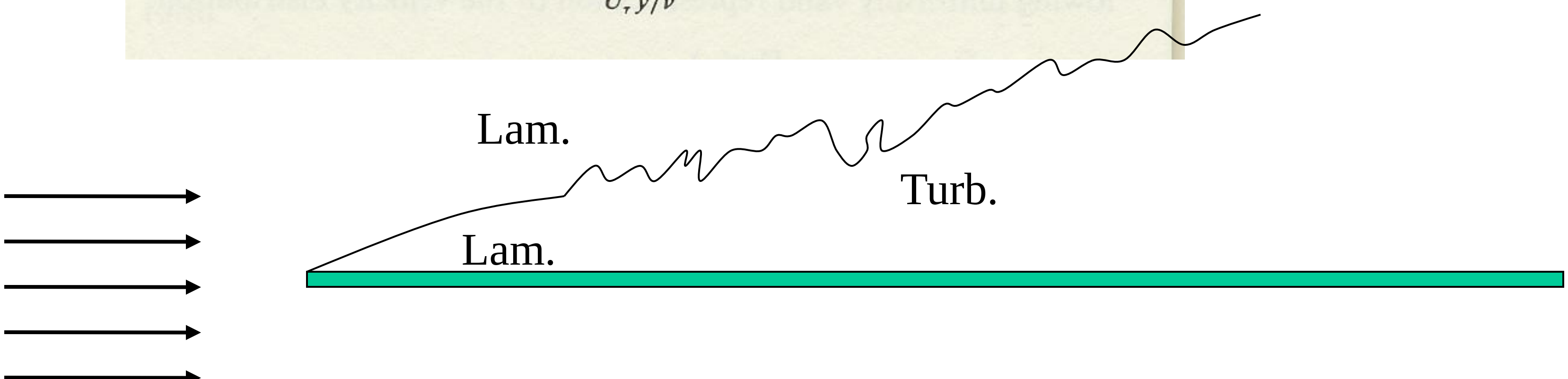
Generally, the motion of fluids is smooth and laminar at low speeds but becomes highly disordered and turbulent as the velocity increases. The transition from laminar to turbulent flow can involve a sequence of instabilities in which the system realizes progressively more complicated states[1], or it can occur suddenly[2,3]. Once the transition has taken place, it is generally assumed that, under steady conditions, the turbulent state will persist indefinitely. The flow of a fluid down a straight pipe provides a ubiquitous example of a shear flow undergoing a sudden transition from laminar to turbulent motion[4–6]. Extensive calculations[7,8] and experimental studies[9] have shown that, at relatively low flow rates, turbulence in pipes is transient, and is characterized by an exponential distribution of lifetimes. They[8,9] also suggest that for Reynolds numbers exceeding a critical value the lifetime diverges (that is, becomes infinitely large), marking a change from transient to persistent turbulence. Here we present experimental data and numerical calculations covering more than two decades of lifetimes, showing that the lifetime does not in fact diverge but rather increases exponentially with the Reynolds number. This implies that turbulence in pipes is only a transient event (contrary to the commonly accepted view), and that the turbulent and laminar states remain dynamically connected, suggesting avenues for turbulence control[10].



“Lei da Parede” da Camada Limite Turbulenta

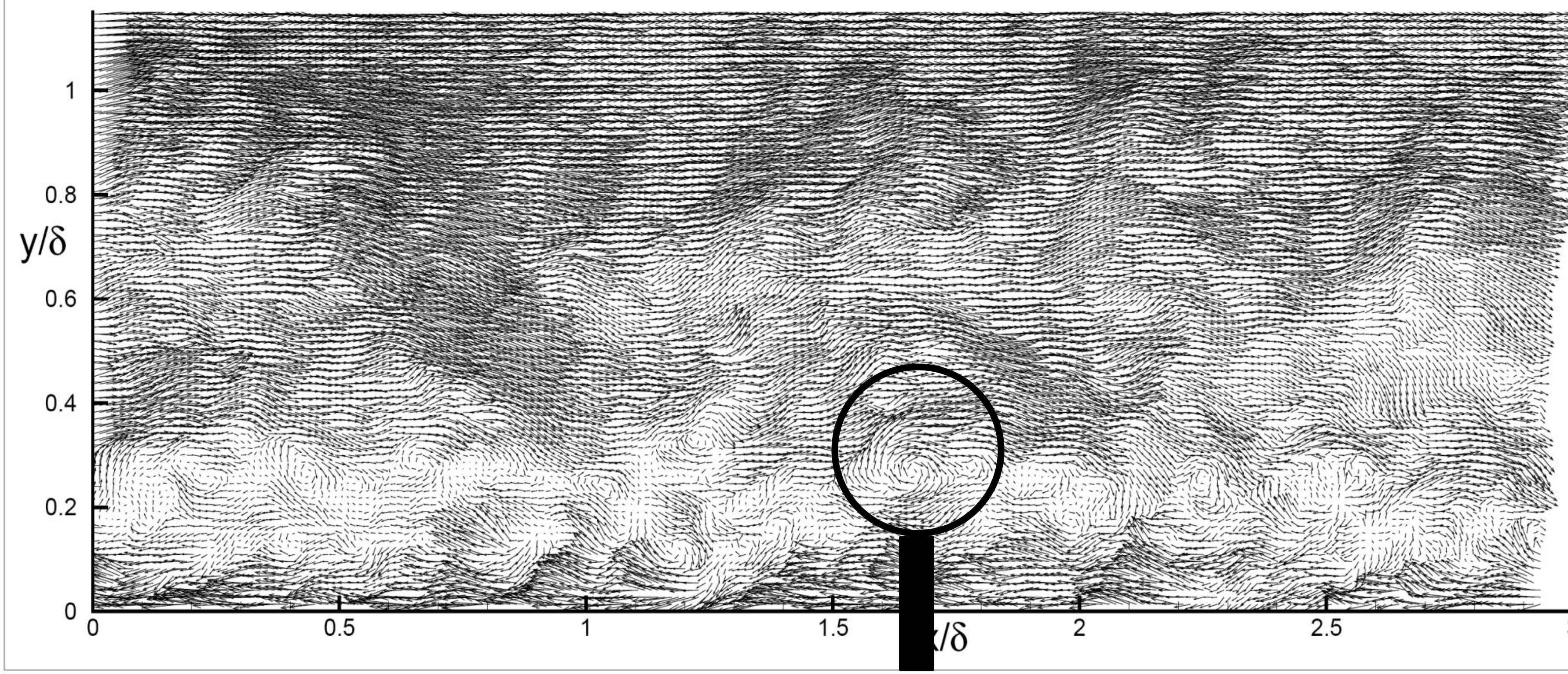


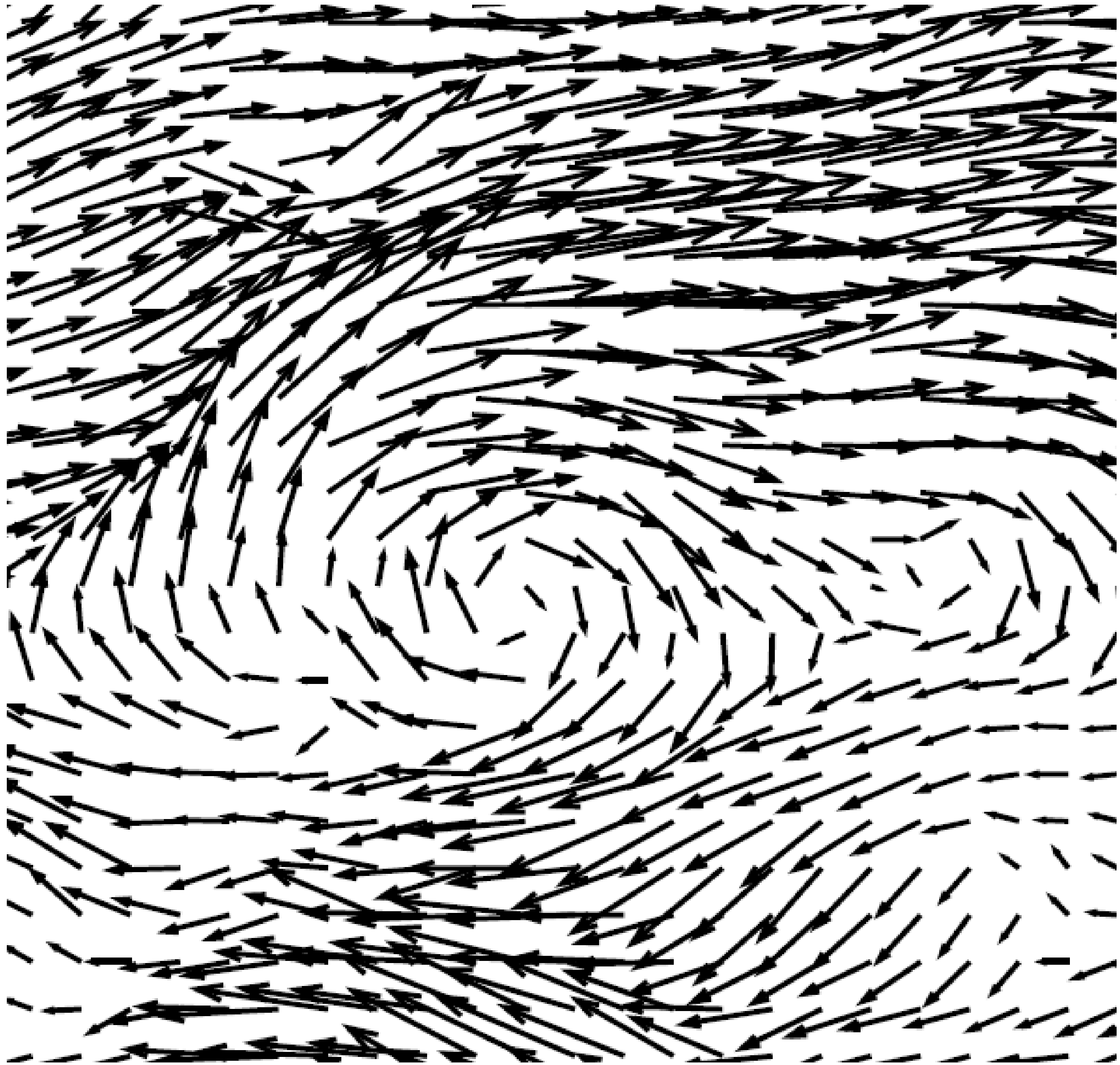
Prandl,
von Karman
~1930



Velocity vectors in a flat plate boundary layer at Re_θ = 7705.

U_c = 0.72 U_∞, flow left to right, realization "High01"; Adrian and Tomkins (1997).

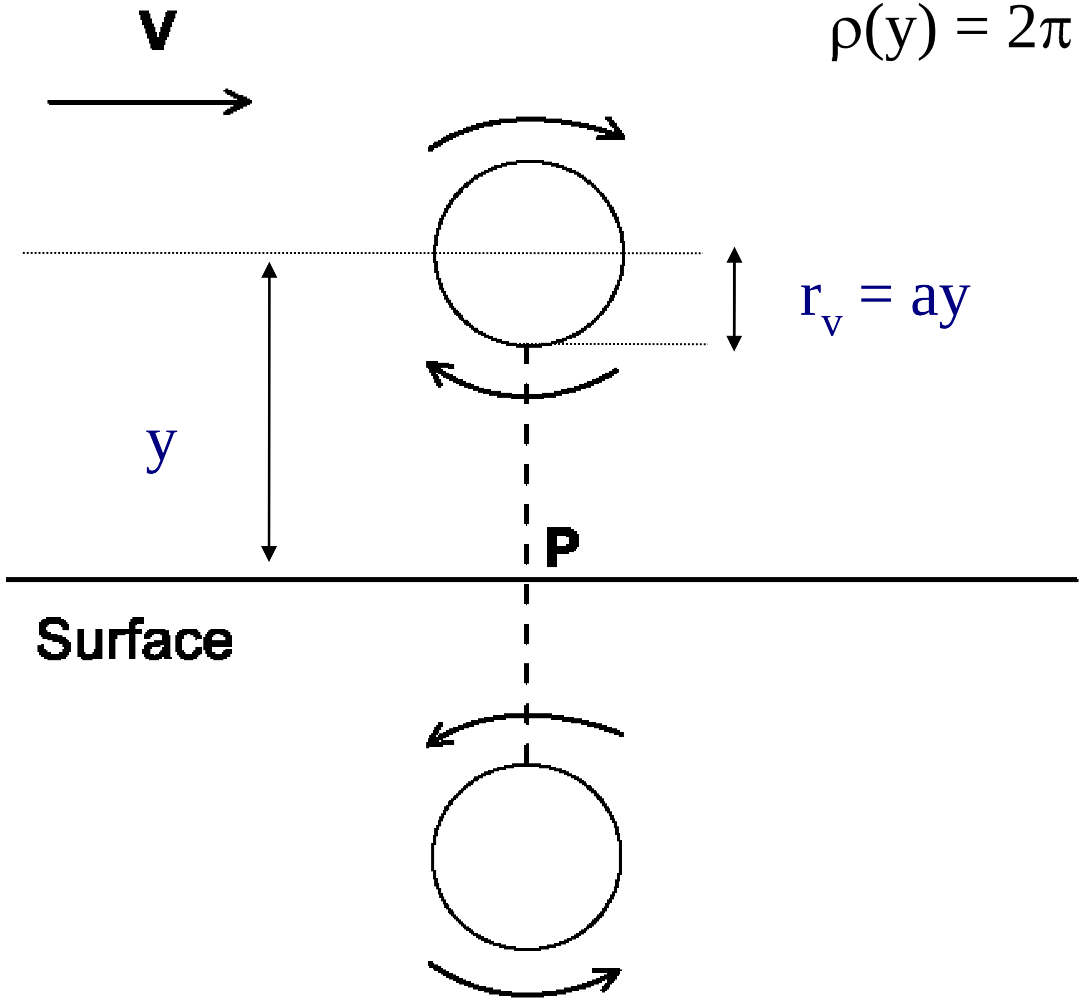




Boundary Layer Modelling
L.M. PRE 2009

a = 1.0, V=1.0

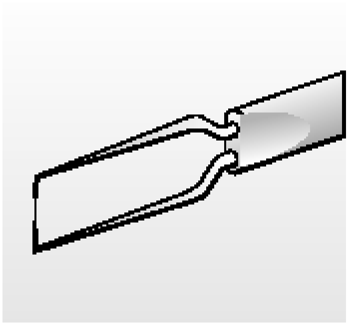
$\rho(y) = 2\pi / (1+y^2)$

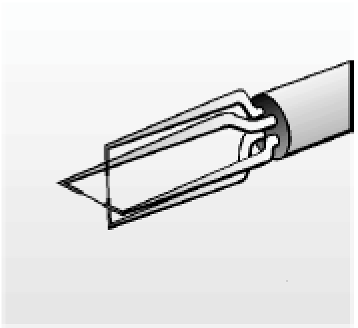


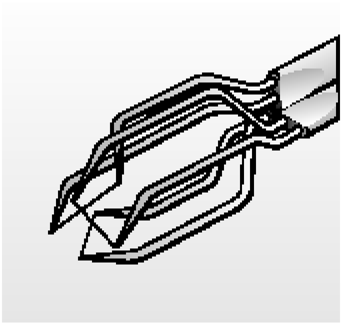
Como se mede Turbulência? Três métodos essenciais: CTA, PIV, LDA

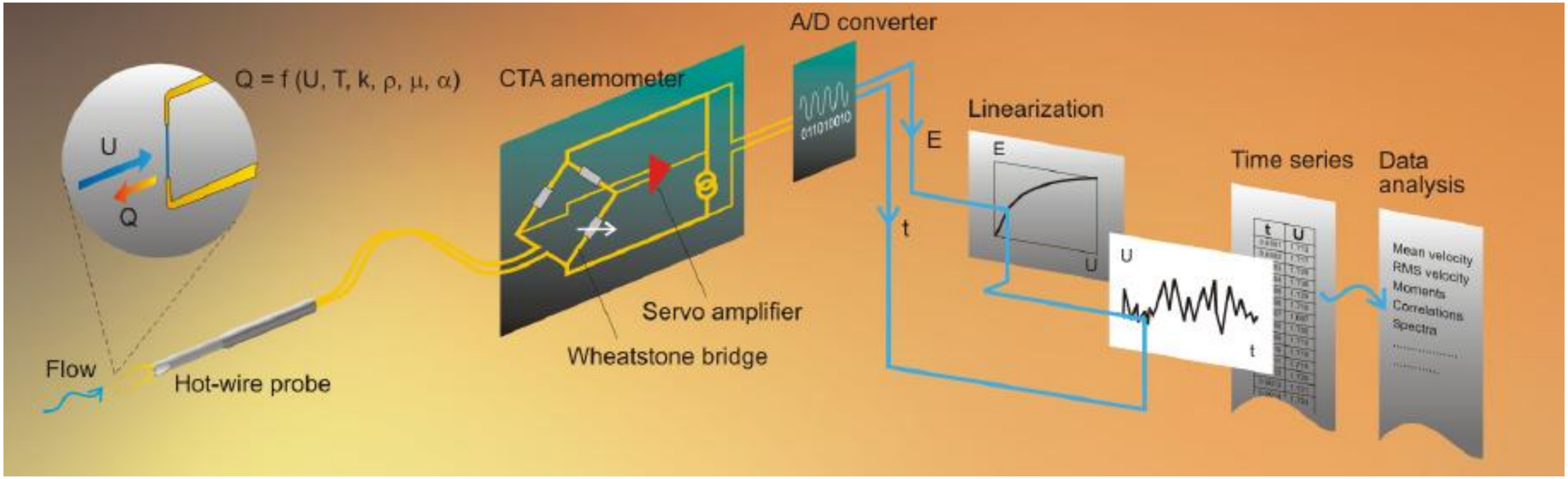
Constant Temperature Anemometry

Parâmetros do fio quente: comprimento = 1.2mm; diâmetro = 0.5μm









Large Scale Intermittency in the Atmospheric Boundary Layer.

multi-hot-wire probe

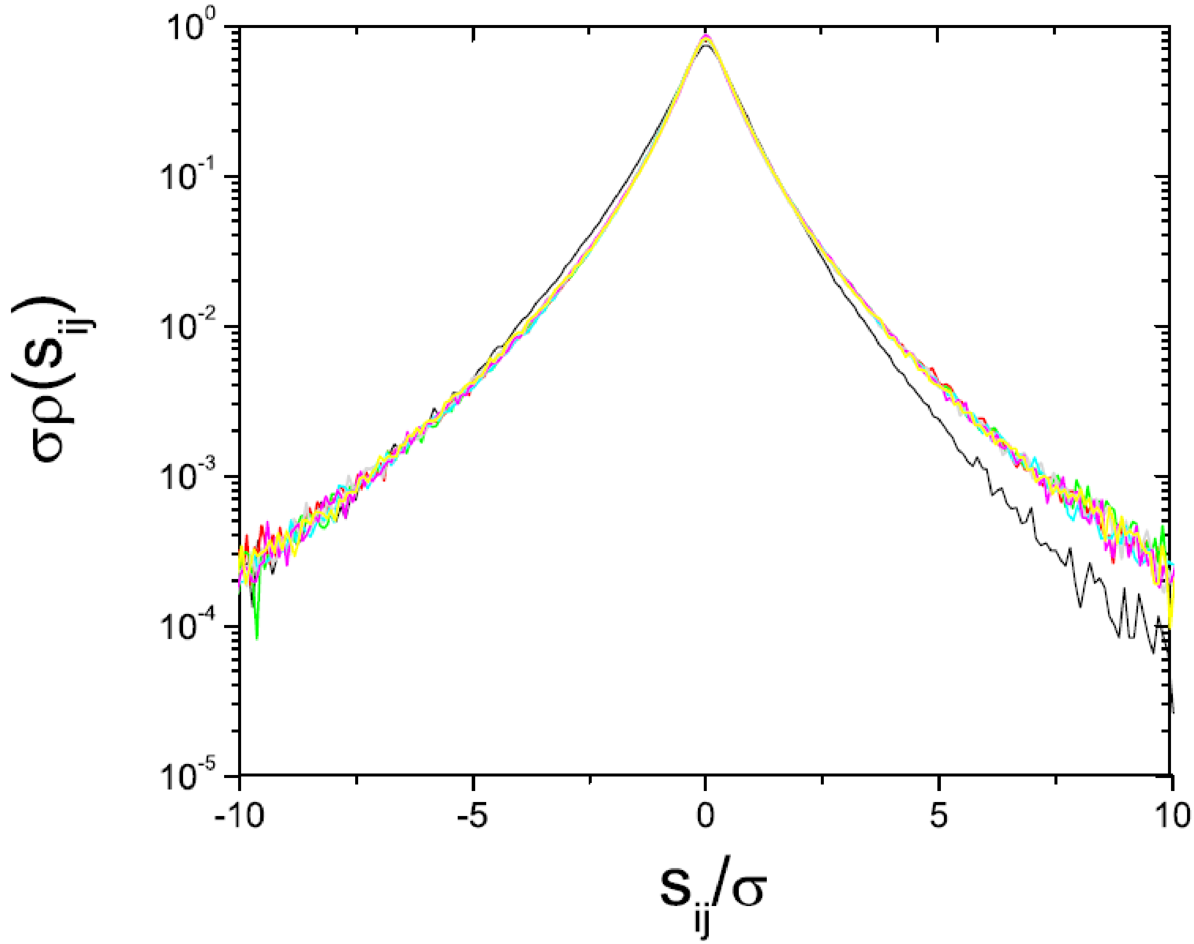
http://www.ethlife.ethz.ch/articles/tages/turbulenzmaloja.html

G. Gulitski, M. Kholmyansky, W. Kinzelbach, B. Luthi, A. Tsinober, And S. Yorish – JFM 2007 (three papers).

Gradientes
de velocidade,

$A_{ij} = \partial_i v_j$

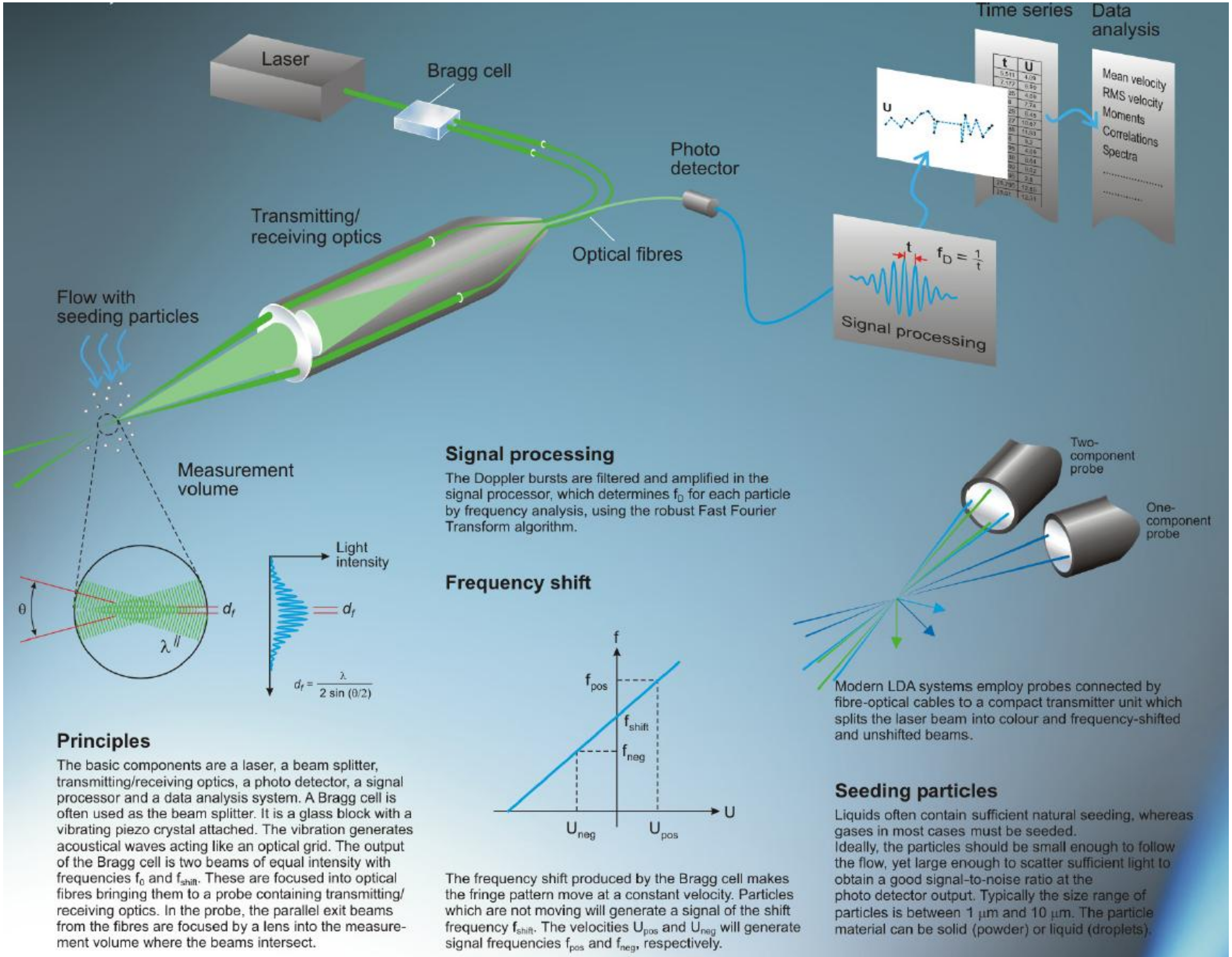
são descritos
por distribuições
não-gaussianas
de probabilidade.



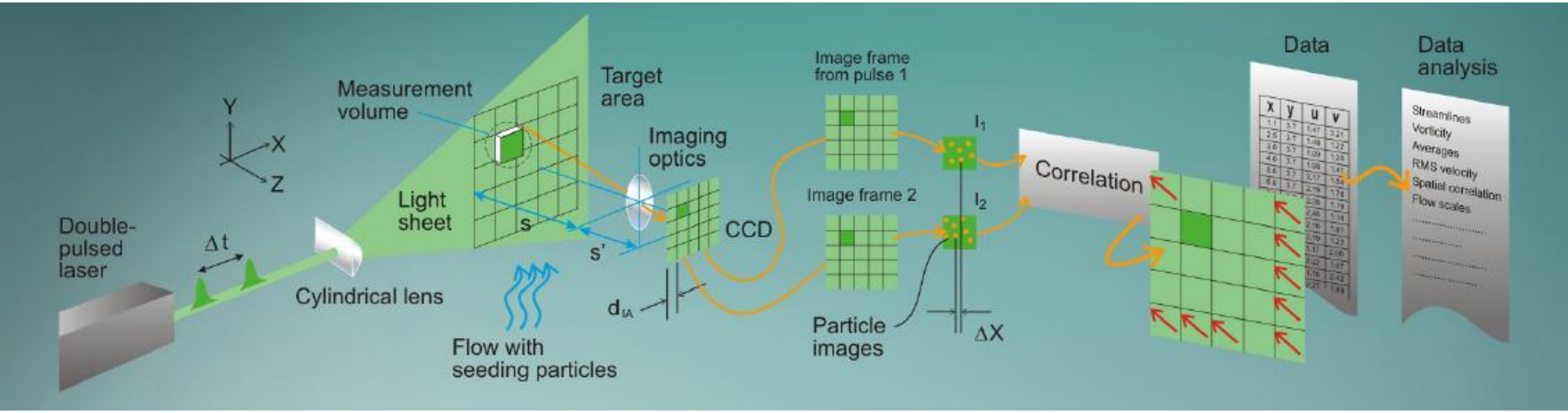
Resultados modelados em:

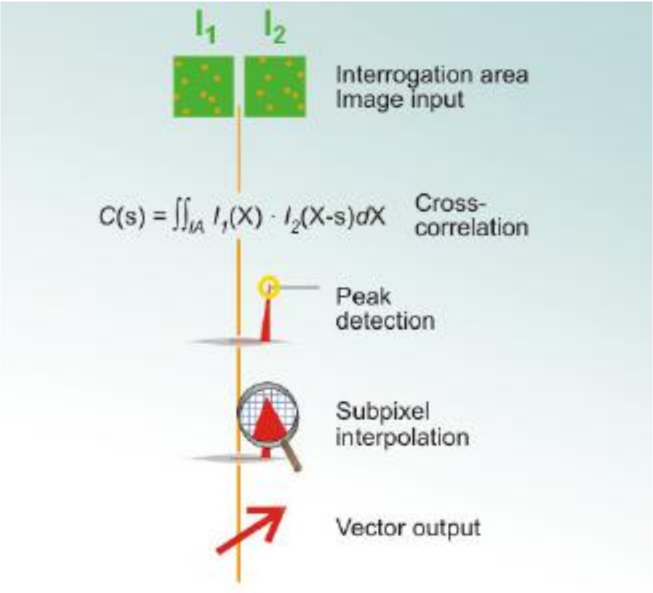
M. Kholmyansky, L. M., R.M. Pereira, and A. Tsinober
Phys. Rev E **80**, 036311 (2009).

Laser Doppler Anemometry



Particle Image Velocimetry





The correlation of the two interrogation areas, I_1 and I_2, results in the particle displacement ΔX, represented by a signal peak in the correlation $C(\Delta X)$.

Equações de Navier-Stokes

$$\partial_t v_\alpha + v_\beta \partial_\beta v_\alpha = -\partial_\alpha P + \nu \partial^2 v_\alpha + f_\alpha$$

Força externa, definida a grandes escalas;

Dissipação viscosa, dominante a escalas pequenas

Termo não-linear de convecção, dominante a escalas intermediárias

Vínculo de incompressibilidade: $\partial_\alpha v_\alpha = 0$

Equações Adimensionais de Navier-Stokes:

$x \longrightarrow Lx$

$t \longrightarrow (L^2/\nu)t$

$v \longrightarrow Vv$

$P \longrightarrow (\nu V/L)\,P$

$$\partial_t v_\alpha + v_\beta \partial_\beta v_\alpha = -\partial_\alpha P + \nu \partial^2 v_\alpha$$

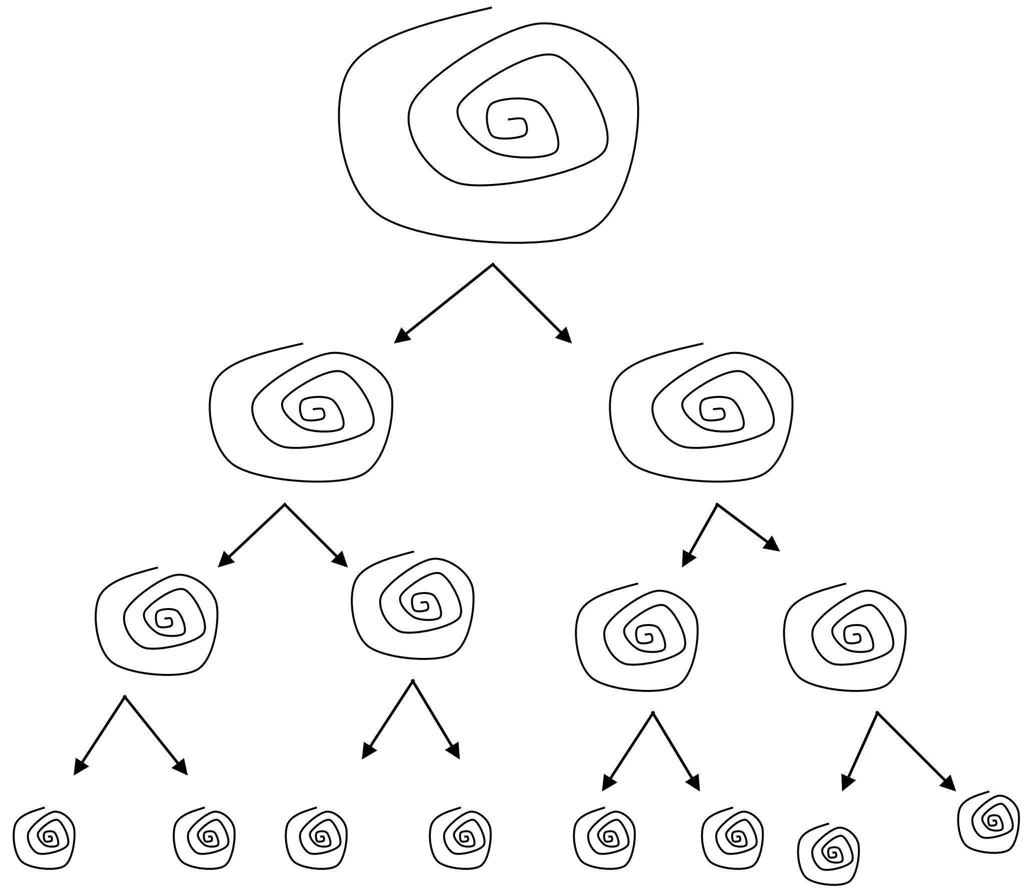
$$\frac{\nu V}{L^2}\partial_t v_\alpha + \frac{V^2}{L} v_\beta \partial_\beta v_\alpha = -\frac{\nu}{L}\partial_\alpha P + \frac{\nu V}{L^2}\partial^2 v_\alpha$$

$$\partial_t v_\alpha + \frac{LV}{\nu} v_\beta \partial_\beta v_\alpha = -\partial_\alpha P + \partial^2 v_\alpha$$

Escoamento turbulento: sistema dinâmico (teoria de campos) de acoplamento extremamente forte.

QED: $g \sim 1/137$; QCD: $g \sim 1$; Turbulência: **$g = R \sim 10^7$**

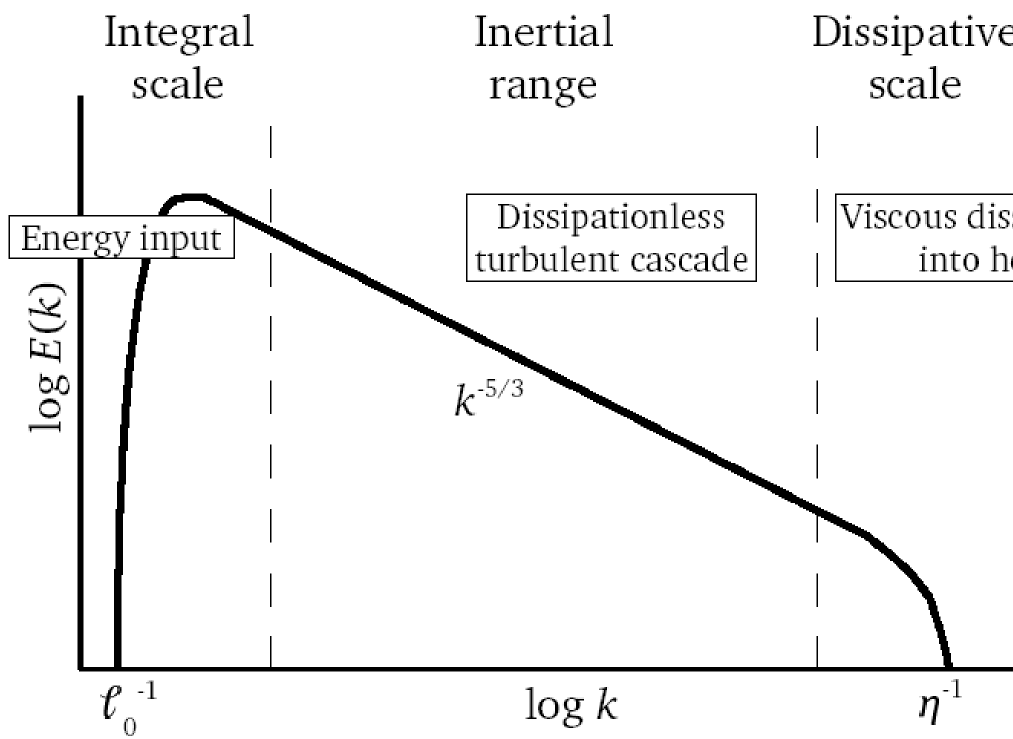
Cascata de Richardson (1922)



"Big whorls have little whorls
that feed on their velocity,
and little whorls have lesser whorls
and so on to viscosity
-- in the molecular sense."

II. A TEORIA K41

A.N. Kolmogorov (1941)



Teoria para o espectro de energia

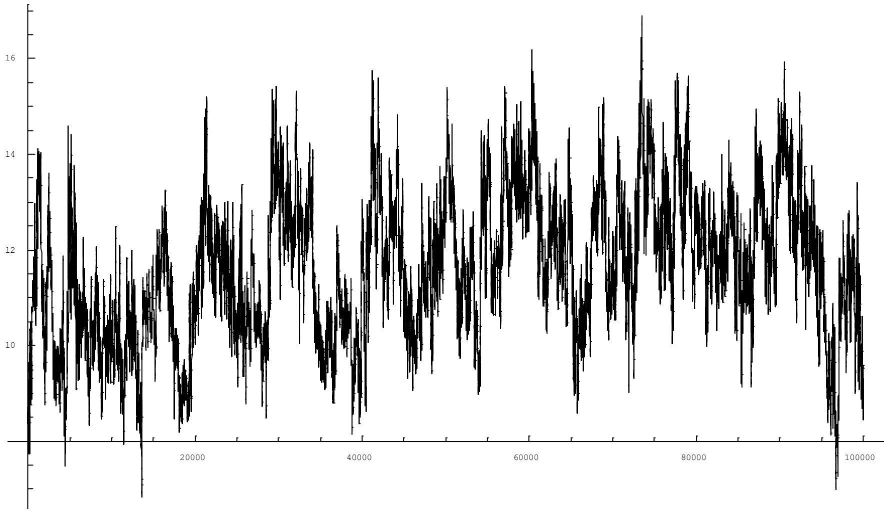
$$\langle v_\alpha(\vec{x},t)v_\beta(\vec{x}',t)\rangle \sim \delta_{\alpha\beta}\int d^3\vec{k}k^{-2}E(k)\exp[i\vec{k}\cdot(\vec{x}-\vec{x}')]$$

Primeira observação: Grant, Stewart e Moillet (1962)

Wind Tunnel Turbulence Data

(Kang, Chester & Meneveau – 2003)

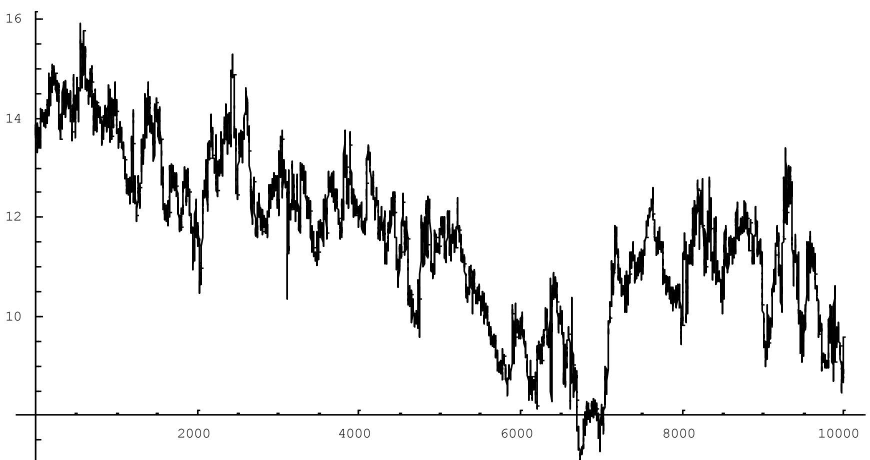
u



2.5 s

t

u

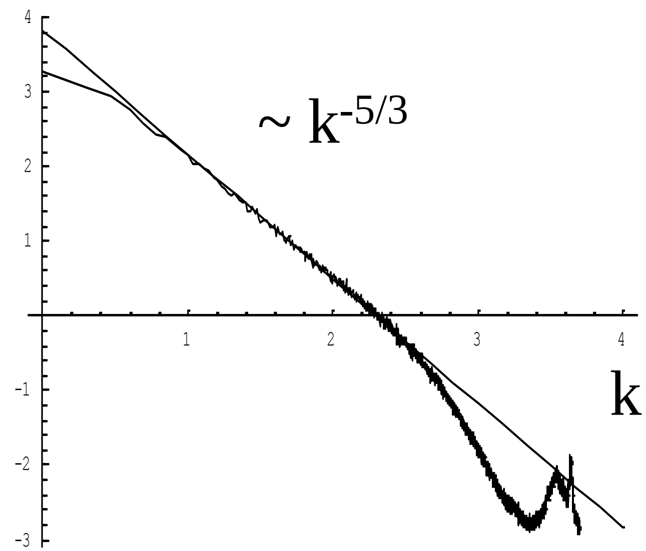


0.25 s

t

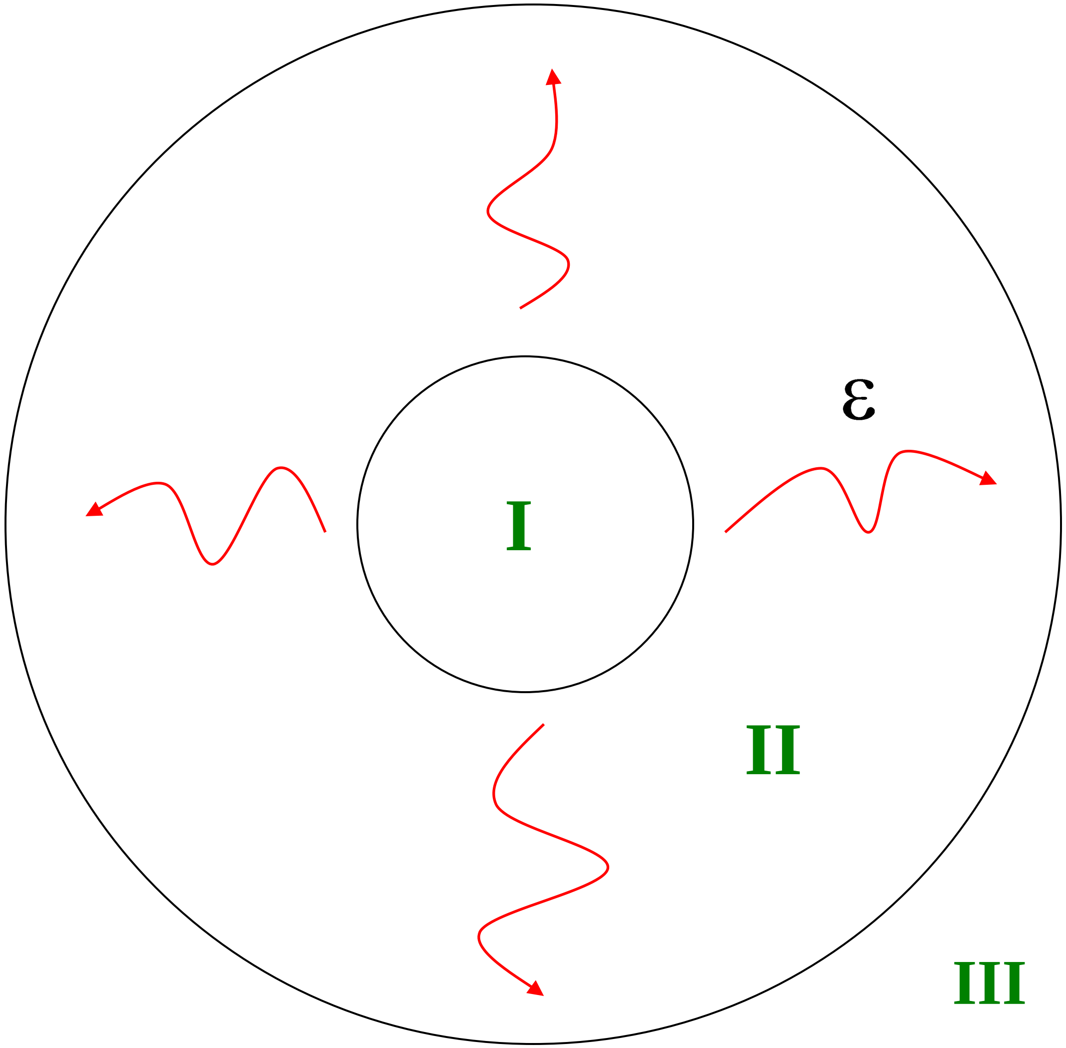
Energy Spectrum

E(k)



R ~ 3 x 10^4

“Espaço k” (número de onda)



I: $0 < k < k_0 \sim 1/L$

Injeção de Energia

II: $k_0 < k < k_\eta \sim 1/\eta$

Transporte de Energia

III: $k > k_\eta \sim 1/\eta$

Dissipação de Energia

ε = taxa de transferência de energia

Vamos supor que na faixa inercial tenhamos

$$E = E(\varepsilon, k) = C_{ko}\, \varepsilon^{\alpha}\, k^{\beta}$$

Análise Dimensional:

$[E] = L^3T^{-2}$

$[\varepsilon] = L^2T^{-3}$ ⇒ $\begin{cases} \alpha = 2/3 \\ \beta = -5/3 \end{cases}$

$[k] = L^{-1}$

De fato, $L^3T^{-2} = (L^2T^{-3})^{2/3}(L^{-1})^{-5/3}$

A teoria fenomenológica de Kolmogorov prevê que

$$\eta \sim \nu^{3/4}\varepsilon^{-1/4}$$

$$\varepsilon \sim V^2/T \sim V^3/L$$

$$R \sim LV/\nu \sim (L/\eta)^{4/3}$$

Simulações numéricas diretas (DNS) exigem redes com $(L/\eta)^3 \sim R^{9/4}$ sítios.

Adicionalmente, para as "funções de estrutura",

$$S_q(|\vec{x} - \vec{x}'|) \equiv \langle|\vec{v}(\vec{x}, t) - \vec{v}(\vec{x}', t)|^q\rangle \sim |\vec{x} - \vec{x}'|^{\zeta(q)}$$

com $\zeta(q) = q/3$. Crítica de Landau: ε flutua!

Desvios são observados! (década de 1980).

III. O FENÔMENO DA INTERMITÊNCIA*

O fenômeno da intermitência foi descoberto por Batchelor e Townsend em 1949 [G.K. Batchelor and A.A. Townsend, Proc. R. Soc. London A 199, 238 (1949)]. Ainda hoje é um dos tópicos centrais de pesquisa em turbulência.

* **Flutuações intensas, não-gaussianas** de observáveis como gradientes/diferenças de velocidades, vorticidade, circulação, etc.

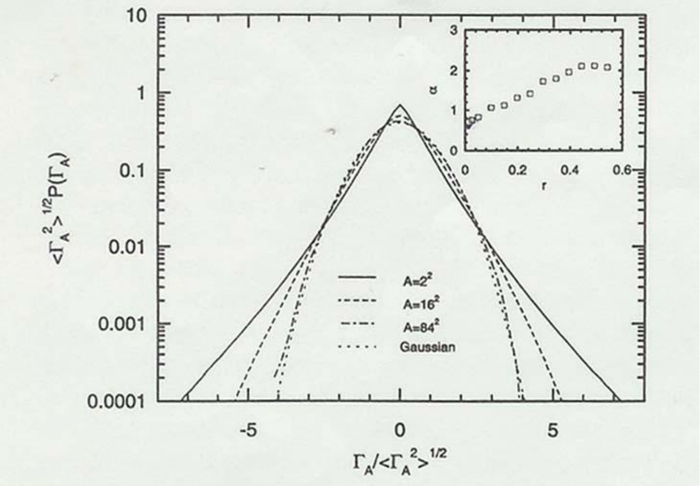


FIG. 3. Normalized PDFs of Γ_A at $R_\lambda = 216$ for $A = r^2$ (square loop in lattice units) for $r = 4$ (dissipation), $r = 16$ (inertial), and $r = 84$ (integral), corresponding to $r = 0.049$, 0.196, and 1.031, respectively. The inset shows the stretching exponent for the tails of the PDF as a function of r.

N. Cao et al., PRL 76, 616 (1996).

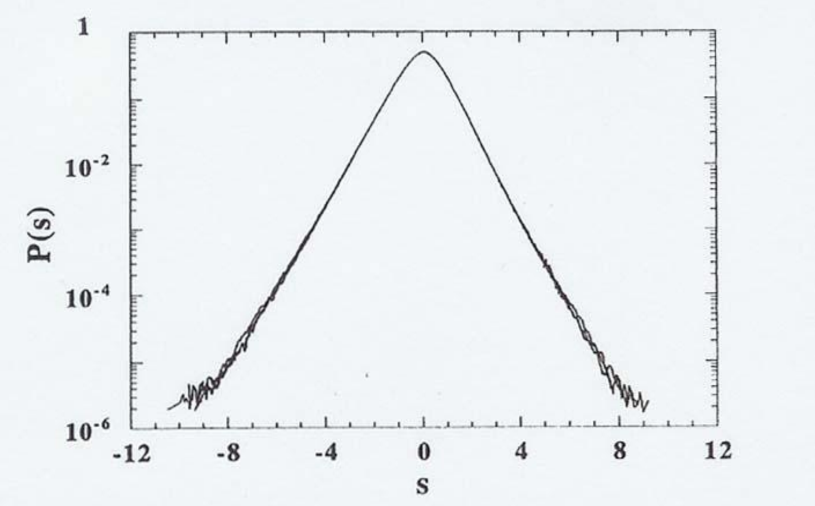
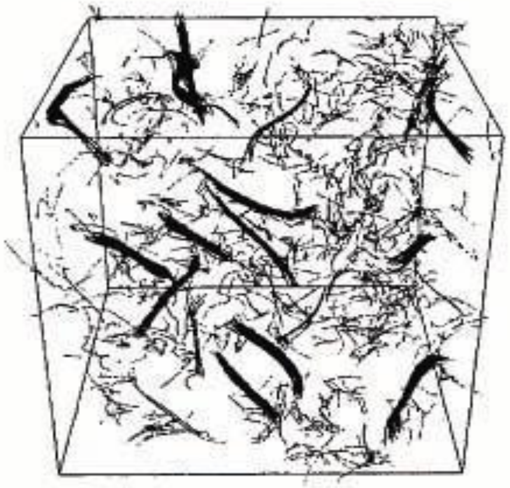
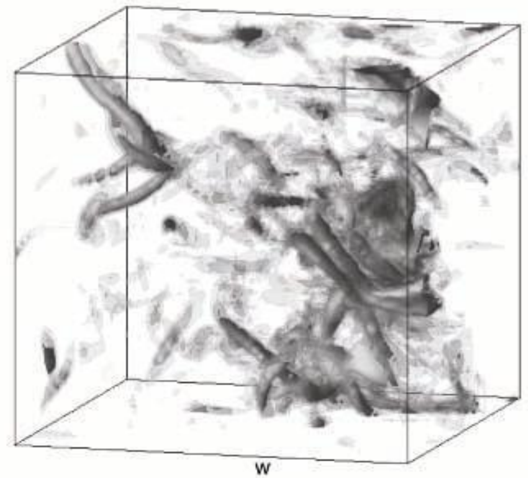


FIG. 8. Log linear plot of the PDF of the longitudinal increments of velocity, calculated for the same scales $r = 490\ \mu$m, and four values of R_λ: $R_\lambda = 1260$, 2300, 2700, and 3700.

P. Tabeling et al., PRE 53, 1613 (1996).



Z.S. She et al., Proc. R. Soc. London Ser. A 434, 101 (1991).
Simulações Numéricas Diretas



M. Farge et al., PRL 87, 054501 (2001).
Análise de Wavelet de dados via DNS

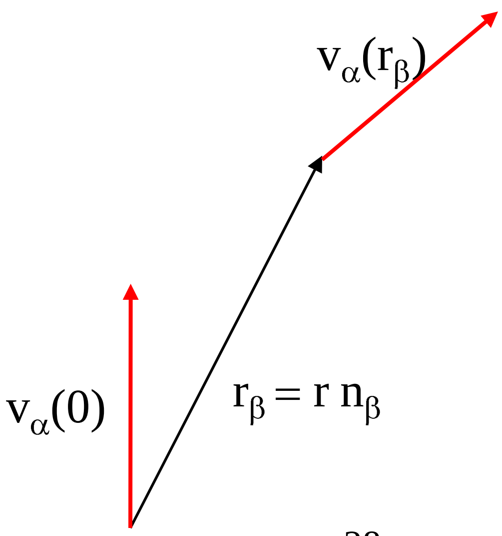
Caracterização quantitativa da intermitência:

- Funções de Estrutura $S_q = < |O(r)|^q >$

(ii) Densidades de Probabilidade

$$\rho = \rho[O(r)]$$

O(r) é algum observável, como $[v_\alpha(r_\beta)-v_\alpha(0)]n_\alpha$



FILMES

Turbulência 2D

Turbulência 3D

Further Direct Numerical Simulations (Earth Simulator)

M. Yokokawa, K. Itakura, A. Uno, T. Ishihara, and Y. Kaneda – Phys. Fluids **15**, L21 (2003).

4096^3 grid points; $R_\lambda \sim 700$

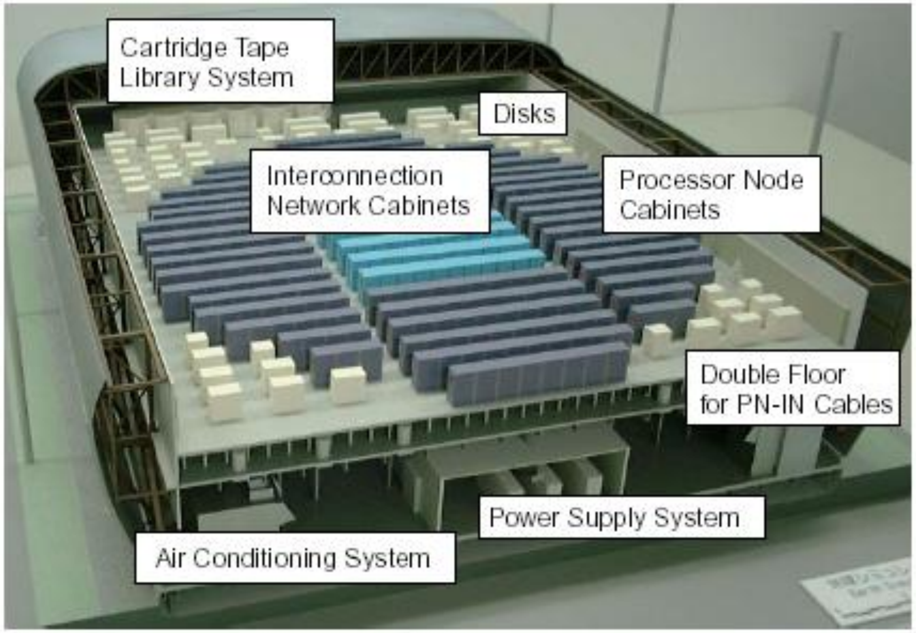


Figure 2: A model of the ES system in the gym-like building. The building is 50m × 65m × 17m and has two stories; it includes a seismic isolation system.

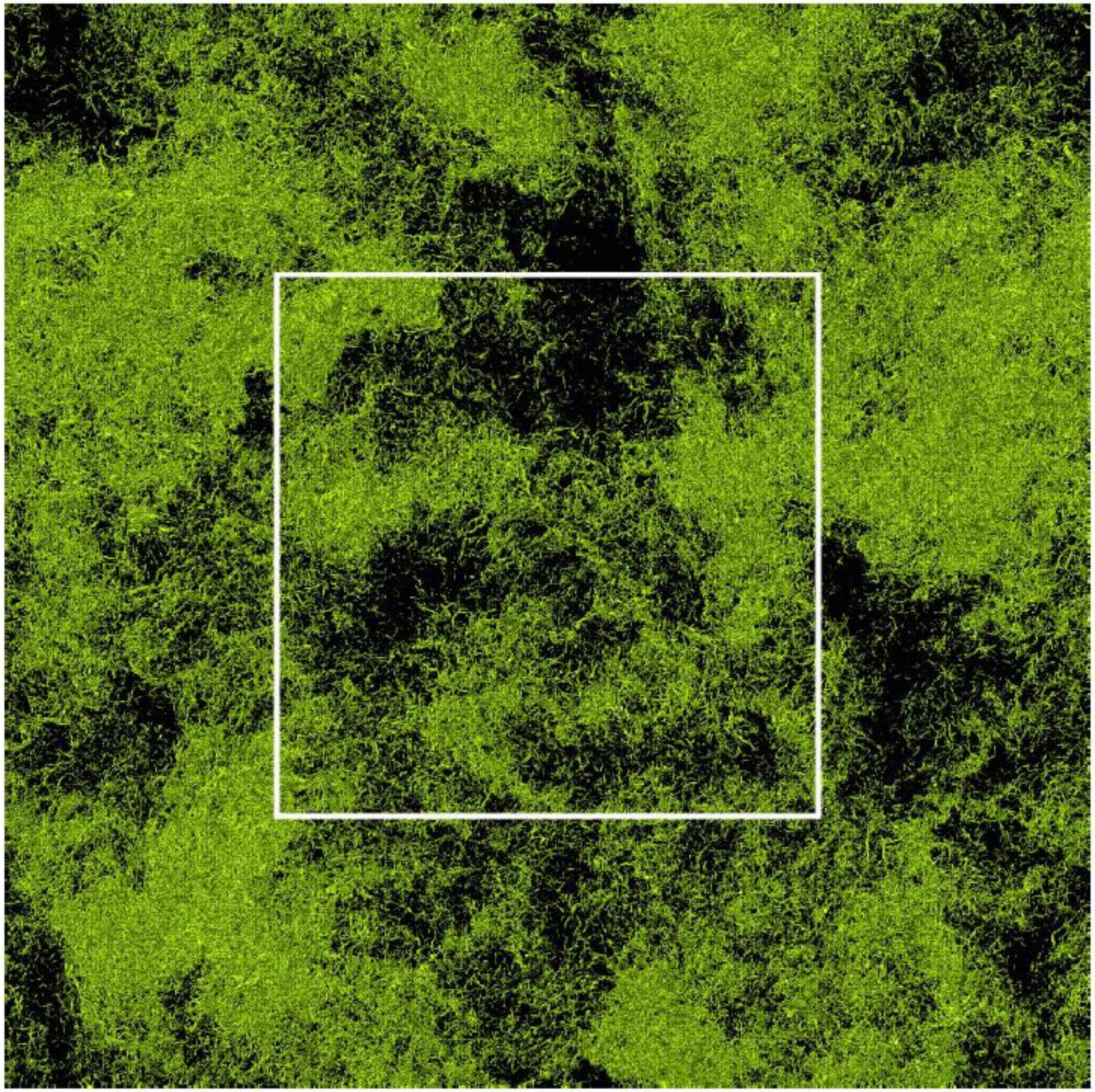


Figure 4: Intense-vorticity isosurfaces showing the region where $|\omega| > \bar{\omega} + 4\sigma$; ω is the vorticity, and $\bar{\omega}$ and σ are the mean and standard deviation of $|\omega|$, respectively. The size of the display domain is $(5984^2 \times 1496)\eta$, periodic in the vertical and horizontal directions. η is the Kolmogorov length scale and $R_\lambda = 732$ (see Table 3).

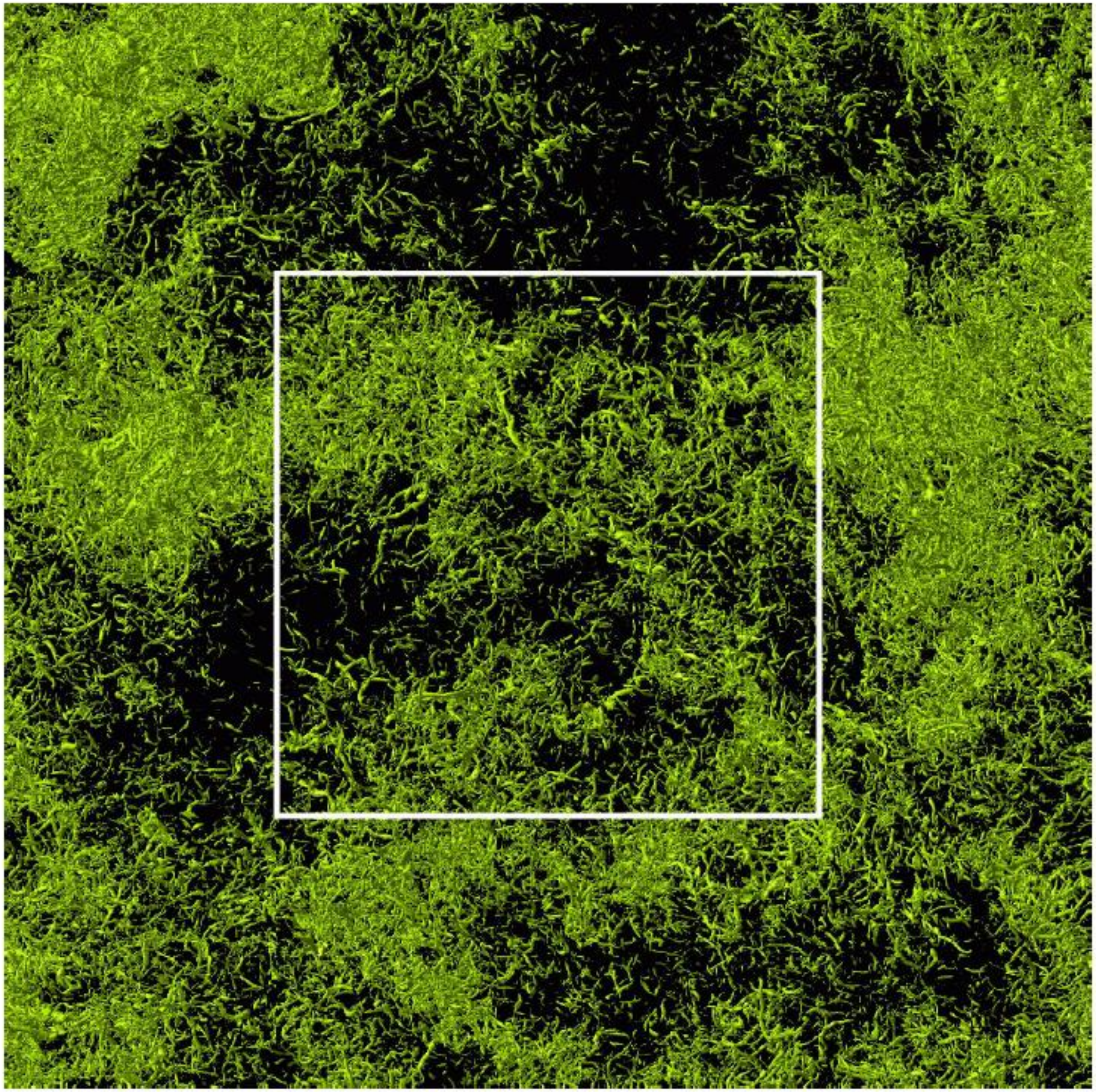


Figure 5: A closer view of the inner square region of Fig. 4; the size of the display domain is $(2992^2 \times 1496)\eta$.

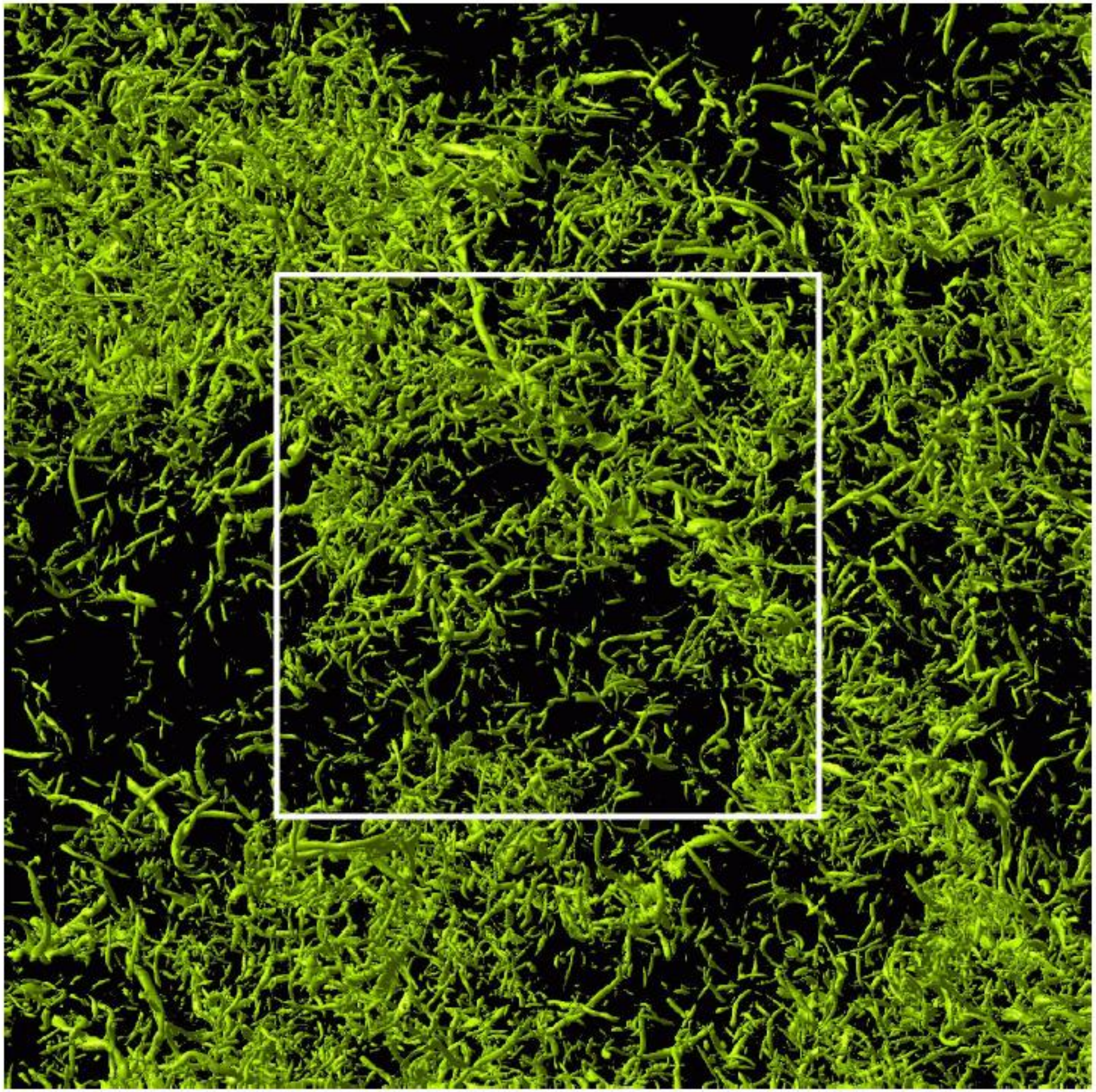


Figure 6: The same isosurfaces as in Fig. 4; a closer view of the inner-square region of Fig. 5. The size of the display domain is $(1496^2 \times 1496)\eta$.

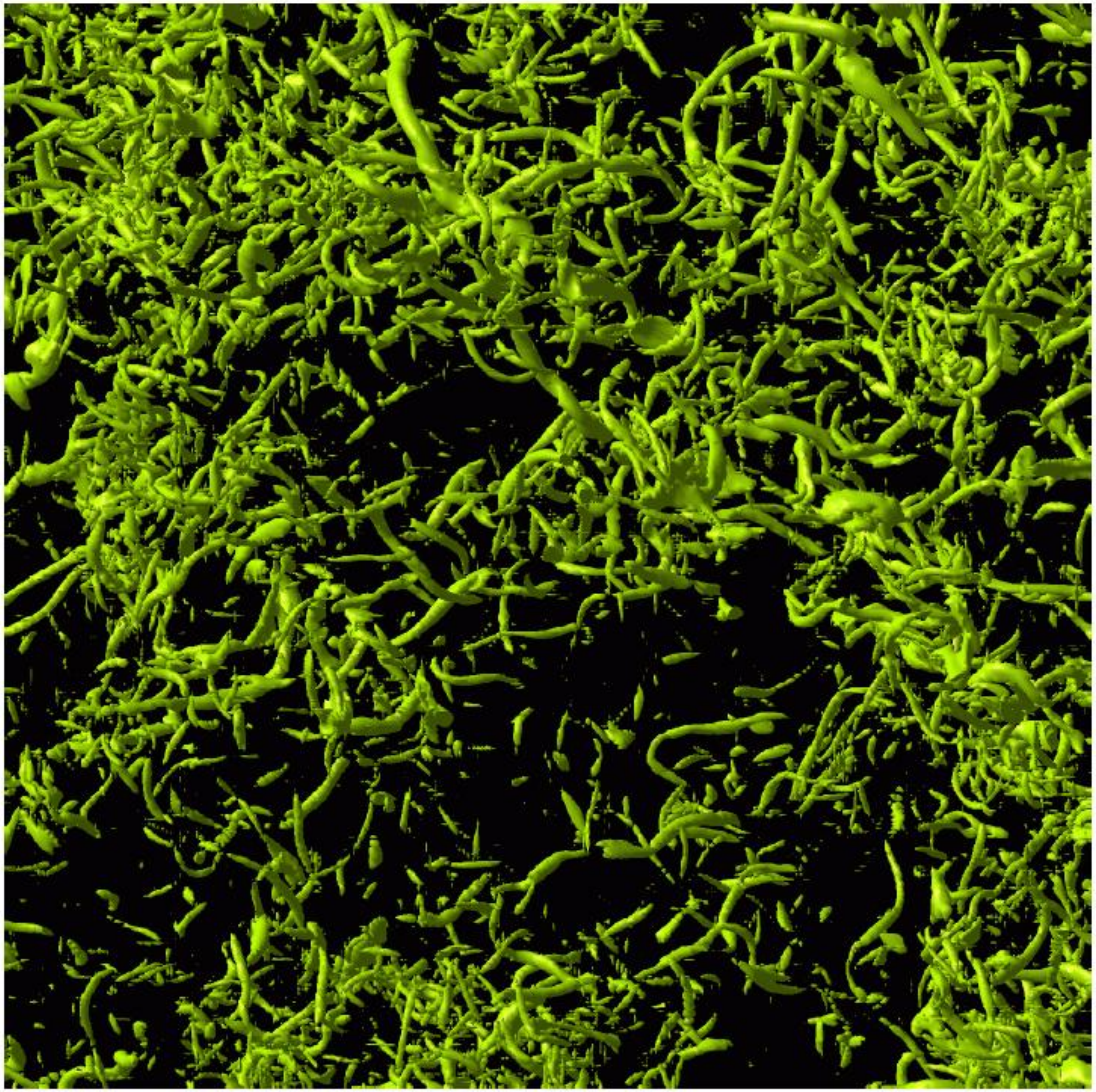


Figure 7: The same isosurfaces as in Fig. 4; a closer view of the inner-square region of Fig. 6. The size of the display domain is $(748^2 \times 1496)\eta$.

Seja

L = Escala Integral;

η = Escala Dissipativa de Kolmogorov;

Para $L >> r >> \eta$ (a “faixa inertial”) temos $S_q \sim r^{\zeta(q)}$

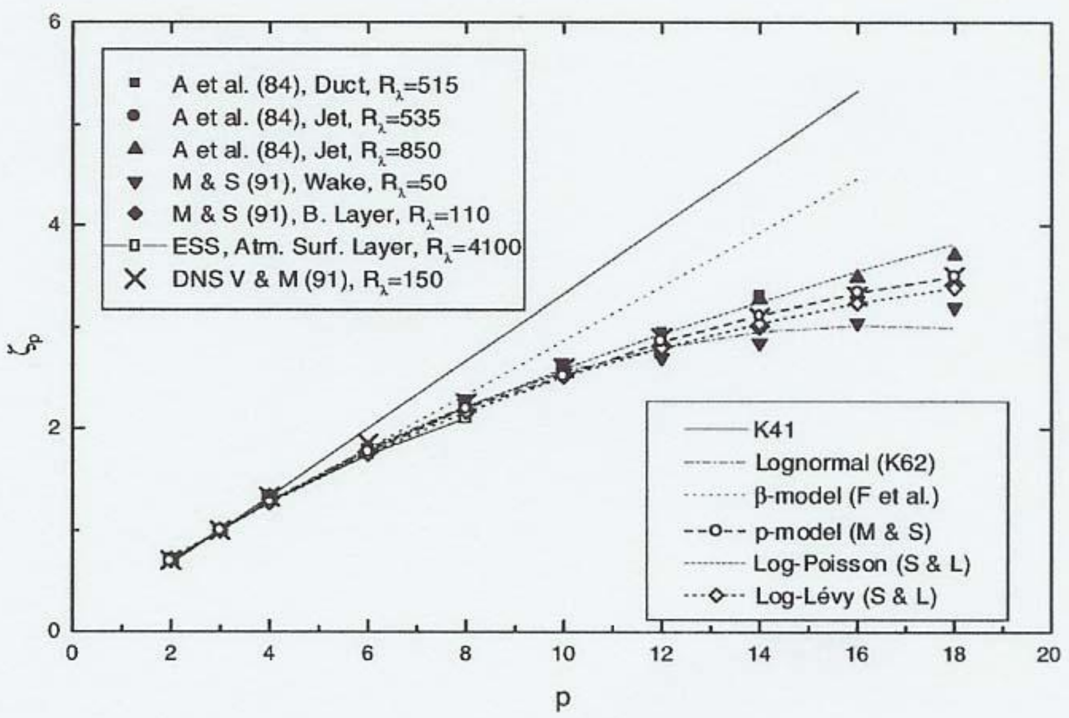


Fig. 11. Power-law exponents ζ_p of the structure functions as a function of the order p, together with the values predicted by K41 and the various intermittency models of Table 1.

O Modelo β-Randômico

O processo de fragmentação de "eddies"gera um conjunto geométrico com dimensão de Hausdorff D ‡ 3.

FRACTAIS: Conjuntos Geométricos Auto-Similares

O que significa dizer que determinada estrutura geométrica possui
<u>d</u> dimensões?

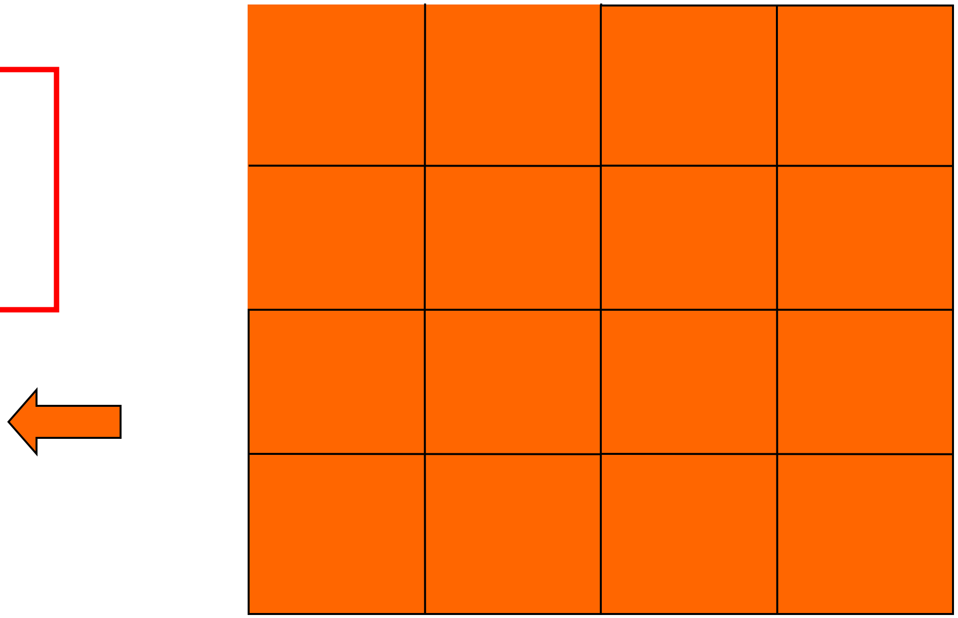
Resposta A: Parametrização (x_1, x_2,…, x_d)

Resposta B (mais geral): Pode-se cobrir um objeto geométrico de dimensão linear L com $N \sim (L/\delta)^D$ objetos menores de dimensão linear δ.

$D = \mathrm{Log}(N)/\mathrm{Log}(L/\delta) =$

“Dimensão de Hausdorff”

$N = 16,\ L/\delta = 4,\ D = 2$



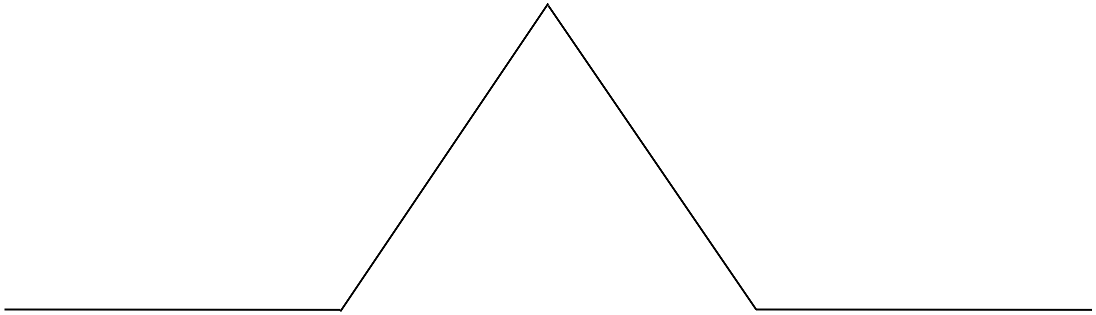
Há um número enorme de exemplos onde D não é um número inteiro!!! (Multi)fractais são onipresentes:

Costas continentais, perfis topográficos, bolas de papel amassado, materiais porosos, física das nuvens, cosmologia, interfaces rugosas, transições de fase, transições quânticas metal-isolante, involuções cerebrais, árvores, raízes, bronquíolos, sistema circulatório, sistema nervoso, dispersão de poluentes, fraturas, polímeros, trajetórias no espaço de fase de sistemas dinâmicos, mapeamentos analíticos não-lineares, turbulência, flutuações de índices financeiros, etc.

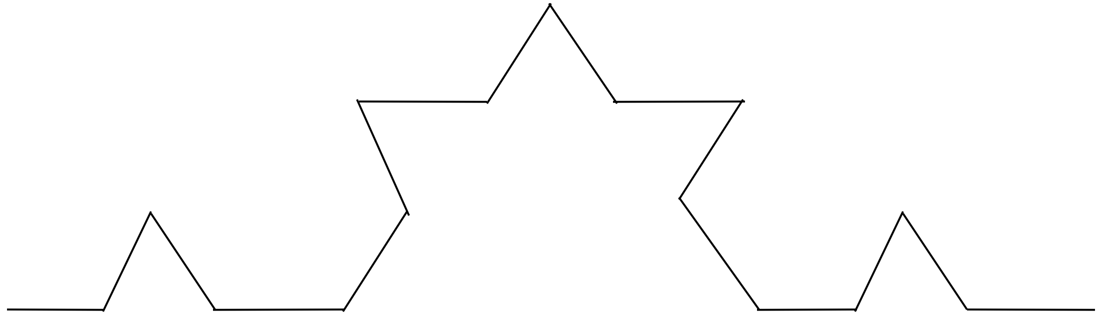
Veja o livro de B. Mandelbrot "The Fractal Geometry of Nature"

Curva de Koch

$L_0=1$, $N_0=1$, $\delta_0=1$



$L_1=4/3$, $N_1=4$, $\delta_1=1/3$



$L_2=16/9$, $N_2=16$, $\delta_2=1/9$

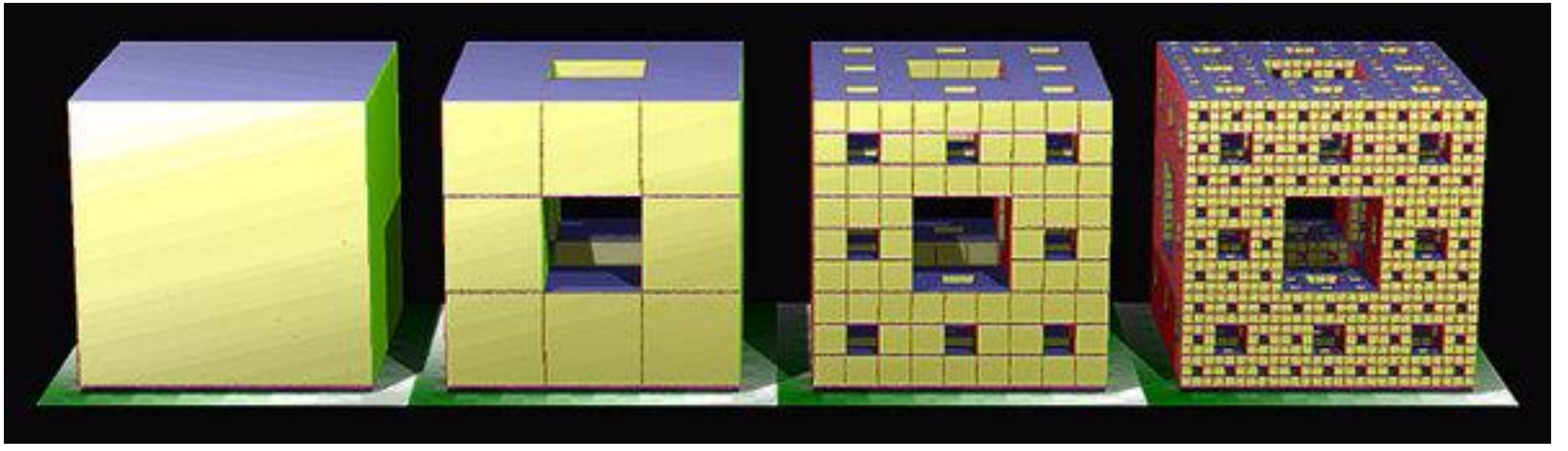
Dessa forma, após n iterações,

$L_n=(4/3)^{n}$, $N_n=4^n$, $\delta_n=(1/3)^n$

Dimensão Fractal:

$(L_0/\delta_n)^D = N_n$ ⇒ $D = \log_3(4) \approx 1.26$

Esponja de Menjer



$$N_n = 20^n = 3^{nD} \Rightarrow \quad D = Log_3(20) = 2.7268...$$

O Modelo β-Randômico

Fator de Redução de Escala: a=2

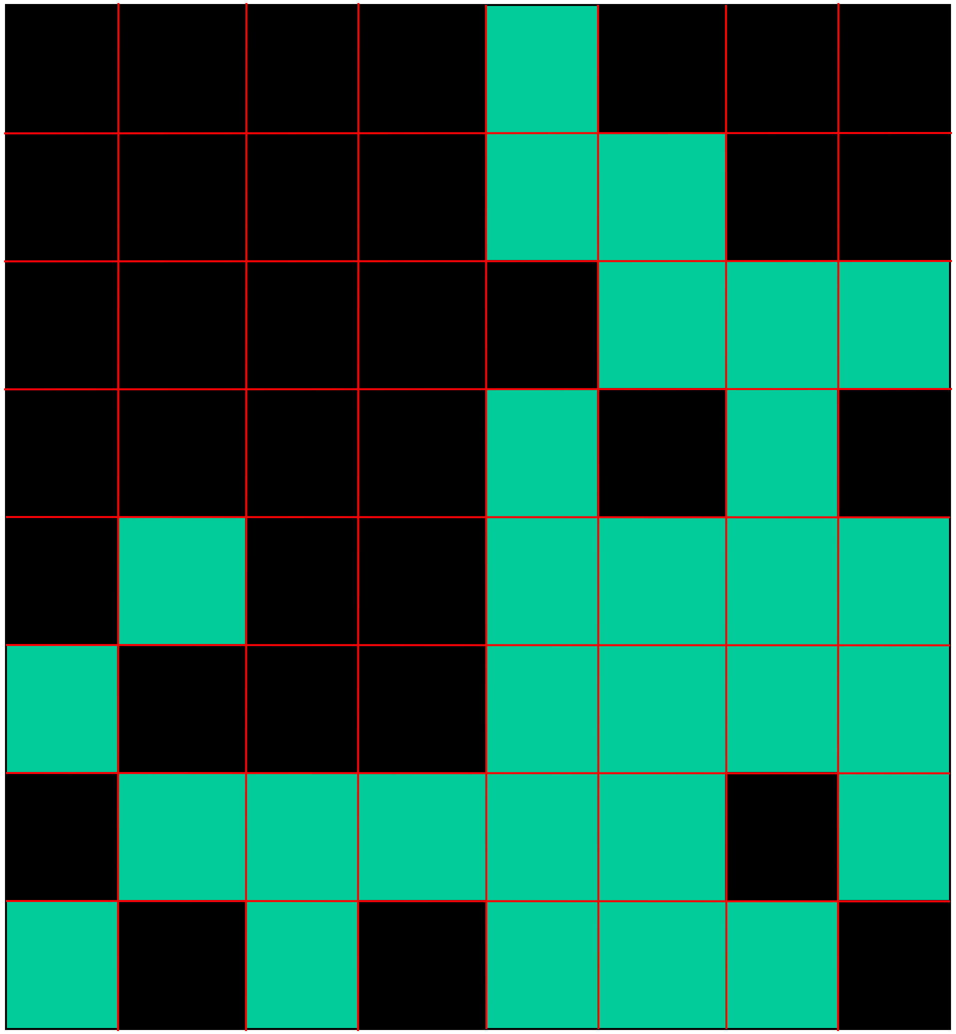
$L \rightarrow L/a \rightarrow L/a^2 \ldots$

$\beta_0=3/4$

$\beta_1=3/4, 1$

$\beta_2=1/2, 3/4, 1$

etc…



Número de “eddies” Fragmentados:

$N(1/2)=4$

$N(3/4)=6$

$N(1)=4$

$N_{tot}=1+3+10=14$

O Modelo β-Randômico

Fator de Redução de Escala: $a=2$

$L \rightarrow L/a \rightarrow L/a^2 \ldots$

Número de "eddies" Fragmentados:

$N(1/2)=4$

$N(3/4)=6$

$N(1)=4$

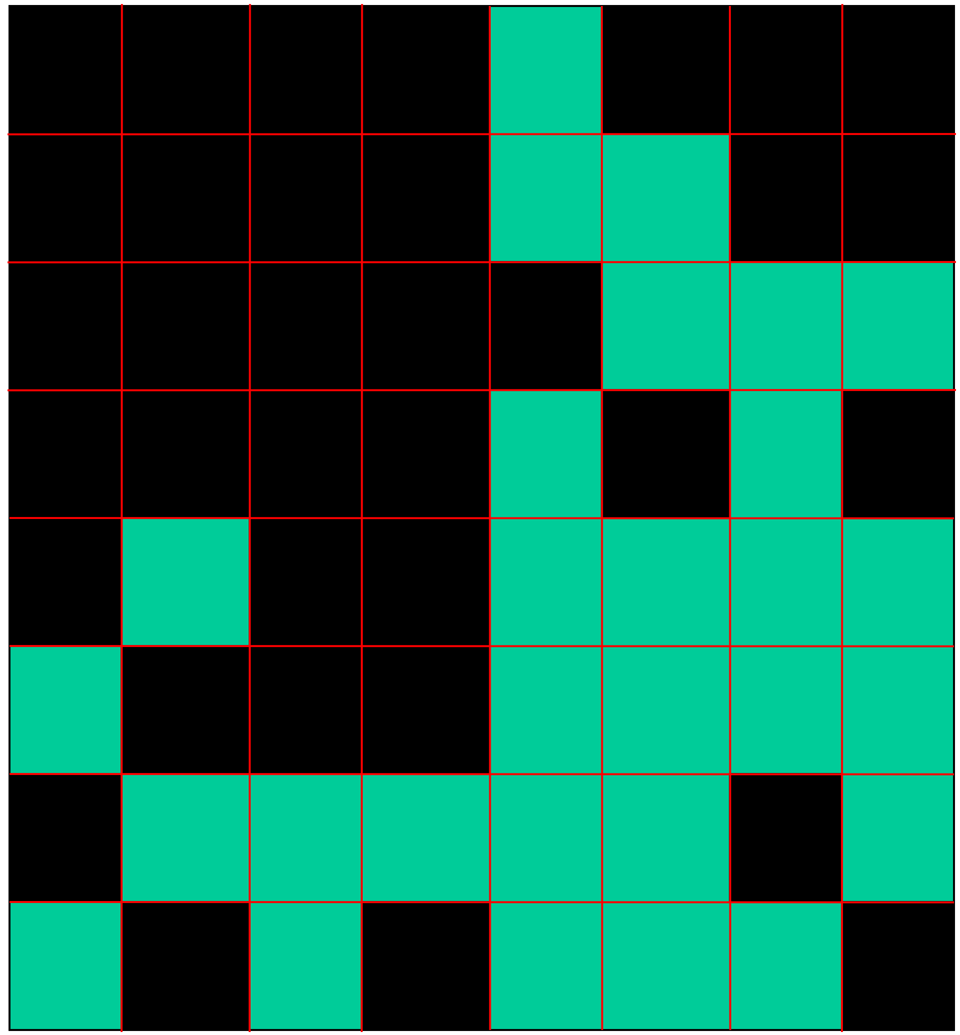
$N_{tot}=1+3+10=14$

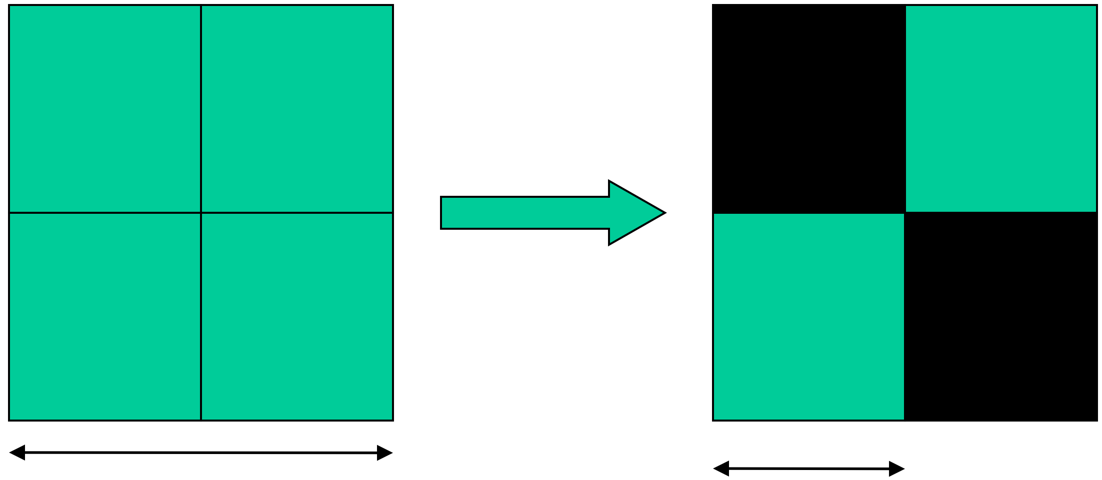
Probabilidades Estimadas:

$P(1/2)=4/14=2/7$

$P(3/4)=6/14=3/7$

$P(1)=4/14=2/7$





L_n $\qquad$ L_{n+1}

Eddy Pai $\qquad$ N_{n+1} Eddies (filhos)

$$\varepsilon_n = (L_n)^3 (V_n)^2 / T_n$$
$$= (L_n)^3 (V_n)^3 / L_n$$

$$\varepsilon_{n+1} = N_{n+1} (L_{n+1})^3 (V_{n+1})^2 / T_{n+1}$$
$$= N_{n+1} (L_{n+1})^3 (V_{n+1})^3 / L_{n+1}$$

Mas $\varepsilon_n = \varepsilon_{n+1}$ …

transfer is constant among $l_n(k)$ and $l_{n+1}(k)$:

$$V_n^3(k)/l_n(k) = \beta_{n+1}(k)\, V_{n+1}^3(k)/l_{n+1}(k). \tag{3.5}$$

This relation implies that the velocity difference $\delta V(l_n) \equiv V_n$ in an eddy generated by a particular set of fragmentations $\beta_1, \beta_2, \ldots, \beta_n$, is

$$V_n \sim l_n^{+1/3}\left(\prod_{i=1,n} \beta_i\right)^{-1/3}. \tag{3.6}$$

The structure functions are then

$$\langle|\delta V(l_n)|^p\rangle = \int \prod_{i=1,n} \mathrm{d}\beta_i P(\beta_1, \ldots, \beta_n)\beta_i |V_n|^p. \tag{3.7}$$

Because we assumed that there are no correlations among different steps of the fragmentation process, it follows that

$$\prod_{i=1,n} \mathrm{d}\beta_i\, P(\beta_1, \ldots, \beta_n) = \prod_{i=1,n} P(\beta_i)\, \mathrm{d}\beta_i.$$

From (3.7) we can compute the exponents ζ_p:

$$\zeta_p = p/3 - \log_2\{\beta^{(1-p/3)}\}. \tag{3.8}$$

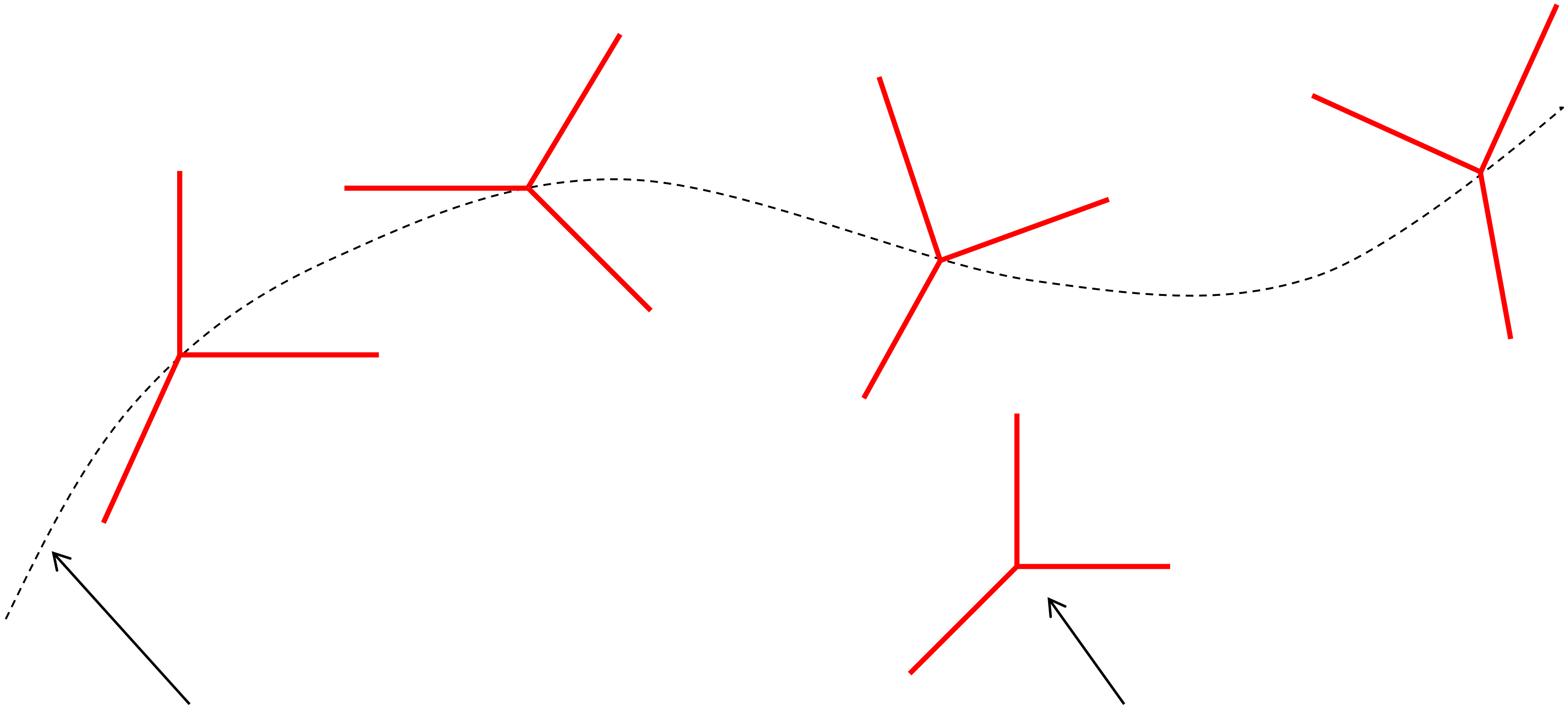
The probability distribution $P(\beta)$ is known in principle from the knowledge of all the β moments, i.e. of all the exponents ζ_p. Figure 1 shows that the simple form

$$P(\beta) = x\delta(\beta - 0.5) + (1-x)\delta(\beta - 1) \tag{3.9}$$

leads (with $x = 0.125$) to a good fit to the available experimental data, x being the only free parameter. There is no good reason to choose a two-step probability distribution for β, of course. We have assumed that an active eddy can generate either velocity sheets ($\beta = 0.5$) or space-filling Kolmogorov-like eddies ($\beta = 1$) (see Saffman 1968). We see, by comparing relation (3.8) and (3.4), that in our model the fractal dimension is

IV. Modelagem Lagrangeana

We work in a lagrangian framework where a local closure procedure can be implement, the so-called "Recent Fluid Deformation Closure" (RFDC)



In the particular case of the RFDC model, we may write, for the conditional probability density function,

$$\rho(\mathbb{A}_1|\mathbb{A}_0,\beta) \equiv \mathcal{N} \int_\Sigma D[\hat{\mathbb{A}}] D[\mathbb{A}] \exp\left\{ - \int_0^\beta dt \left[i \mathrm{Tr}[\hat{\mathbb{A}}^{\mathrm{T}} L(\mathbb{A})] + \frac{g^2}{2} G_{ijkl} \hat{A}_{ij} \hat{A}_{kl} \right] \right\}$$

where the boundary conditions are given by

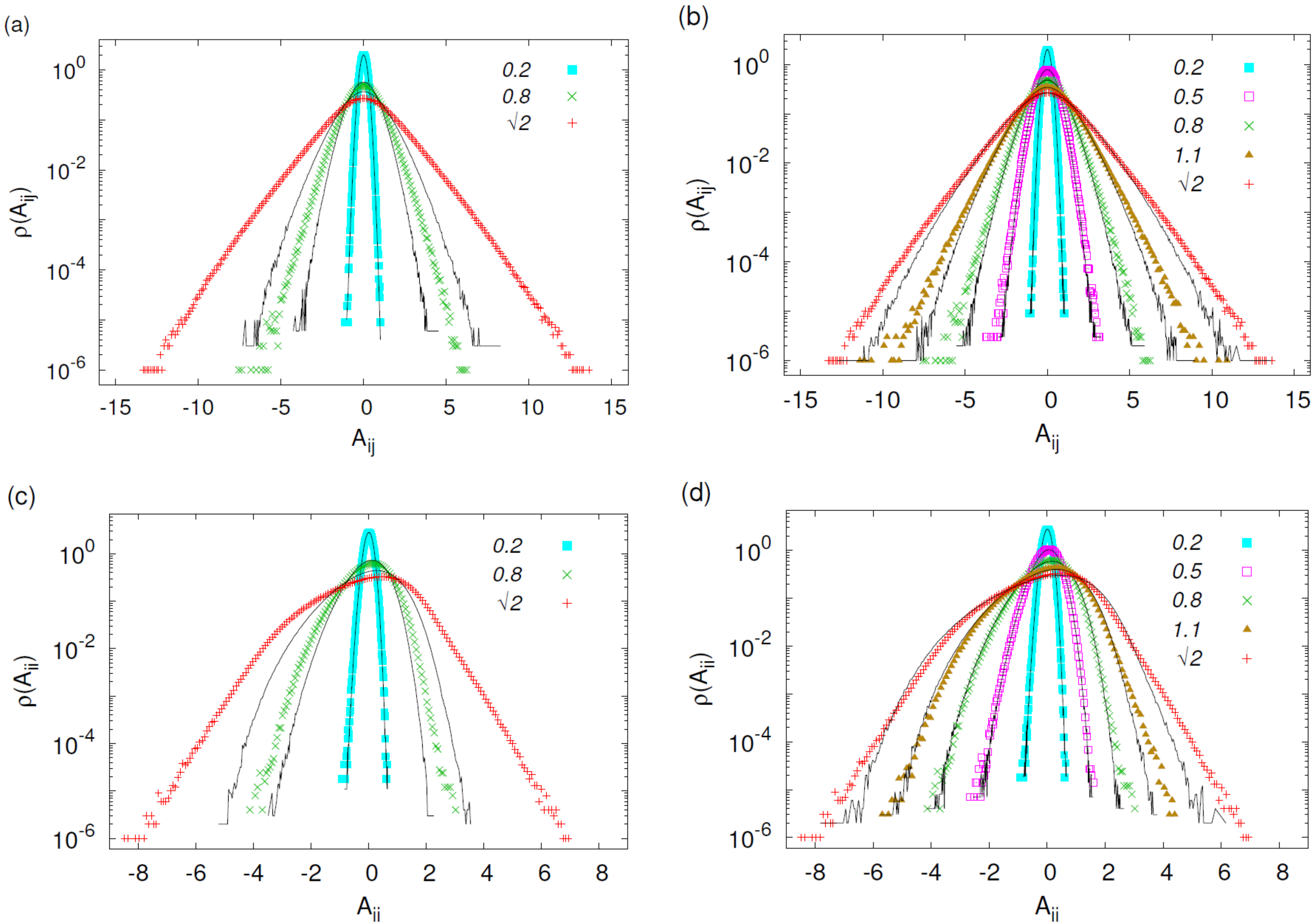
$$\Sigma = \{\mathbb{A}(0) = \mathbb{A}_0 \ , \ \mathbb{A}(\beta) = \mathbb{A}_1\}$$

and

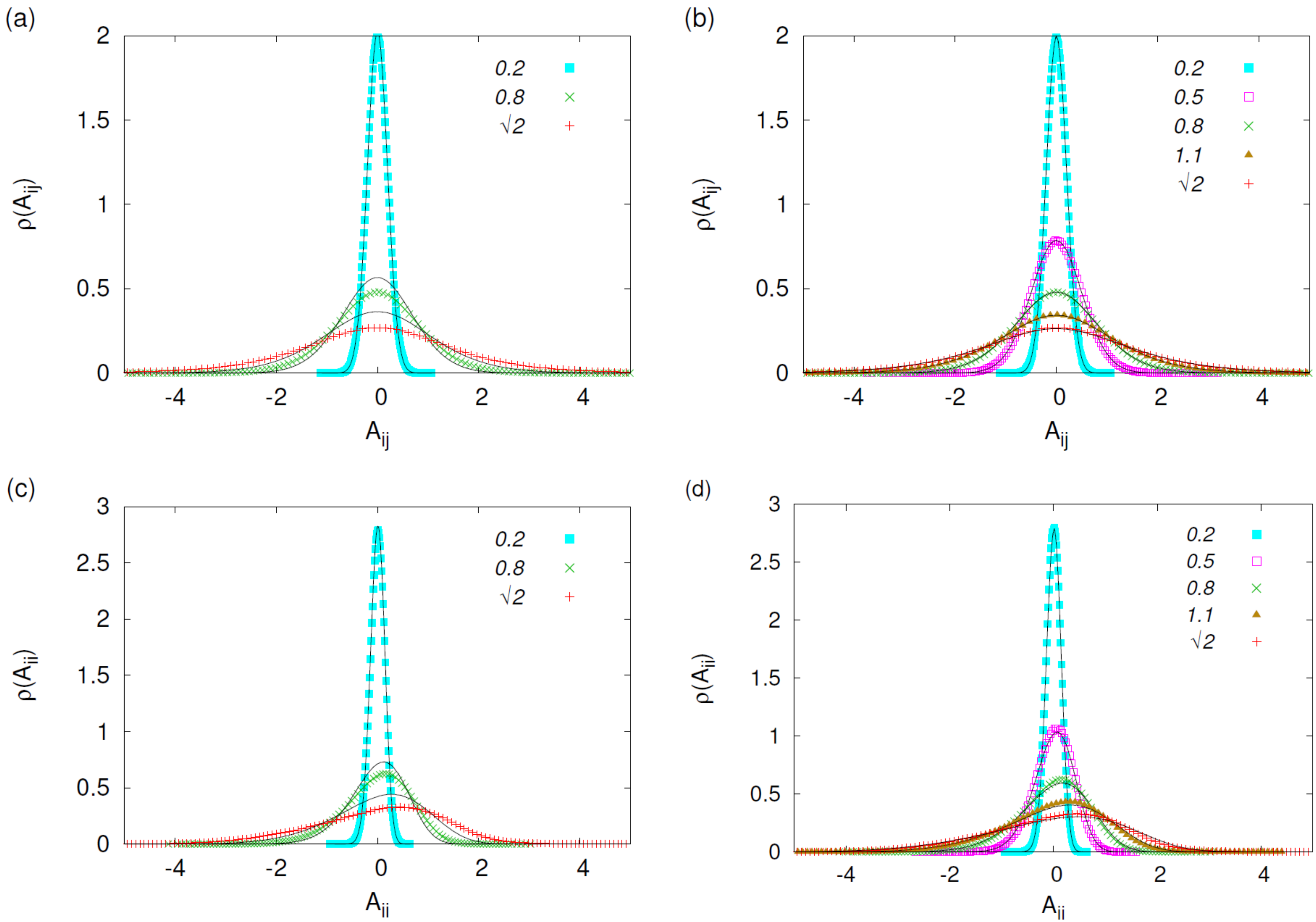
$$L(\mathbb{A}) \equiv \dot{\mathbb{A}} - V(\mathbb{A})$$

We are interested to compute the above path-integral in the large β limit in order to obtain the vgPDFs. That is, we assume that there are no long-range memory effects associated to arbitrary choices of the initial conditions.

L. M., R. Pereira e L.S. Grigorio, JSTAT 2014.



(a)
0.2
0.8
√2
ρ(A_ij)
A_ij
(b)
0.2
0.5
0.8
1.1
√2
ρ(A_ij)
A_ij
(c)
0.2
0.8
√2
ρ(A_ii)
A_ii
(d)
0.2
0.5
0.8
1.1
√2
ρ(A_ii)
A_ii



(a)
(b)
(c)
(d)
0.2
0.5
0.8
1.1
√2
ρ(A_ij)
ρ(A_ii)
A_ij
A_ii

V. CONCLUSÕES

- Apesar dos problemas fundamentais em turbulência ainda estarem essencialmente abertos, têm ocorrido progressos sem paralelo histórico em anos recentes, principalmente relacionados à compreensão do fenômeno da intermitência e à física da camada limite turbulenta;

- Há uma forte conexão com outros tópicos fronteiriços de pesquisa, como localização eletrônica, turbulência quântica, cosmologia, física de hadrons, e até mesmo econofísica;

- O problema da turbulência homogênea e isotrópica pode ser formulado na linguagem de teoria de campos. Trata-se de um sistema dinâmico em regime altamente não-perturbativo;

- O formalismo lagrangeano apresenta-se como um caminho promissor no contexto da teoria estatística da turbulência.

• A pesquisa em turbulência é altamente interdisciplinar, envolvendo a interação entre engenharia, física, matemática, meteorologia; atividade experimental e numérica intensa;

Artigo Divulgativo:

CH Outubro de 2008