Instituto de Matemática

Introdução à Teoria Estatística da Turbulência

COLMEA

UFRJ

2016

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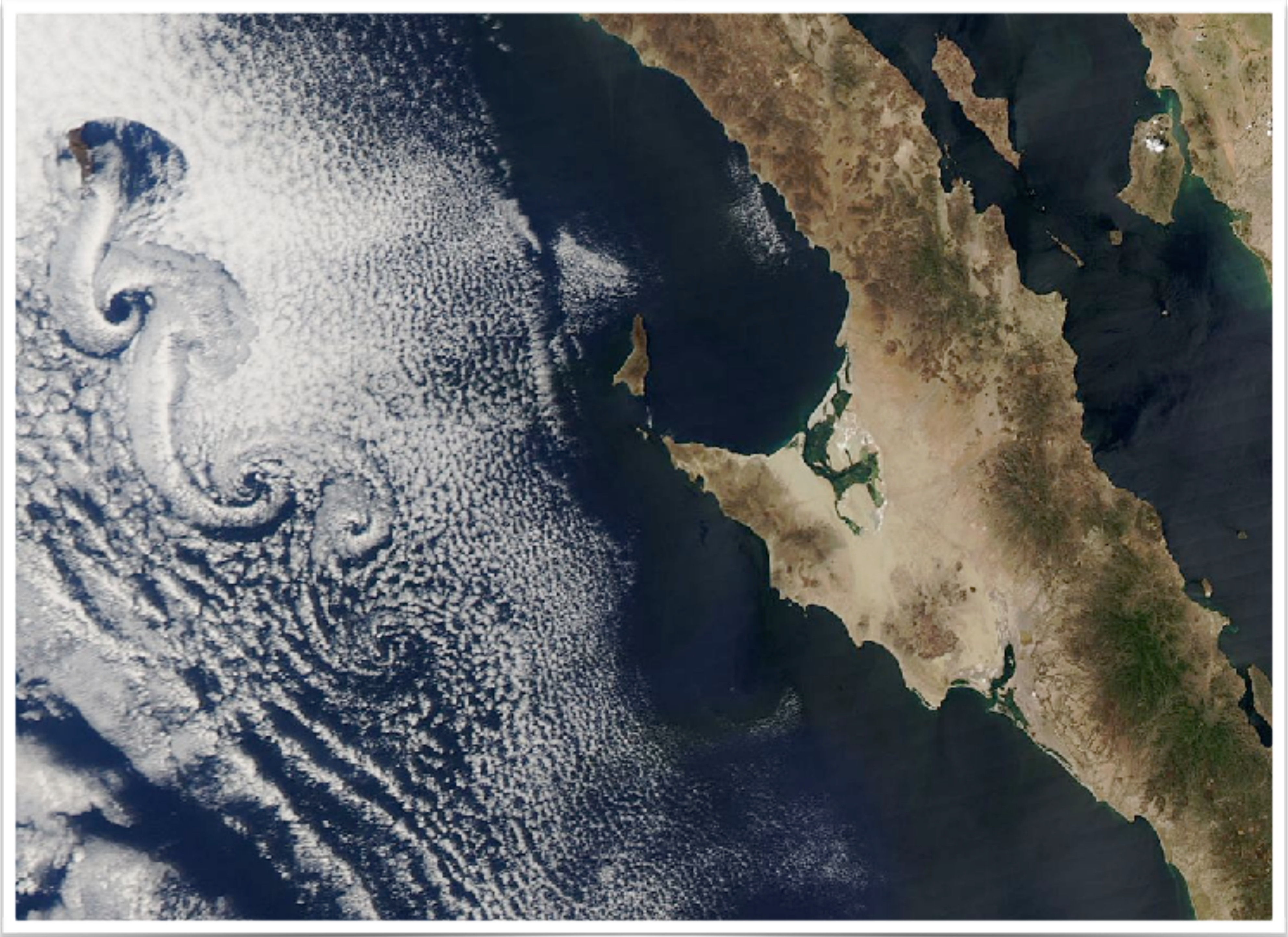
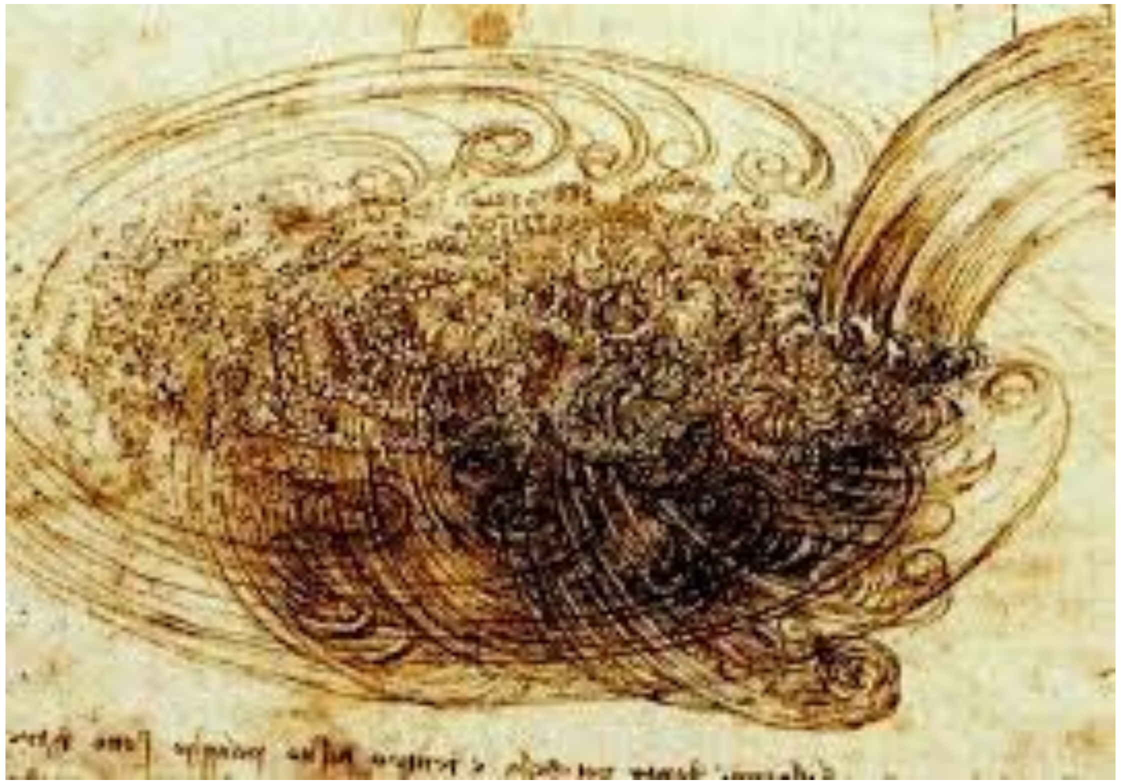
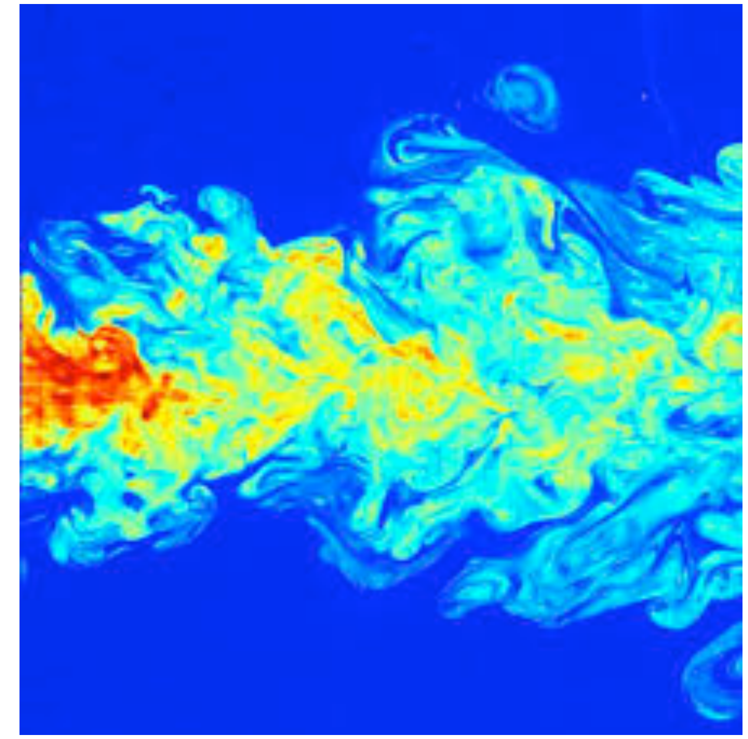
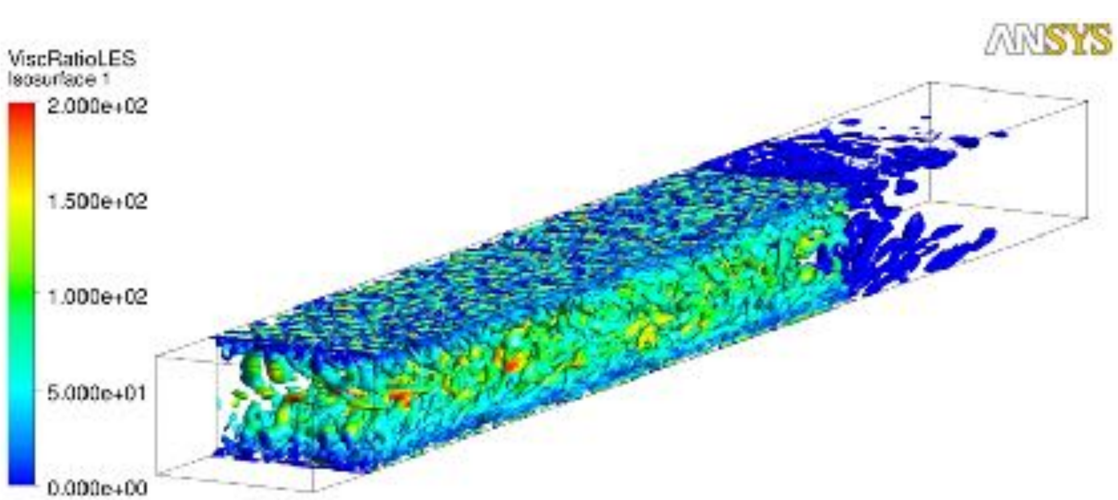


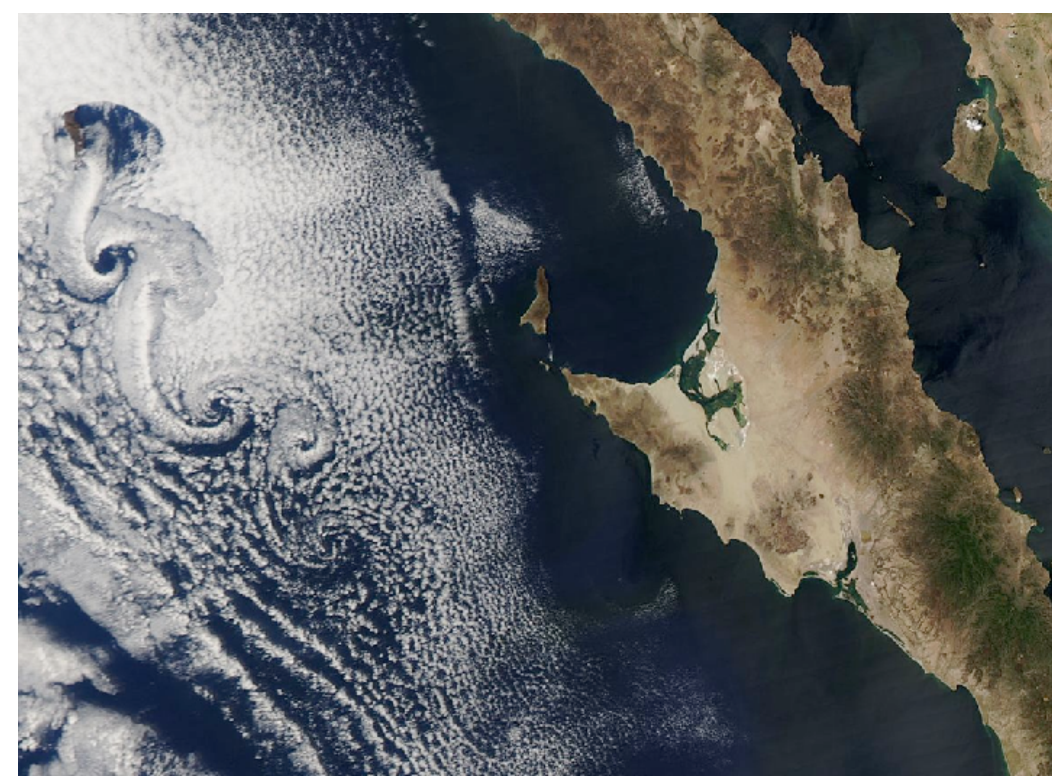
Turbulence

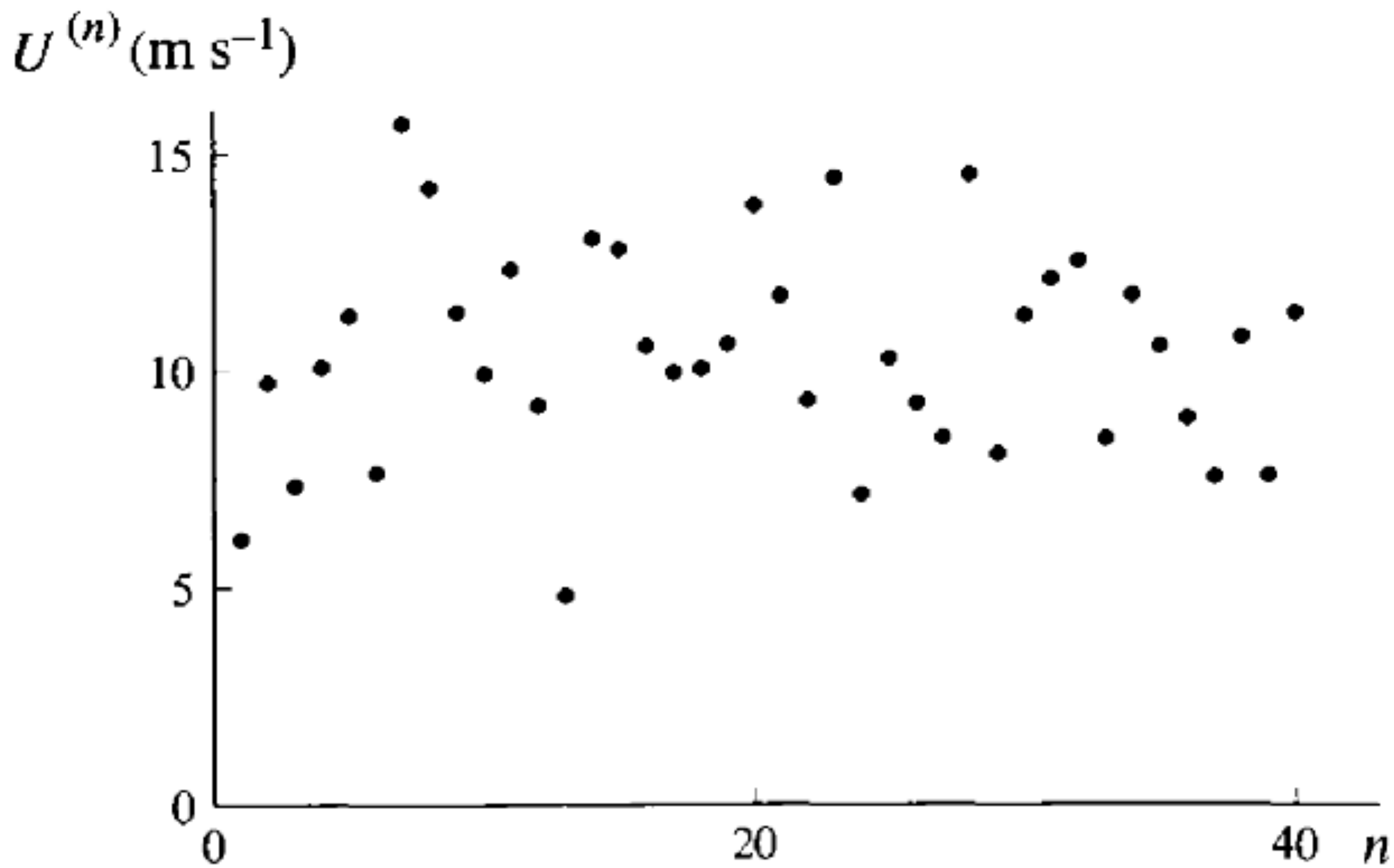




1500 - “Doue la turbolenza dellacqua rigenera, doue la turbolenza dellacqua simantiene plugho, doue la turbolenza dellacqua siposa"

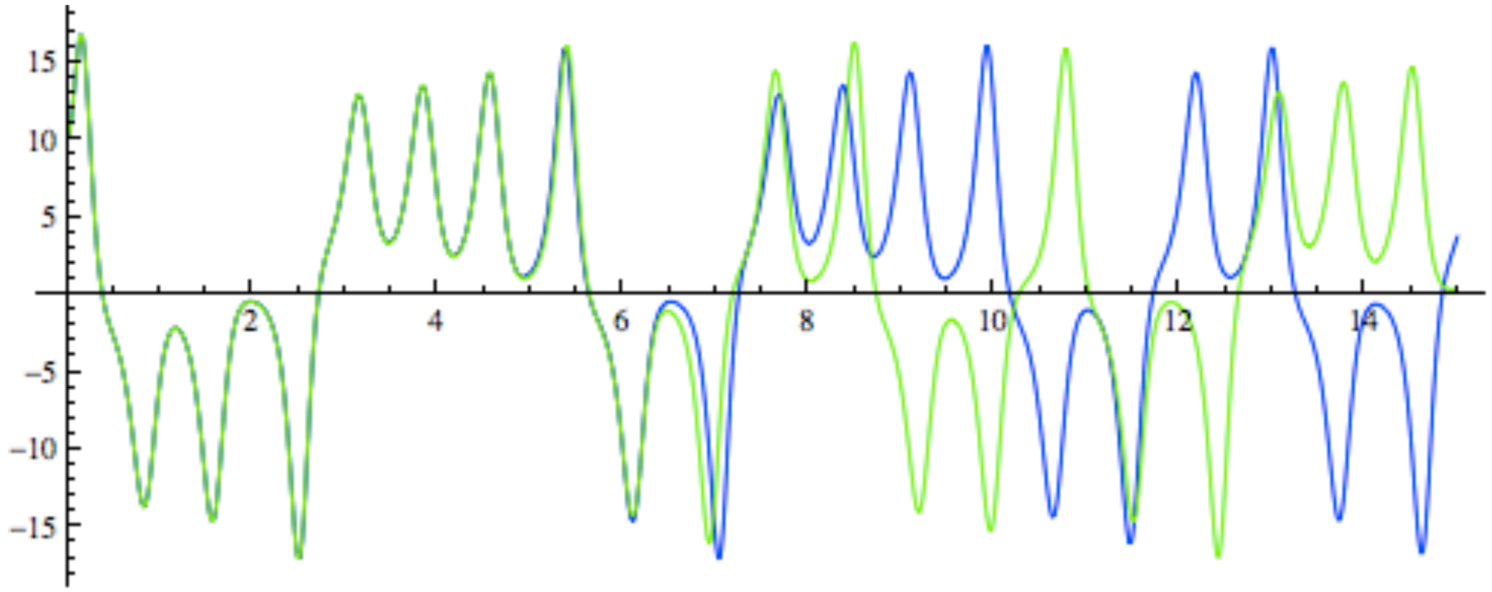




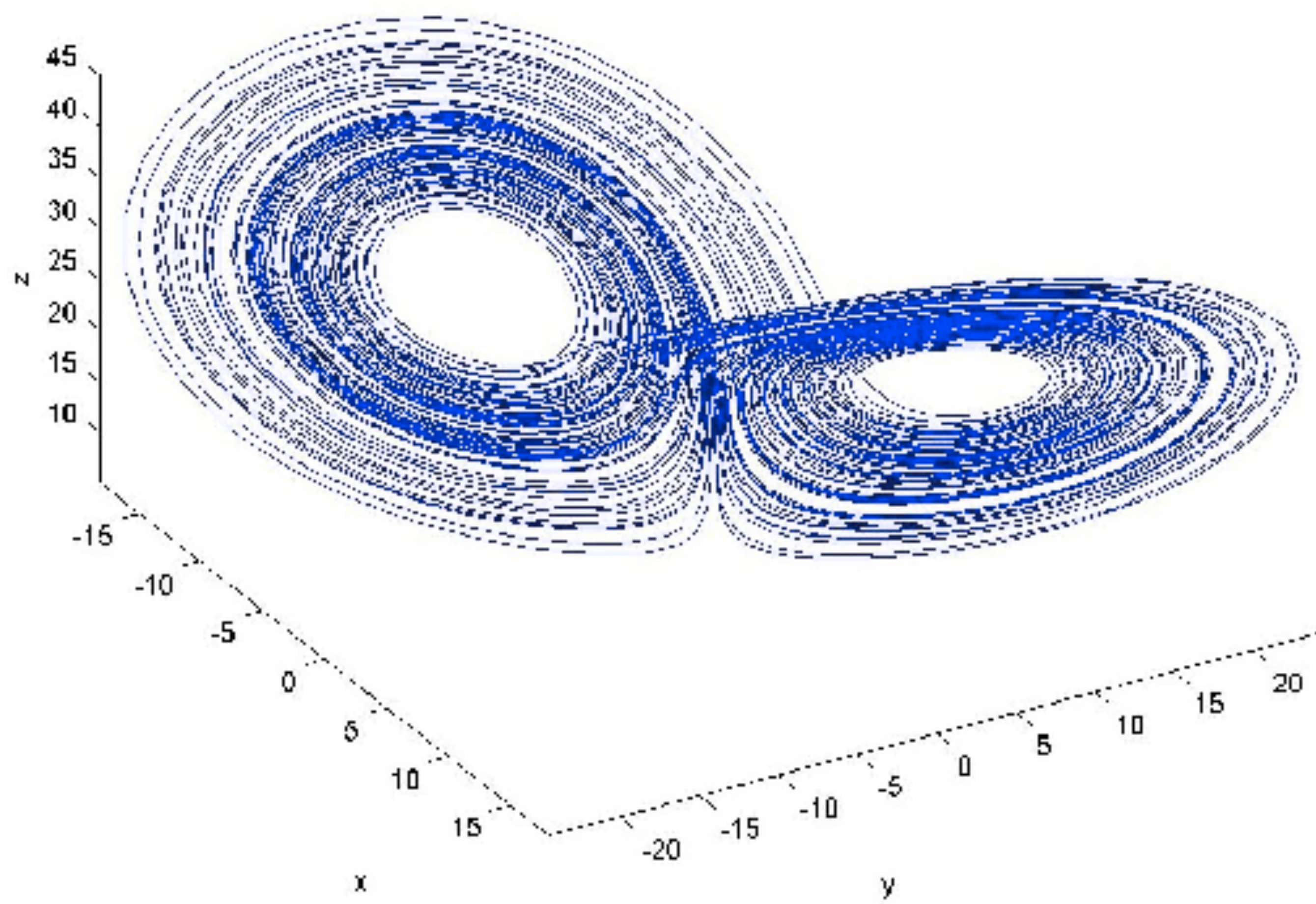


$U^{(n)}$ (m s^{-1})
15
10
5
0
0
20
40
n

A metáfora de Lorenz



$$\frac{dx}{dt} = \sigma(y - x)$$
$$\frac{dy}{dt} = x(\rho - z) - y$$
$$\frac{dz}{dt} = xy - \beta z$$



Turbulência
(uma torre de metáforas)

Intermitência
Multi-Escala

Caos

Teoria da
perturbação

Estruturas coerentes

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.
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Singularidades

Dissipação Anômala

Teorias de
Renormalização

Leis de parede

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.

Plano da nossa viagem

- Turbulence and transition
- Equations of motion and wall effects
- Spectrum and Inertial range
- singularity signature and singular limits
- Reynolds decomposition and Prandtl revolution
- Non-newtonian rheology and drag reduction
- Kolmogorov meets Prandtl

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Conceitos elementares

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Conceitos elementares

Soluções Selvagens (Math alert)

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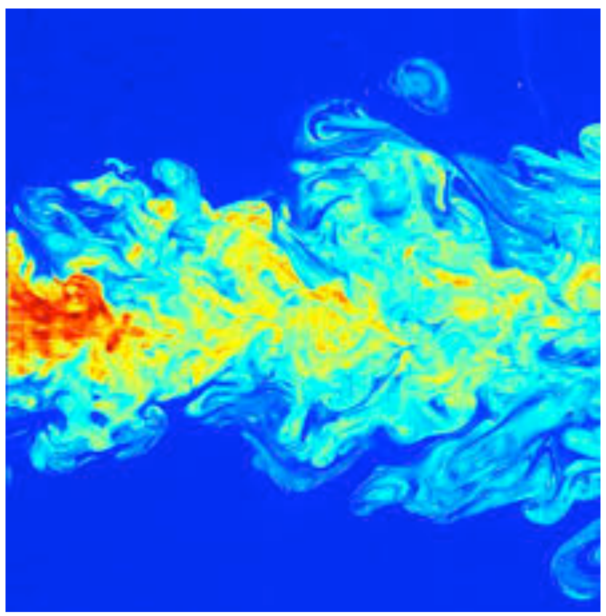
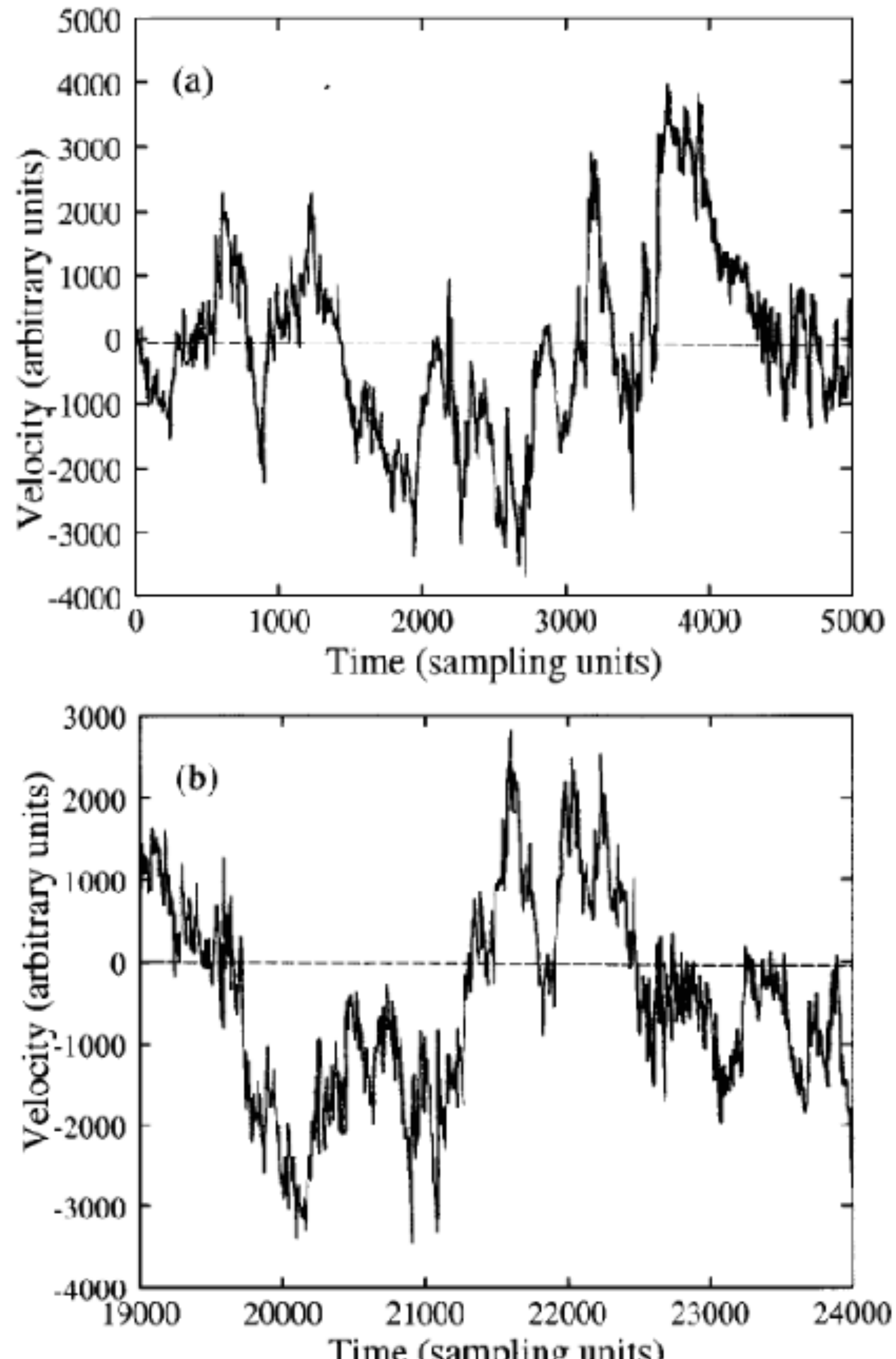
Conceitos elementares

Soluções Selvagens (Math alert)

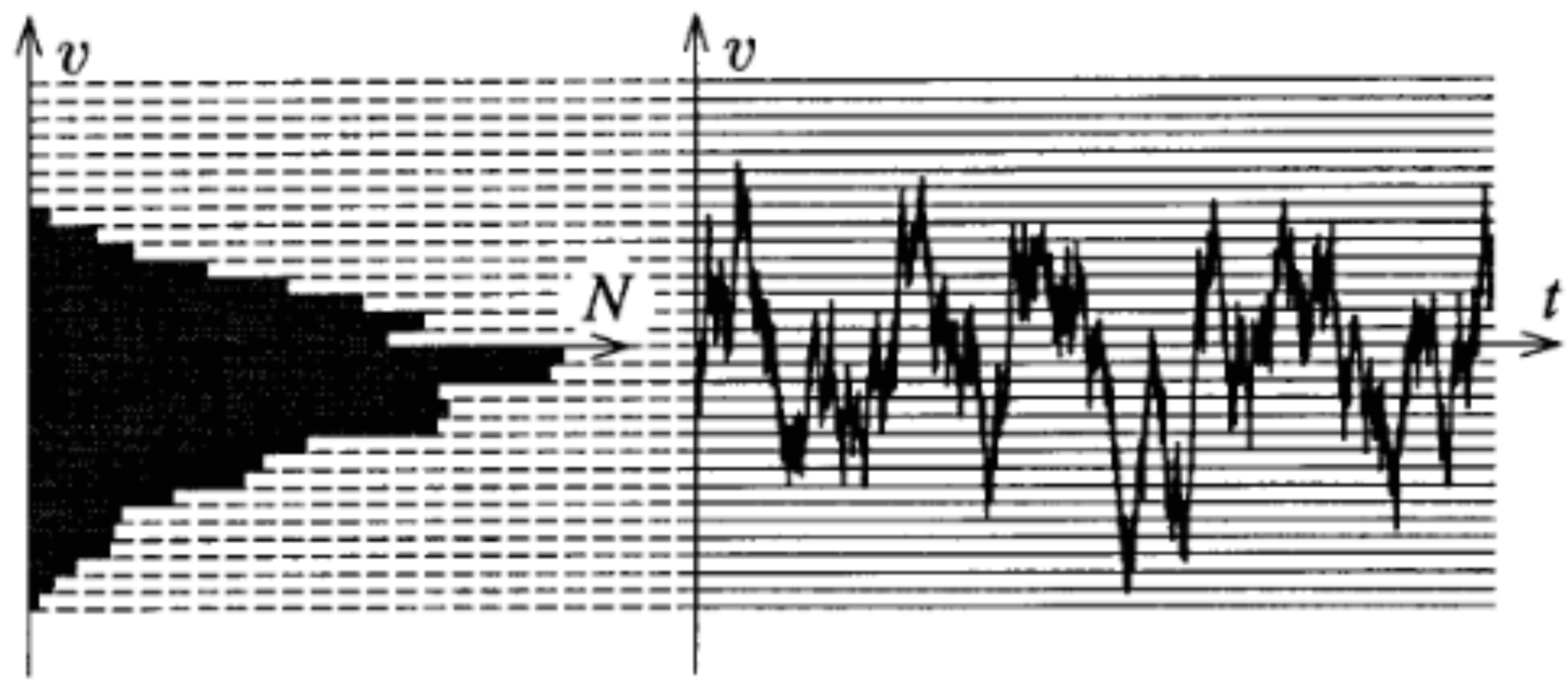
O mundo perto do parede

Can we actually hope to find a good model for turbulence ?

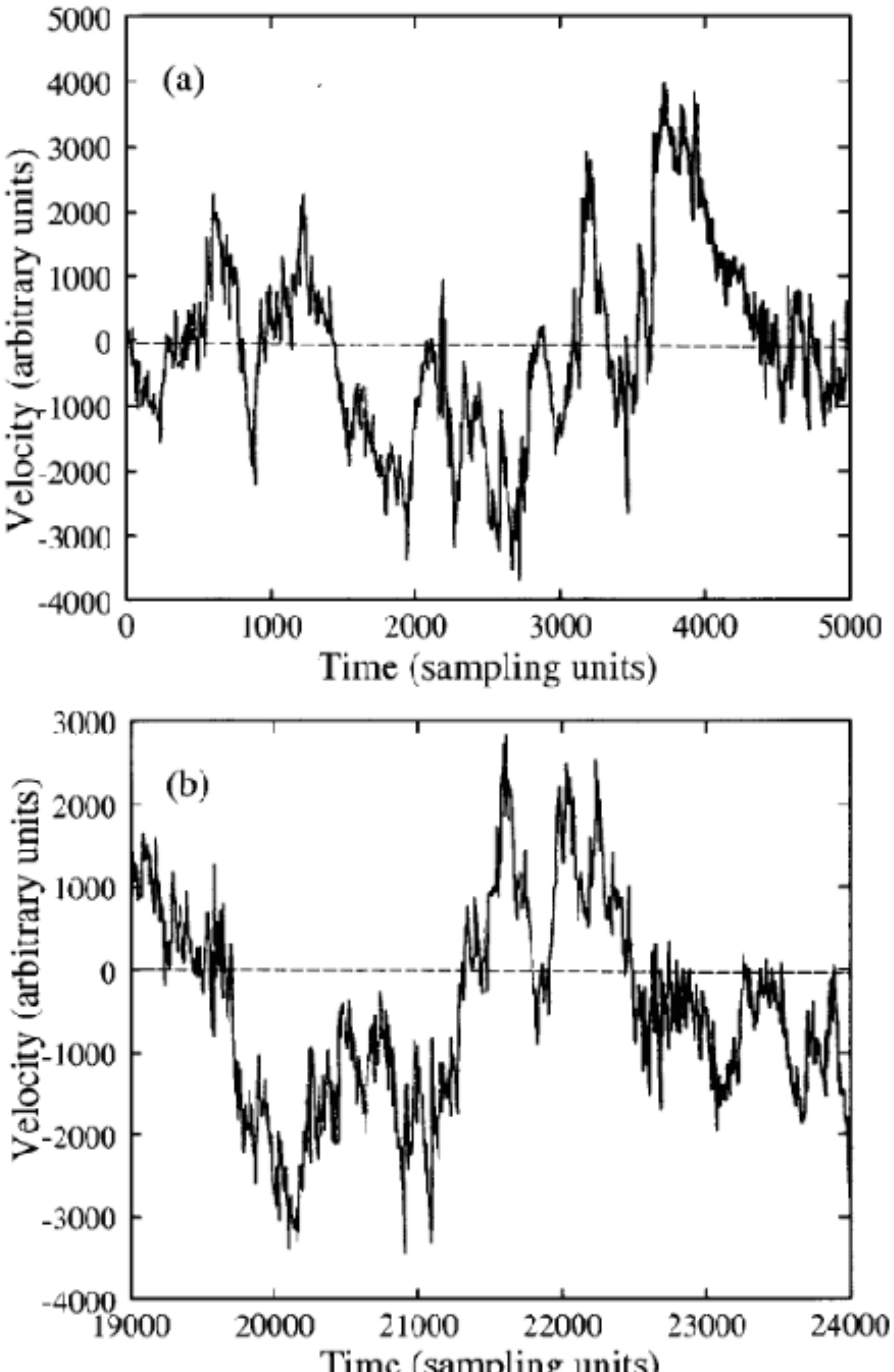


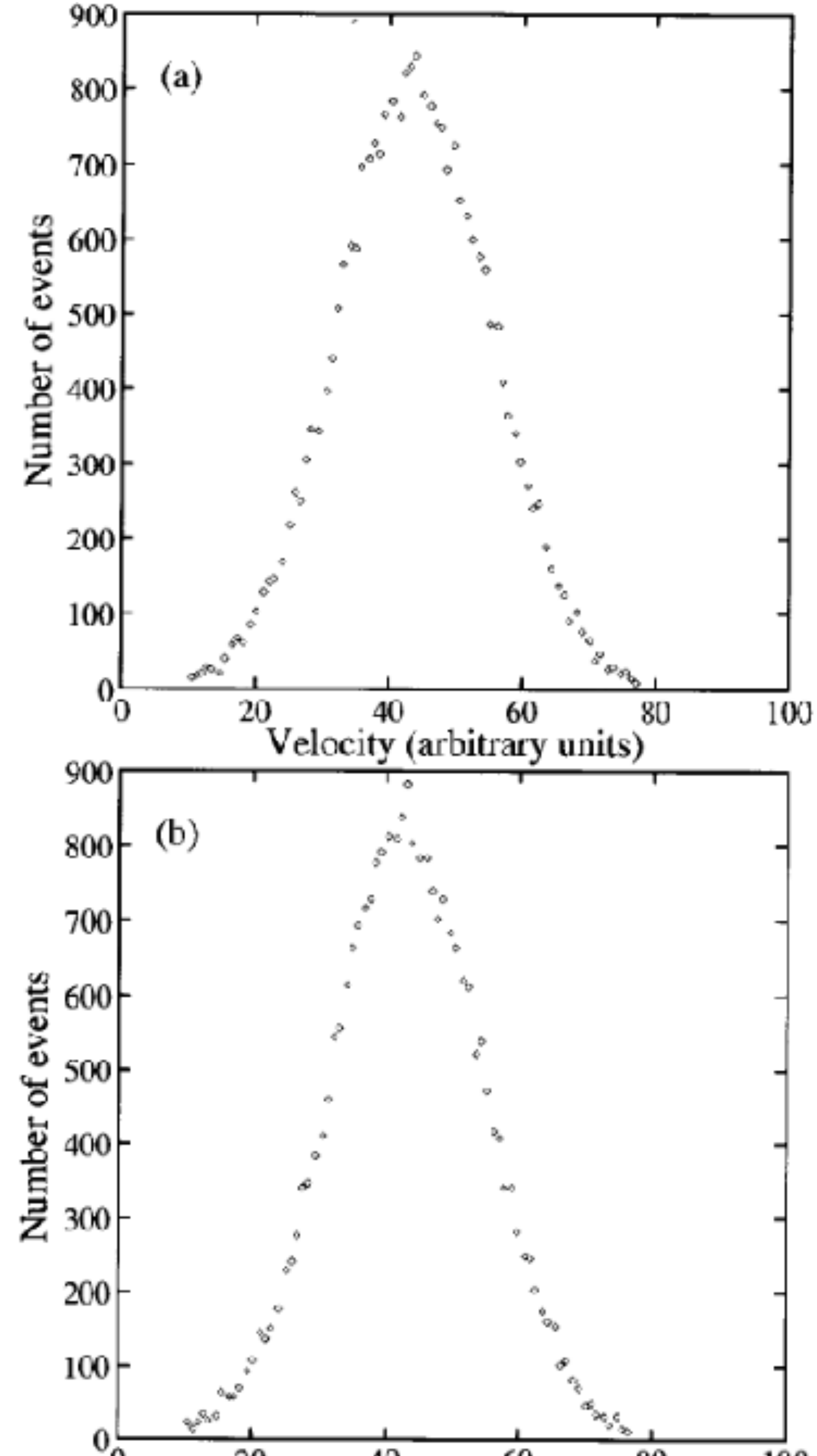


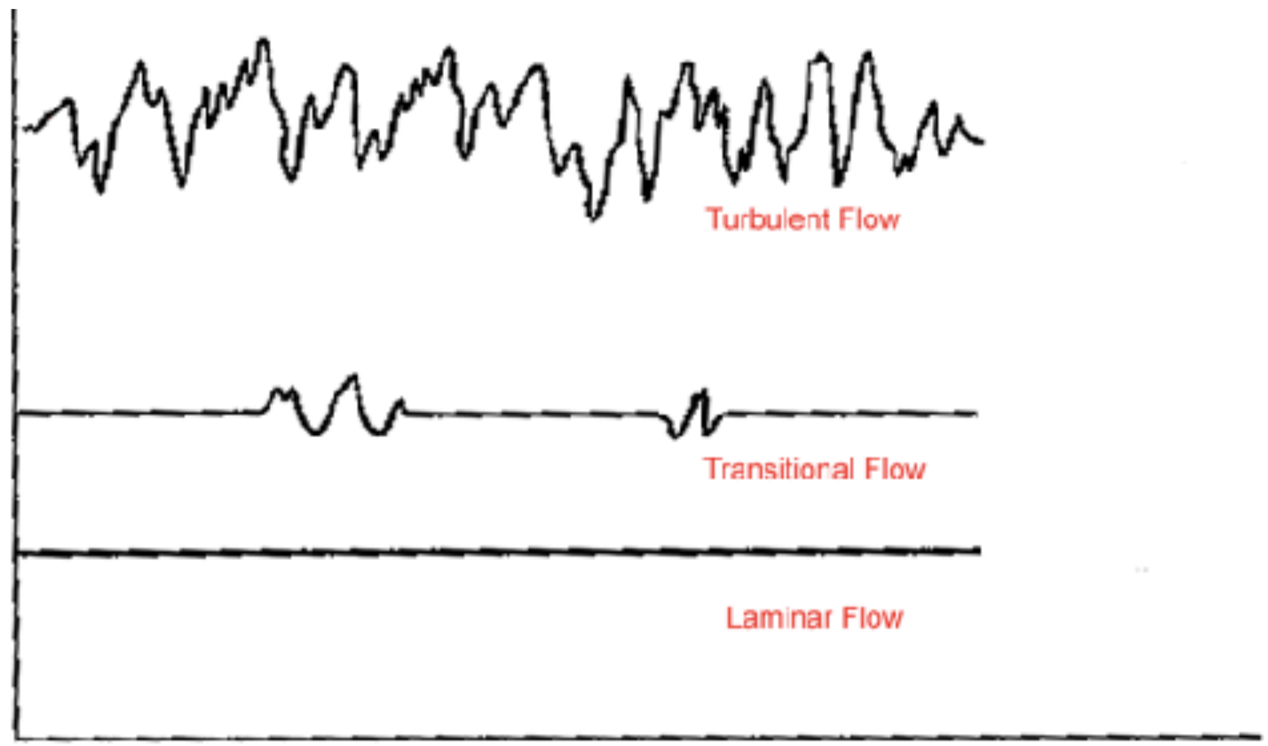
One-dimensional statistics

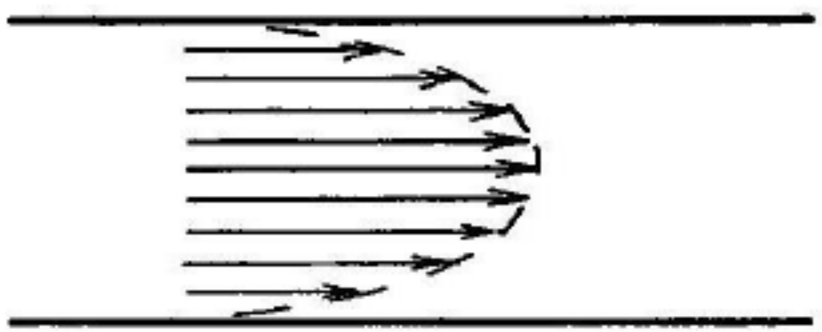


Stationarity

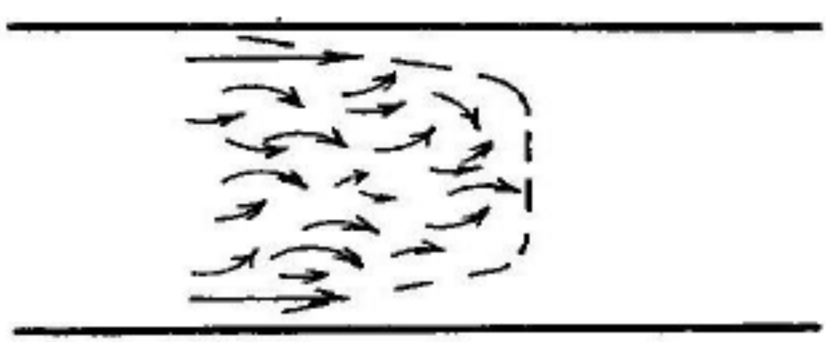




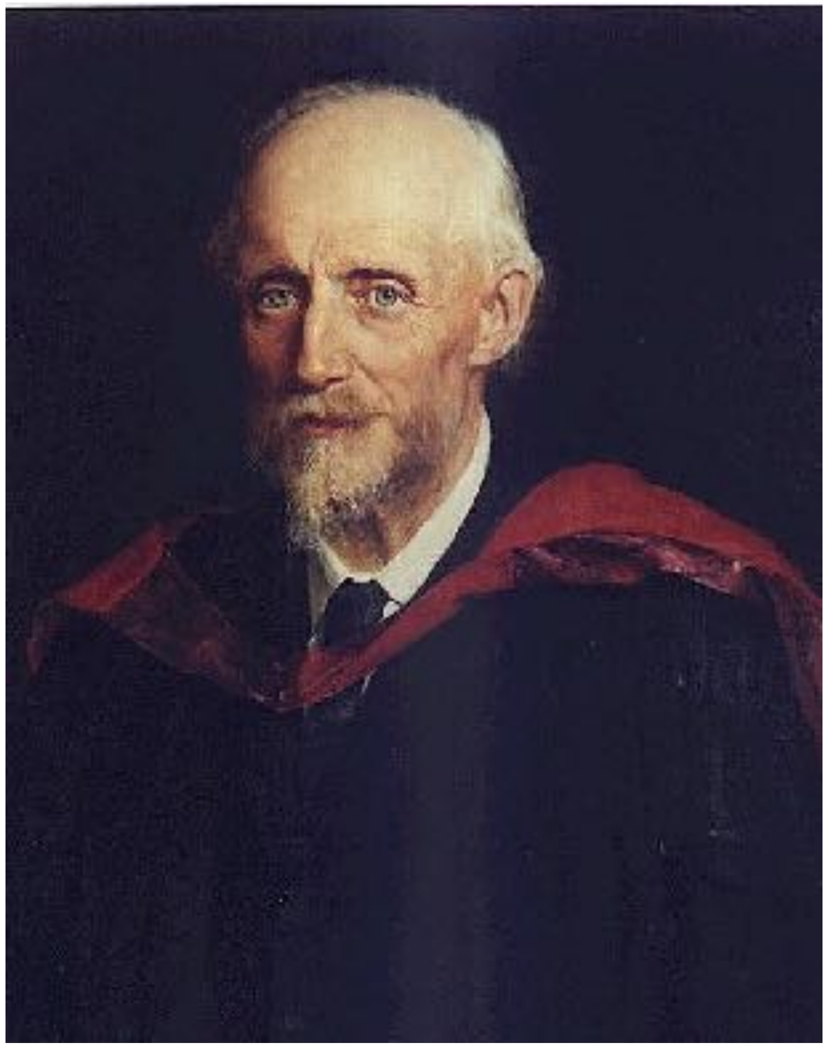




laminar

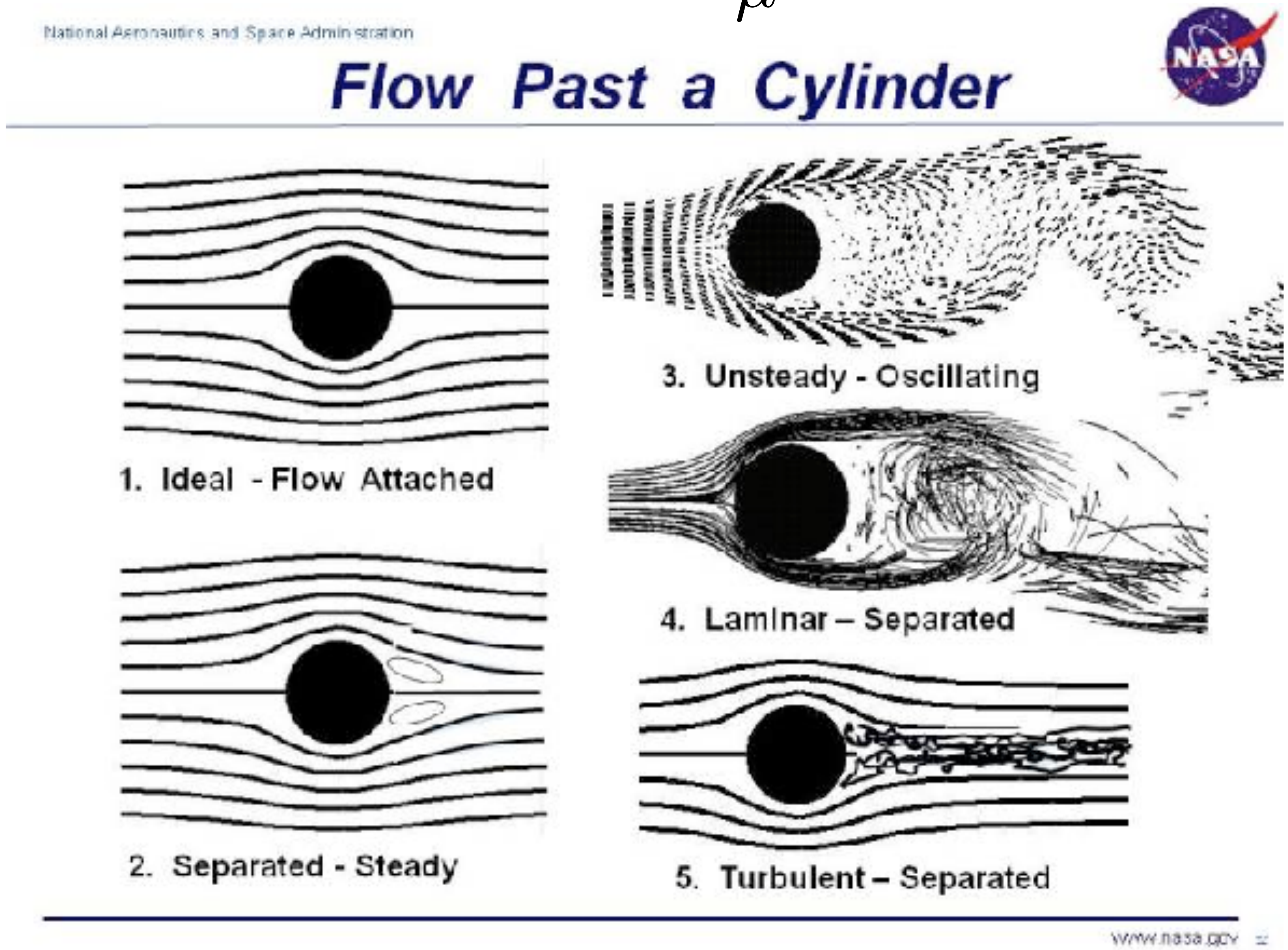


turbulent

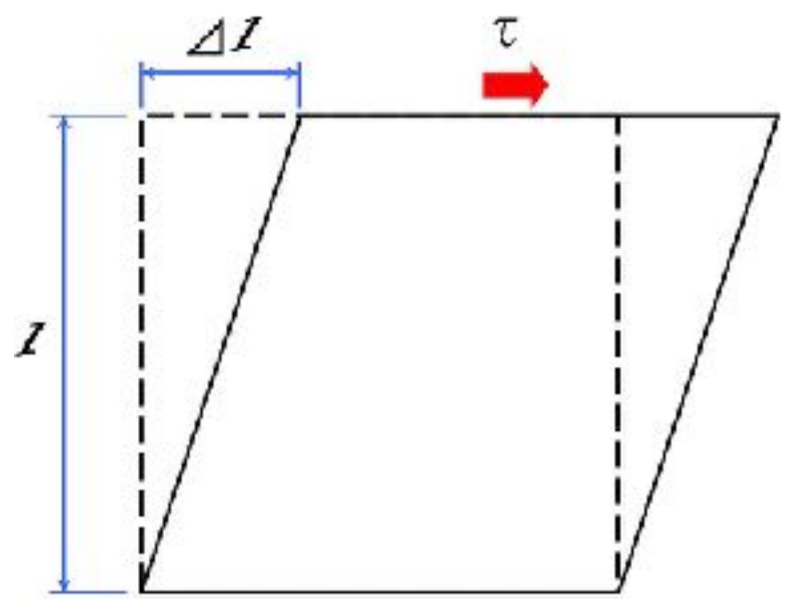


Osborne Reynolds
1842-1912

$$Re = \frac{\rho U L}{\mu}$$



Organism	Reynolds Number
A whale swimming at 10m/s	300,000,000
A duck flying at 20m/s	300,000
A human swimming in water at 2m/s	10,000
Invertebrate larva, 0.3mm long at 1mm/s	0.3
A bacterium swimming at 0.01mm/s	0.00001



$$[\mu] = \frac{M}{LT}$$

$$\tau = \mu \frac{dU}{dx}$$

$$Re = \frac{\rho U L}{\mu}$$

"The mountains flow before the Lord"

Deborah, Book of Judges, Cap.5, vers. 5

Como escoar mel ?

$$Re = \frac{\rho U L}{\mu}$$

The Longest Experiment - 1927->

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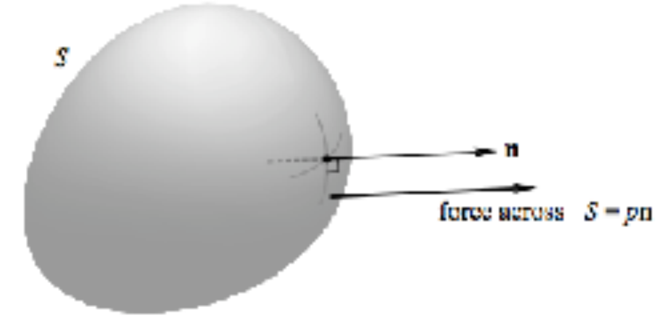
O mundo sem viscosidade- EULER, 1757

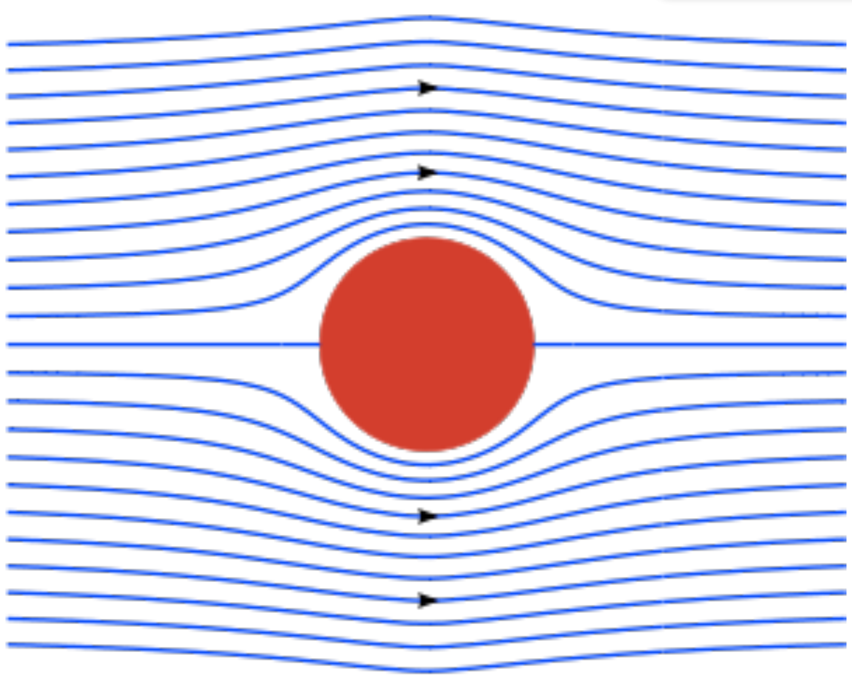


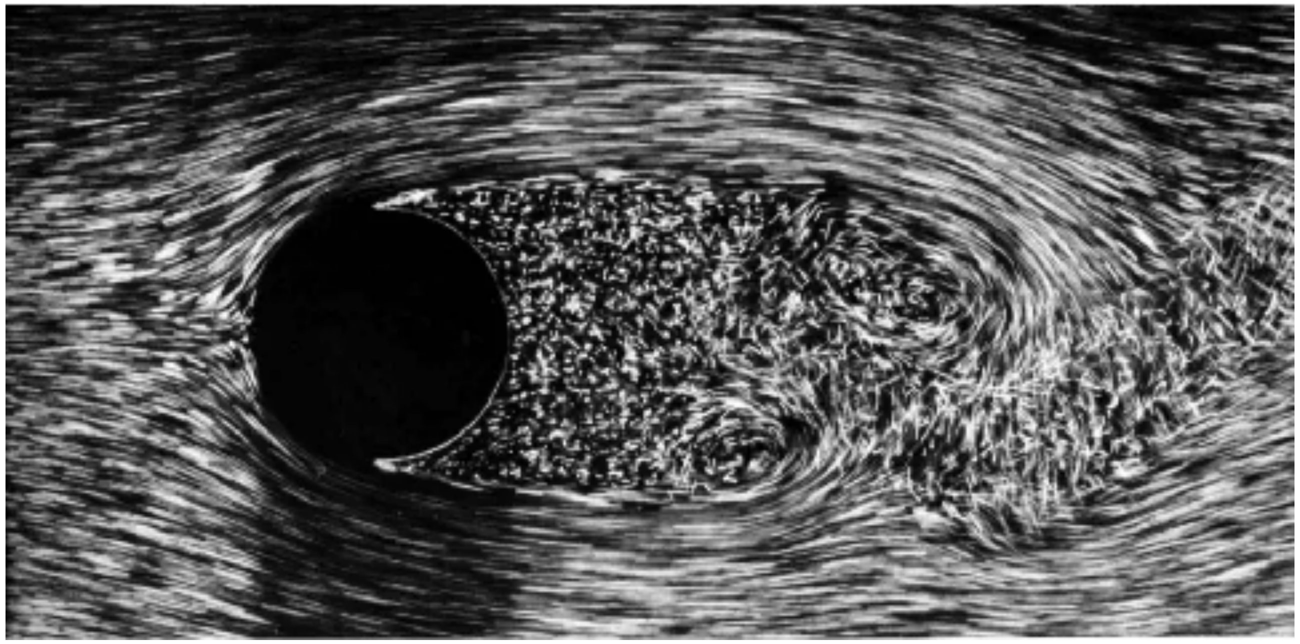
$$\mathbf{S}_W = -\int_{\partial W} grad\, p\, dV$$

$$B_W = \int_{\partial W} \rho \mathbf{b}\, dV$$

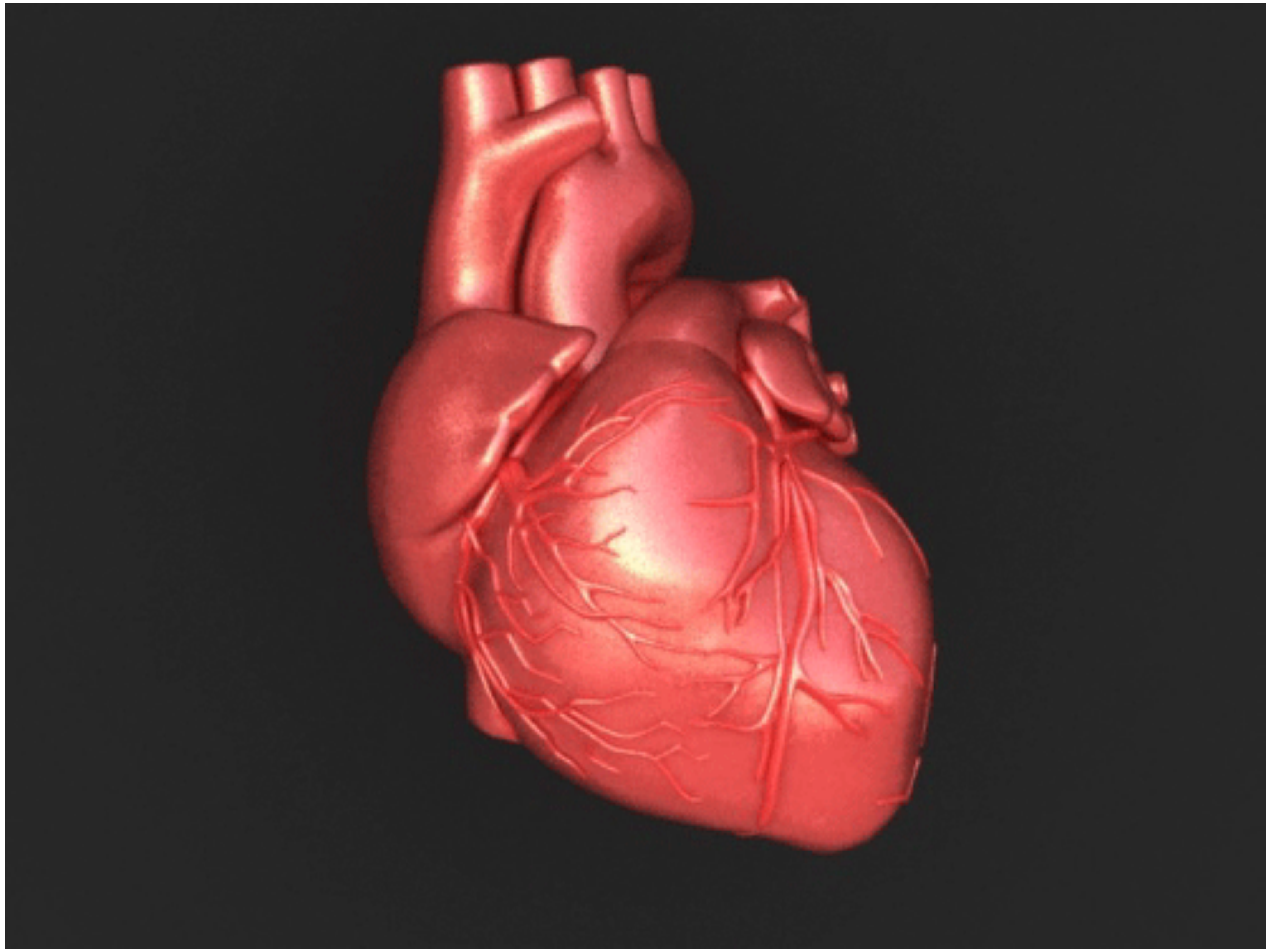
Euler Equations

$$\rho \frac{D\mathbf{u}}{Dt} = -grad\, p + \rho \mathbf{b}$$





Viscosidade e transferência de momento

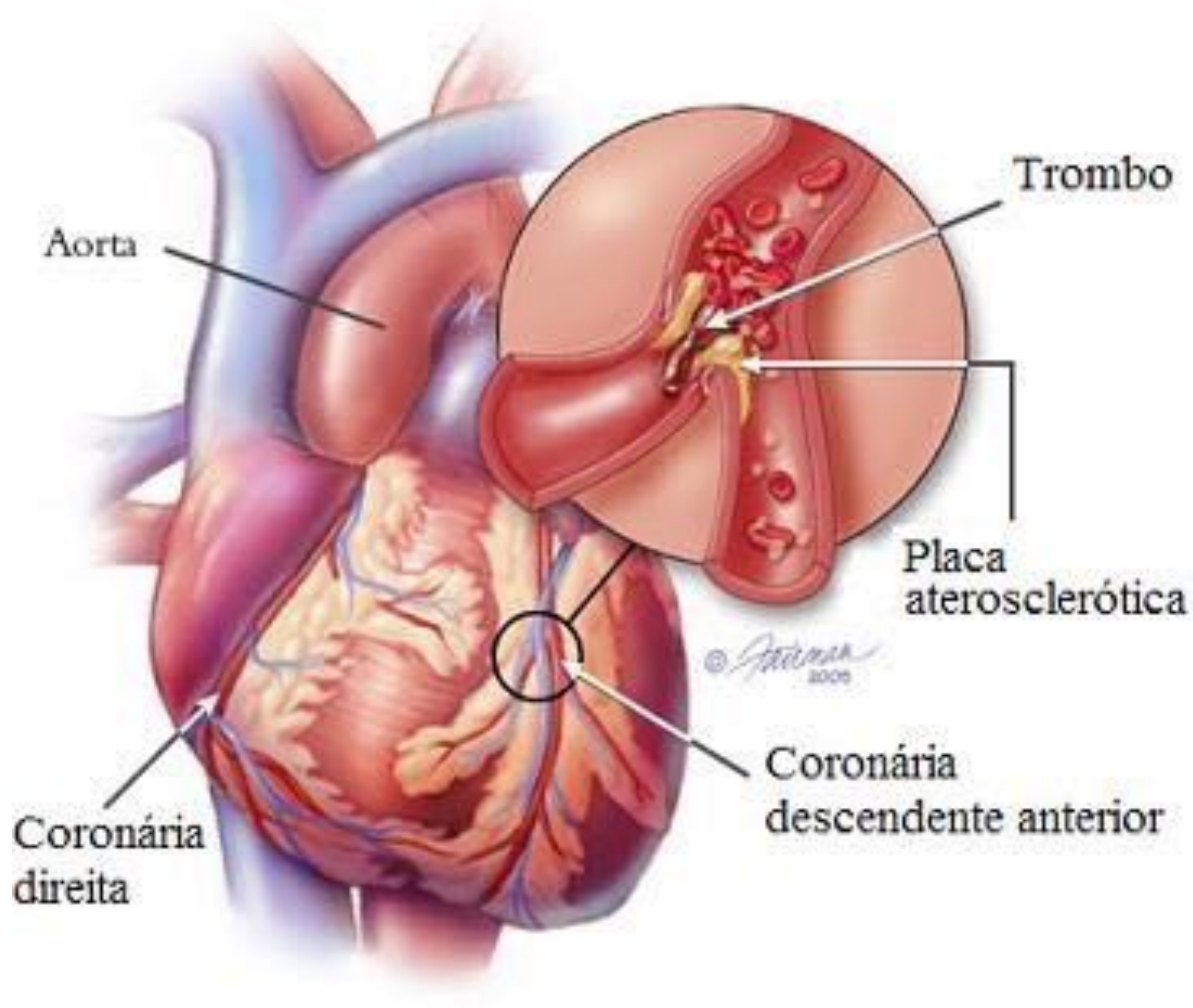


$$[P] = [\text{force}][\text{area}]^{-1}L^{-1} = (MLT^{-2})(L^2)^{-1}L^{-1} = ML^{-2}T^{-2}.$$

$$[Q] = L^3T^{-1},\ [a] = L,\ [\eta] = ML^{-1}T^{-1}$$

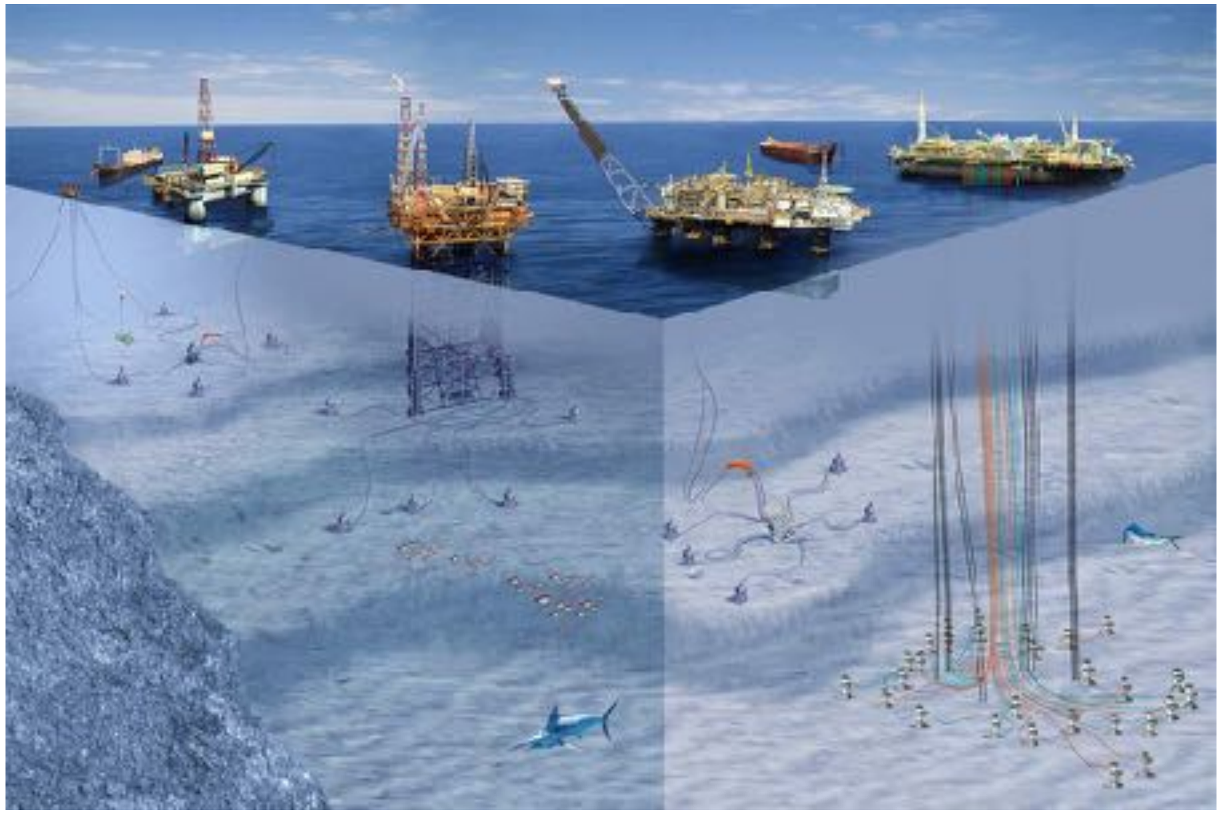
Adimensional → $P\eta^{-1}Q^{-1}a^4,$

$$Q = \frac{Ca^4P}{\eta}$$

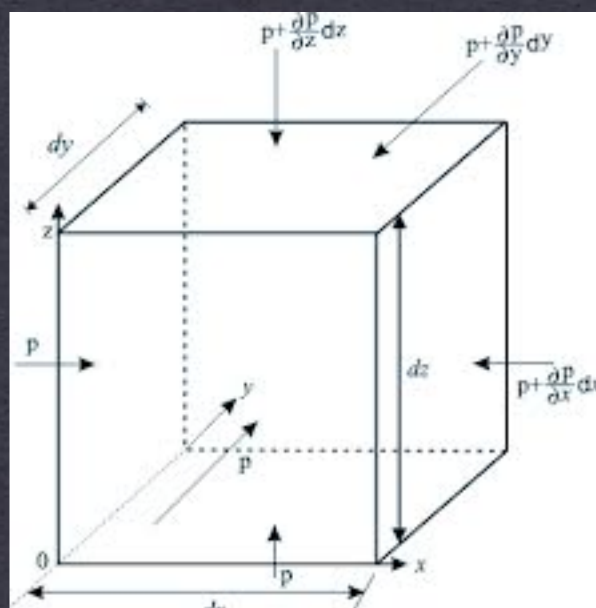


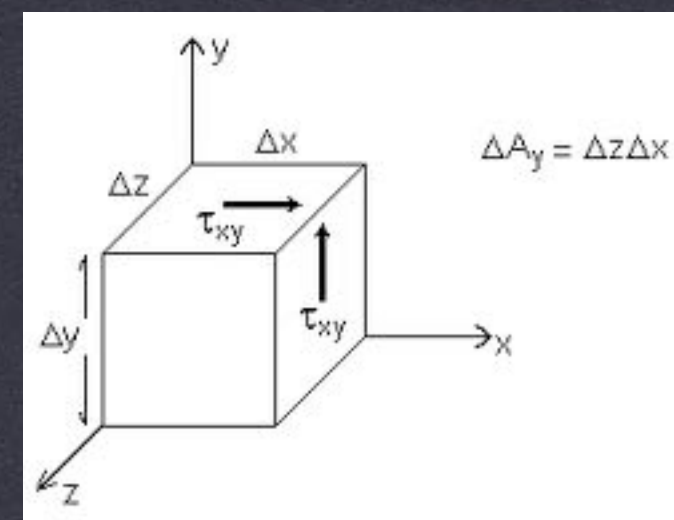
$$Q = \frac{Ca^4 P}{\eta}$$

A viscosidade vai sempre te lembrar da parede



Equações de Navier-Stokes incompressíveis





$$\nu = \frac{\mu}{\rho}$$

$$\begin{aligned} \frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{u} + \nabla p = \nu \Delta \mathbf{u} & \qquad \Omega \\ \nabla \cdot \mathbf{u} = 0 & \qquad \Omega \\ \mathbf{u} = 0 & \qquad \partial\Omega \end{aligned}$$

Non-dimensionalization and scaling of the Navier–Stokes equations

Reynolds Number

$$Re = \frac{UL}{\nu}$$

Non-Dimensional Incompressible Navier-Stokes

$$\partial_t u + u.\nabla u + \nabla p = \frac{1}{Re}\Delta u$$

$$\nabla \cdot u = 0$$

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Definições/Notações

Autocovariance

$$R(s) = \langle u(t)u(t+s) \rangle$$

Autocorrelation

$$\rho(s) = \frac{\langle u(t)u(t+s) \rangle}{u(t)^2}$$

Integral timescale

$$\bar{\tau} \equiv \int_0^\infty \rho(s)\, ds$$

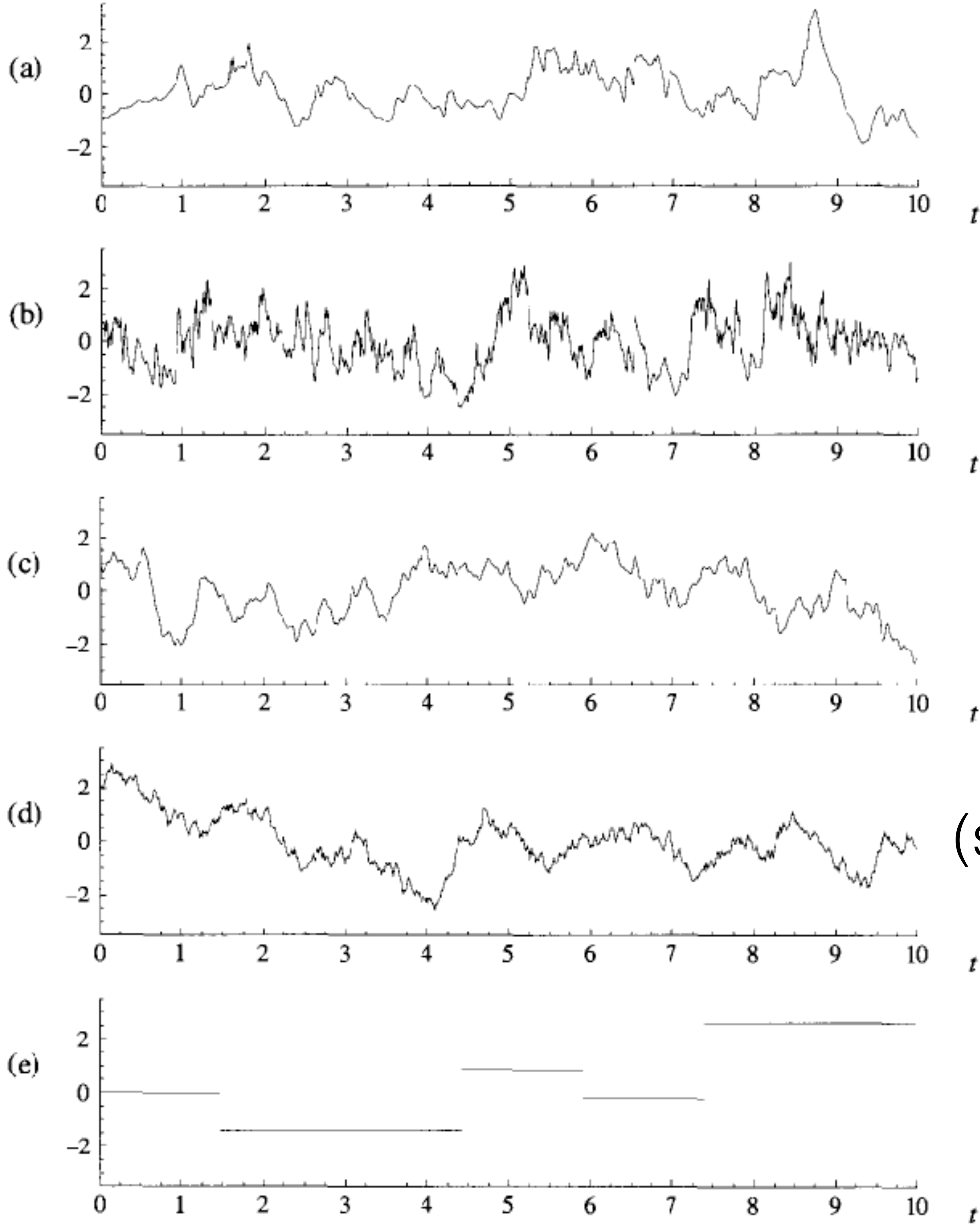
Frequency spectrum

$$E(\omega) \equiv \frac{1}{\pi} \int_{-\infty}^{\infty} R(s) e^{-i\omega s}\, ds$$

Contribution to the variance of all modes within this range

$$\int_{\omega_a}^{\omega_b} E(\omega)\, ds$$

$$R(0) = \langle u(t)^2 \rangle = \int_0^\infty E(\omega)\, d\omega$$



a) Turbulent Flow

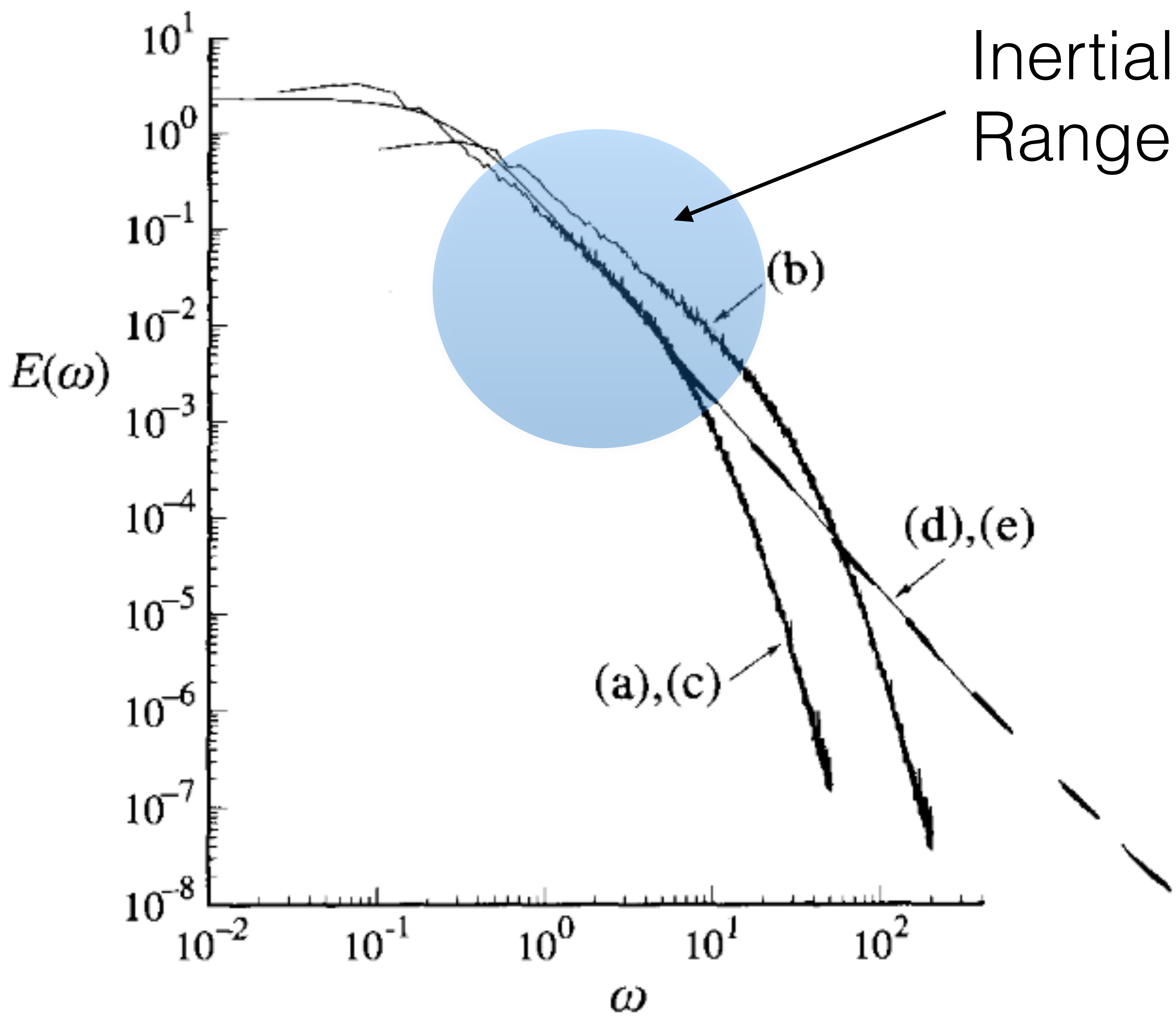
b) Turbulent Flow
(higher Reynolds)

c) Gaussian Process
(same spectrum as (a))

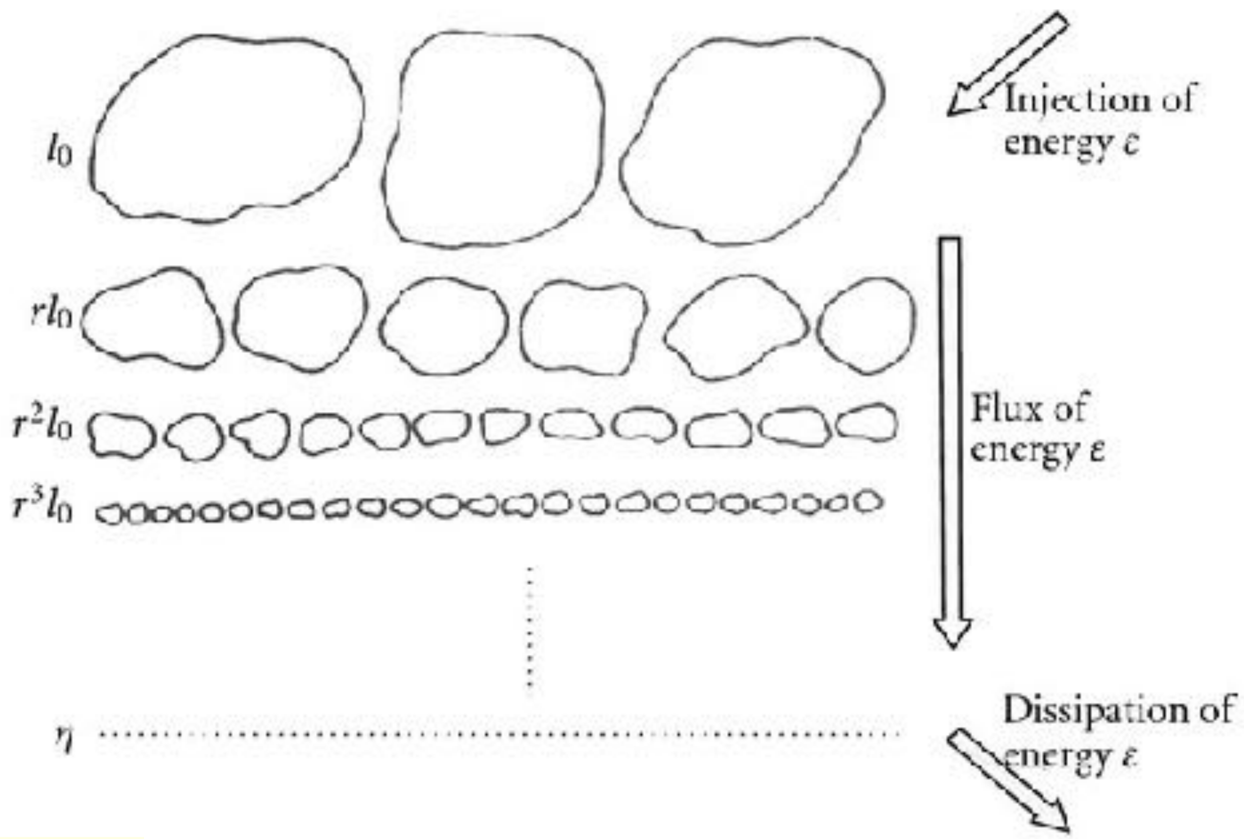
d) Ornstein -Uhlenbeck
(same integral time-scale as (a))

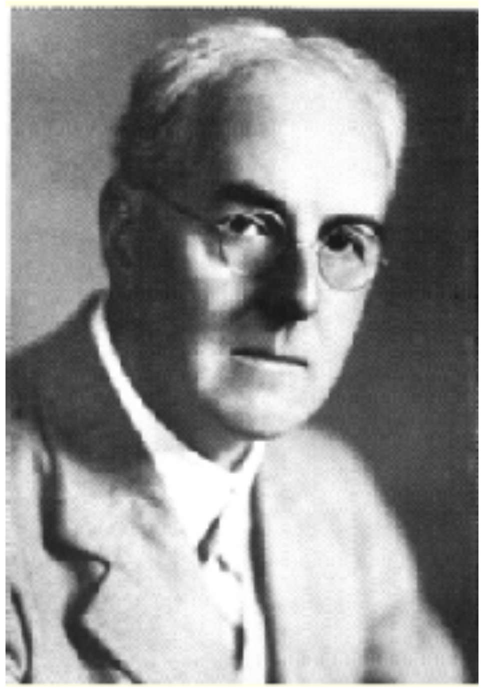
e) Jump Process
(same spectrum as (d))

Spectral signature -> Power Law
(Self-similarity / Phase-transition)



Does wind have a velocity ?
Self-similarity





Lewis Fry Richardson
1881-1953

Energy Dissipation Rate

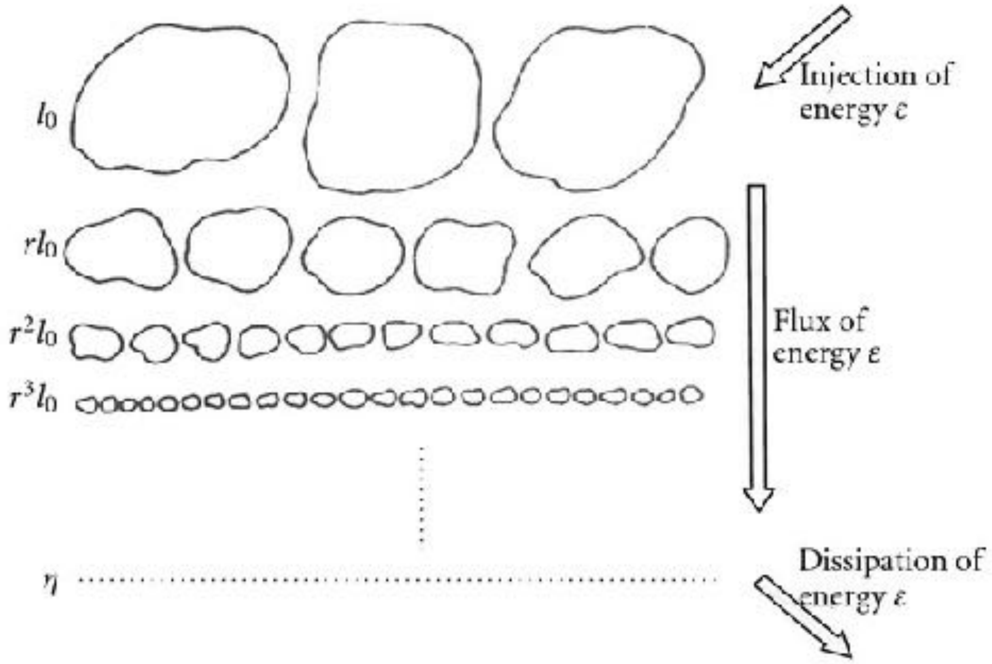
$$[\epsilon] \sim [U]^2/[T] \sim [U]^3/[L]$$

Big whorls have little whorls
That feed on their velocity.
And little whorls have lesser world
And so on to viscosity

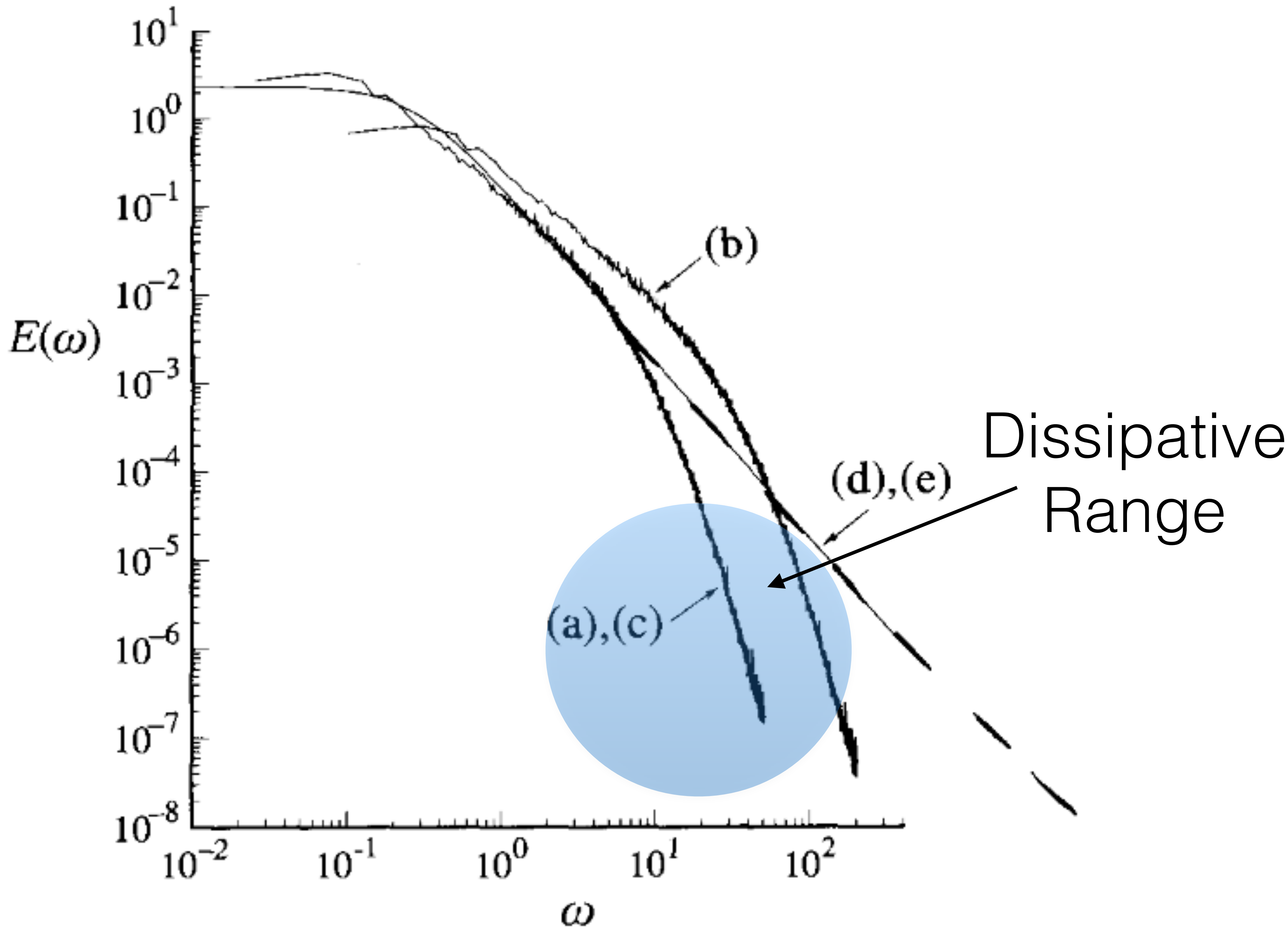
Richardson cascade

Π_ℓ energy flux at scale ℓ

$$u_\ell \sim \epsilon^{1/3}\ell^{1/3} \longleftarrow \Pi_\ell \sim u_\ell^3/\ell \sim \epsilon$$



Spectral signature -> Power Law
(Self-similarity / Phase-transition)



Richardson cascade

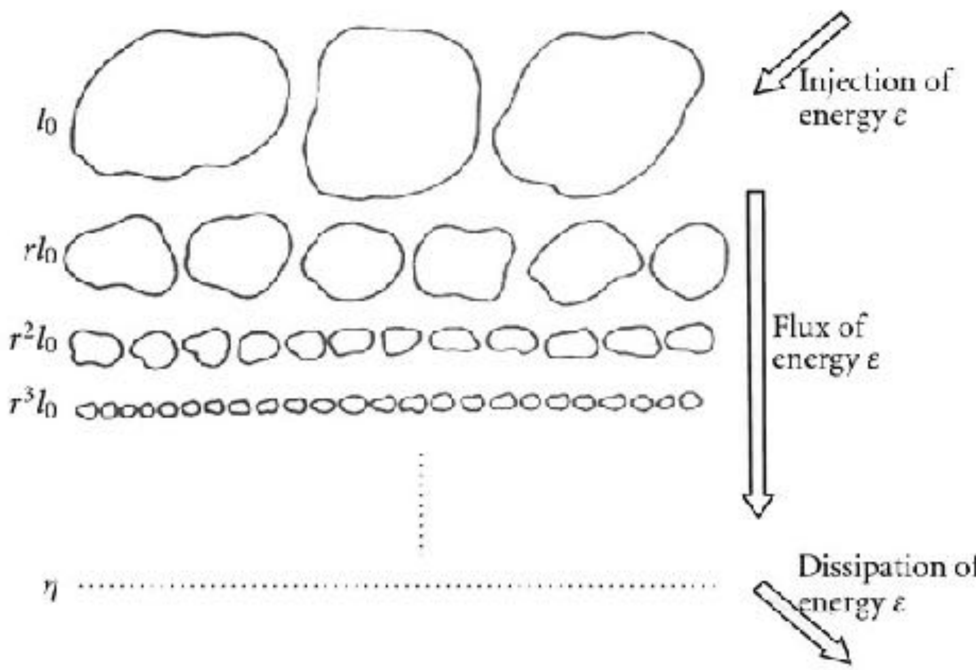
Π_ℓ energy flux at scale ℓ

$$u_\ell \sim \epsilon^{1/3}\ell^{1/3} \longleftarrow \Pi_\ell \sim u_\ell^3/\ell \sim \epsilon$$

$$t_\ell^I \sim \epsilon^{-1/3}\ell^{2/3}$$

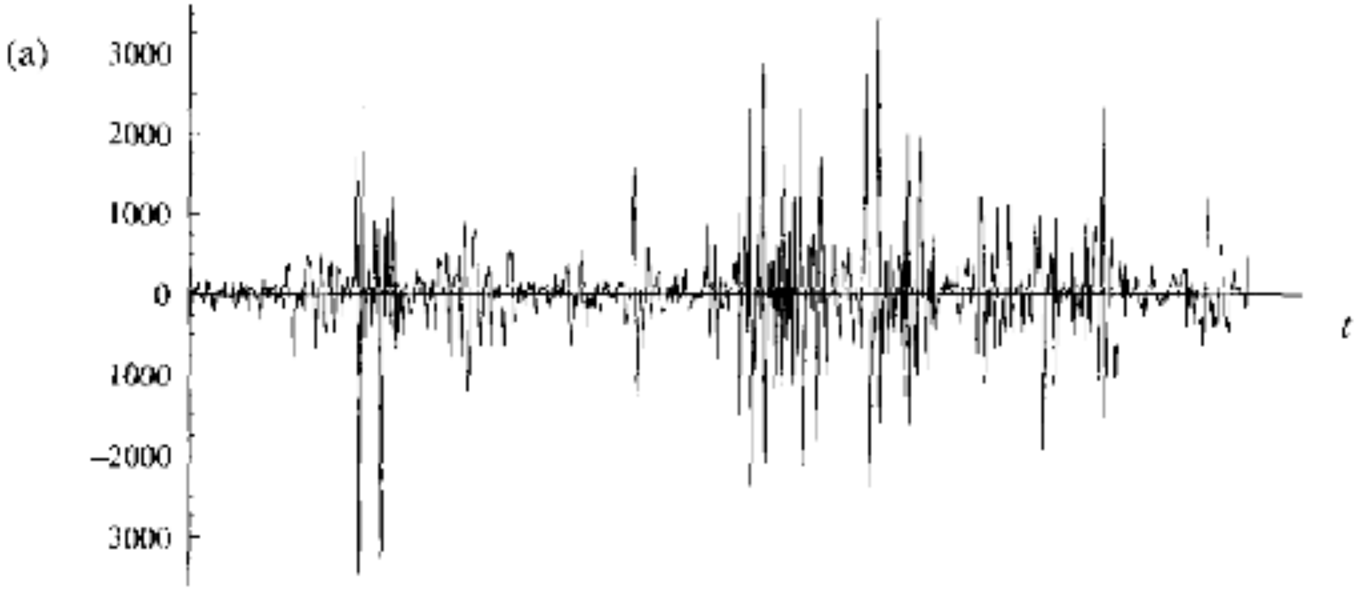
$$\sim \longrightarrow \ell_d \sim (\nu^3/\epsilon)^{1/4}$$

$$t_\ell^\nu \sim \ell^2/\nu$$



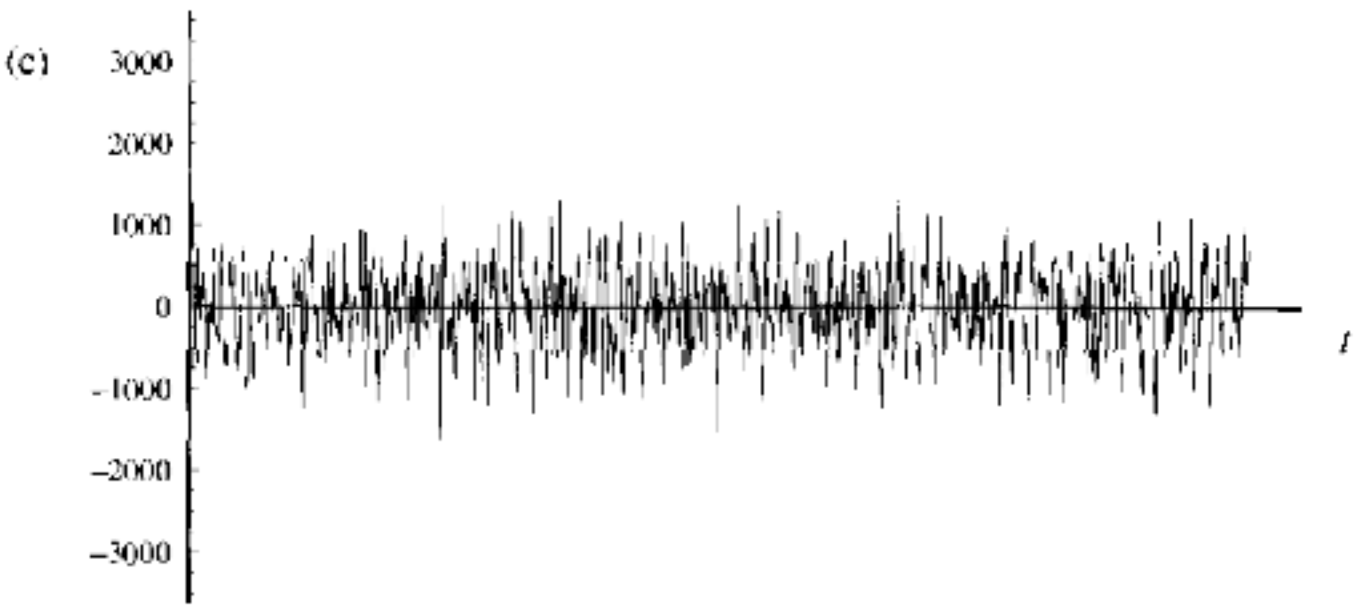
Top of the Inertial Range $\quad \epsilon \sim U^3/L$

$$\ell_d/L \sim 1/Re^{3/4} \longrightarrow u_d/U \sim 1/Re^{1/4}$$



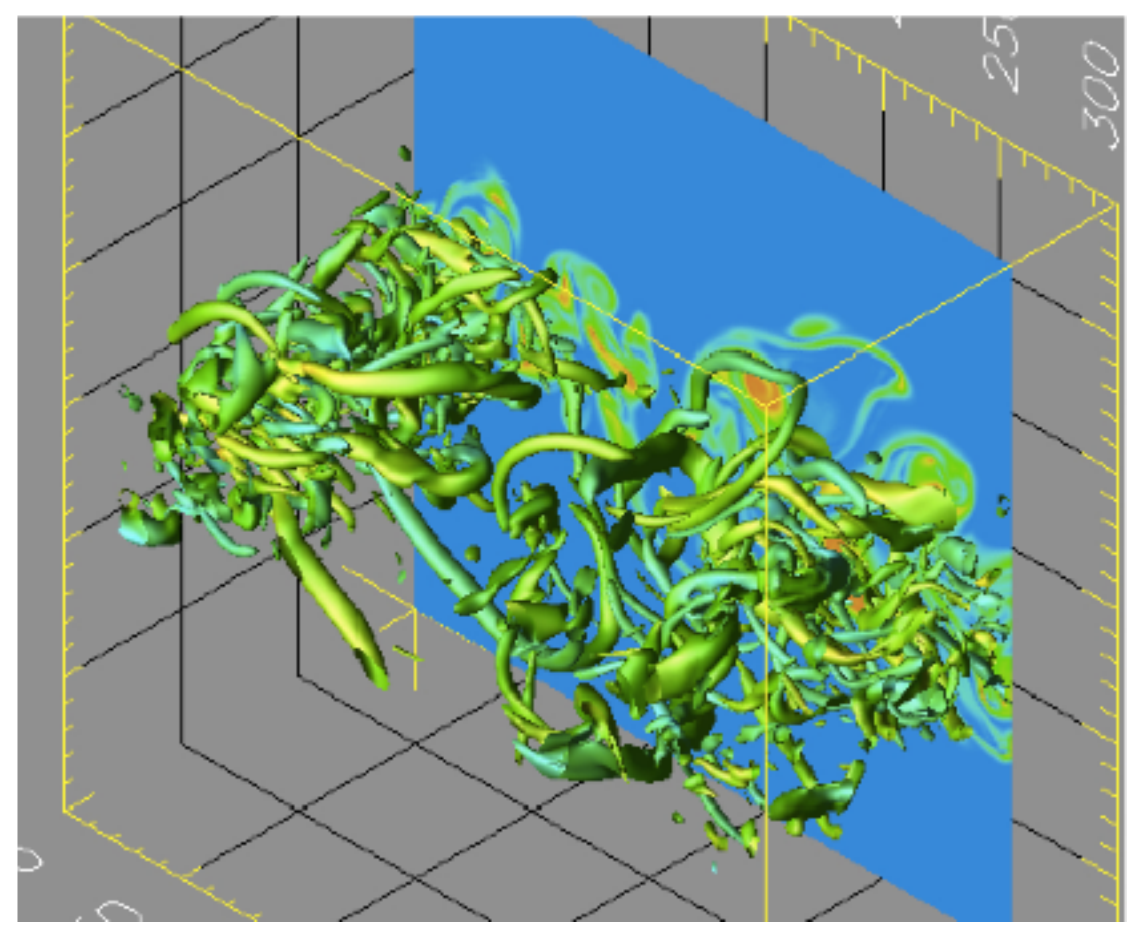
(a)

high-pass
Filtering



(c) G.P.

Intermittency



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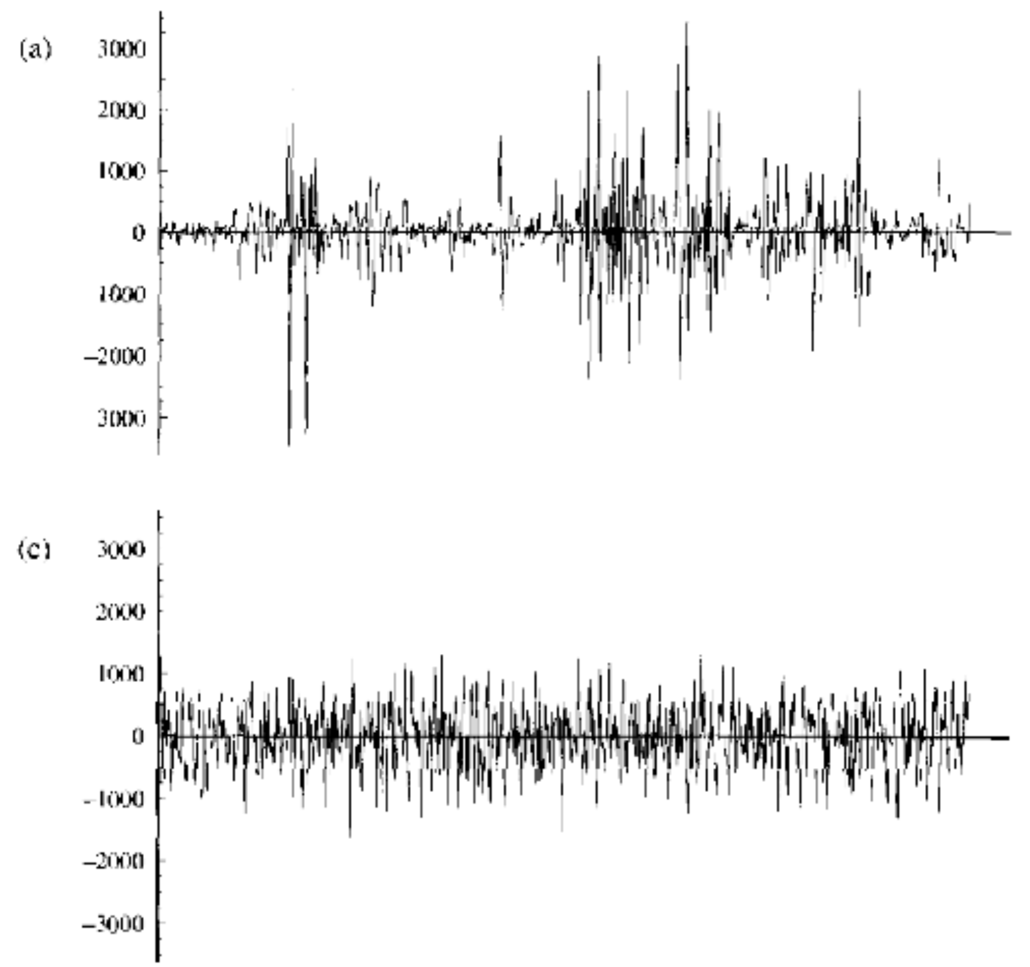
Soluções Fracas
(Turbulência = singularidade ?)

Jean-Leray



Millennium Problems

Yang–Mills and Mass Gap

Experiment and computer simulations suggest the existence of a "mass gap" in the solution to the quantum versions of the Yang-Mills equations. But no proof of this property is known.

Riemann Hypothesis

The prime number theorem determines the average distribution of the primes. The Riemann hypothesis tells us about the deviation from the average. Formulated in Riemann's 1859 paper, it asserts that all the 'non-obvious' zeros of the zeta function are complex numbers with real part 1/2.

P vs NP Problem

If it is easy to check that a solution to a problem is correct, is it also easy to solve the problem? This is the essence of the P vs NP question. Typical of the NP problems is that of the Hamiltonian Path Problem: given N cities to visit, how can one do this without visiting a city twice? If you give me a solution, I can easily check that it is correct. But I cannot so easily find a solution.

Navier–Stokes Equation

This is the equation which governs the flow of fluids such as water and air. However, there is no proof for the most basic questions one can ask: do solutions exist, and are they unique? Why ask for a proof? Because a proof gives not only certitude, but also understanding.

Hodge Conjecture

The answer to this conjecture determines how much of the topology of the solution set of a system of algebraic equations can be defined in terms of further algebraic equations. The Hodge conjecture is known in certain special cases, e.g., when the solution set has dimension less than four. But in dimension four it is unknown.

Poincaré Conjecture

In 1904 the French mathematician Henri Poincaré asked if the three dimensional sphere is characterized as the unique simply connected three manifold. This question, the Poincaré conjecture, was a special case of Thurston's geometrization conjecture. Perelman's proof tells us that every three manifold is built from a set of standard pieces, each with one of eight well-understood geometries.

Birch and Swinnerton-Dyer Conjecture

Supported by much experimental evidence, this conjecture relates the number of points on an elliptic curve mod p to the rank of the group of rational points. Elliptic curves, defined by cubic equations in two variables, are fundamental mathematical objects that arise in many areas: Wiles' proof of the Fermat Conjecture, factorization of numbers into primes, and cryptography, to name three.

CLAY MILLENNIUM PROBLEMS

Uma maneira bem difícil de ficar rico

(A) Existence and smoothness of Navier–Stokes solutions on $\mathbb{R}^3$. Take $\nu > 0$ and $n = 3$. Let $u^\circ(x)$ be any smooth, divergence-free vector field satisfying (4). Take $f(x,t)$ to be identically zero. Then there exist smooth functions $p(x,t), u_i(x,t)$ on $\mathbb{R}^3 \times [0,\infty)$ that satisfy (1), (2), (3), (6), (7).

(B) Existence and smoothness of Navier–Stokes solutions in $\mathbb{R}^3/\mathbb{Z}^3$. Take $\nu > 0$ and $n = 3$. Let $u^\circ(x)$ be any smooth, divergence-free vector field satisfying (8); we take $f(x,t)$ to be identically zero. Then there exist smooth functions $p(x,t)$, $u_i(x,t)$ on $\mathbb{R}^3 \times [0,\infty)$ that satisfy (1), (2), (3), (10), (11).

(C) Breakdown of Navier–Stokes solutions on $\mathbb{R}^3$. Take $\nu > 0$ and $n = 3$. Then there exist a smooth, divergence-free vector field $u^\circ(x)$ on $\mathbb{R}^3$ and a smooth $f(x,t)$ on $\mathbb{R}^3 \times [0,\infty)$, satisfying (4), (5), for which there exist no solutions (p,u) of (1), (2), (3), (6), (7) on $\mathbb{R}^3 \times [0,\infty)$.

(D) Breakdown of Navier–Stokes Solutions on $\mathbb{R}^3/\mathbb{Z}^3$. Take $\nu > 0$ and $n = 3$. Then there exist a smooth, divergence-free vector field $u^\circ(x)$ on $\mathbb{R}^3$ and a smooth $f(x,t)$ on $\mathbb{R}^3 \times [0,\infty)$, satisfying (8), (9), for which there exist no solutions (p,u) of (1), (2), (3), (10), (11) on $\mathbb{R}^3 \times [0,\infty)$.

Escalas Espacias
Definições/Notações

Two-point correlation

$$R_{ij}(\mathbf{r},\mathbf{x},t) \equiv \langle u_i(\mathbf{x},t) u_i(\mathbf{x}+\mathbf{r},t \rangle$$

velocity spectrum spectrum tensor

$$\Phi_{ij}(\mathbf{k},t) = \frac{1}{(2\pi)^3} \int\int\int e^{-i\mathbf{k}\cdot r} R_{ij}(\mathbf{r},t) d\mathbf{r}$$

Decomposition of the covariance

$$R_{ij}(0,t) = \langle u_i u_j \rangle = \int\int\int \Phi_{ij}(\mathbf{k},t) d\mathbf{k}$$

Escalas espacias
Definições/Notações

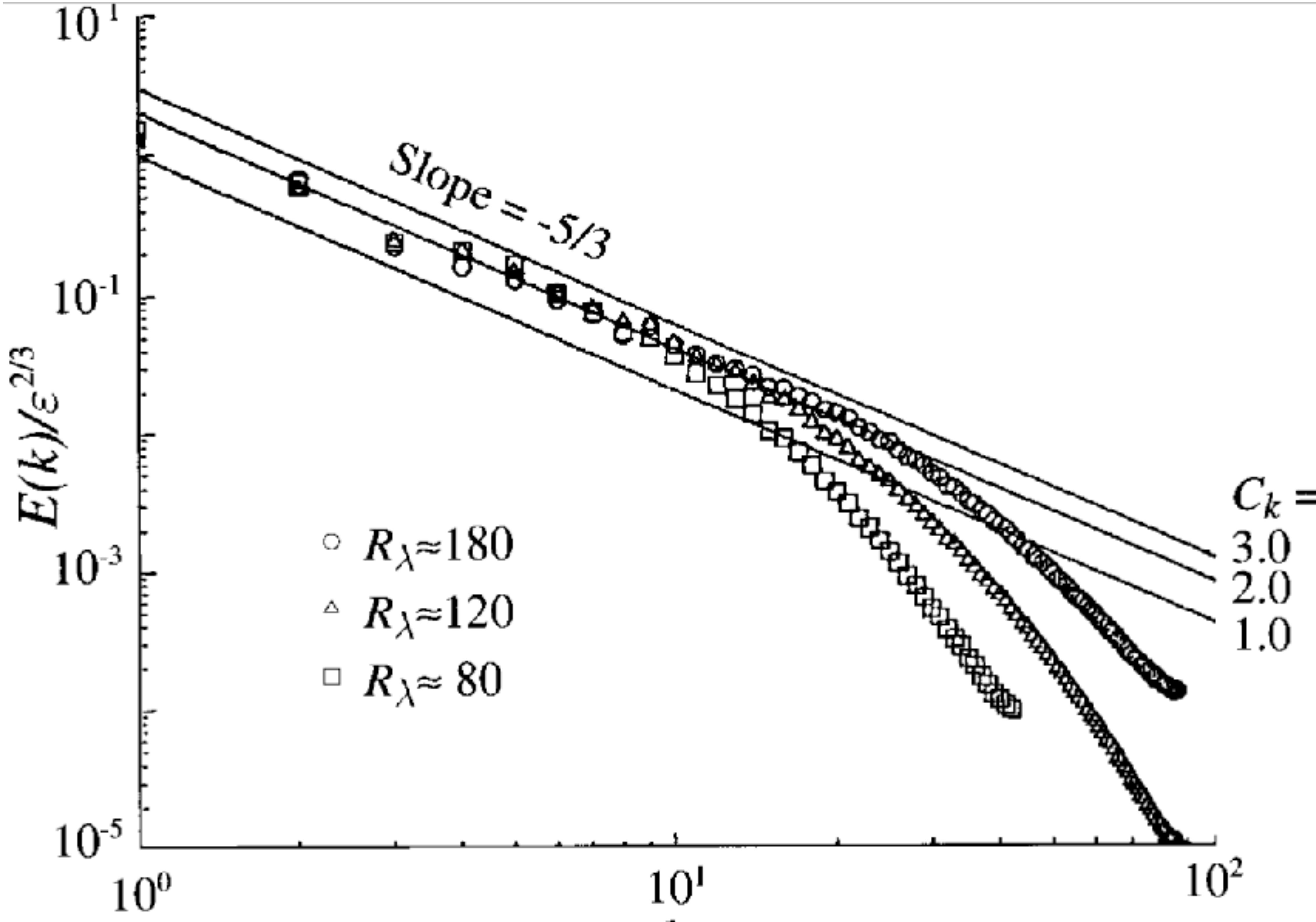
Energy Spectrum Function

$$E(k,t) = \int\int\int \frac{1}{2}\Phi_{ii}(\mathbf{k},t)\delta(|\mathbf{k}| - k)d\mathbf{k}$$

Decomposition of turbulent kinetic energy

$$\int_0^\infty E(k,t)d\mathbf{k} = \frac{1}{2}R_{ii}(0,t) = \frac{1}{2}\langle u_i u_i \rangle$$

Energy spectrum



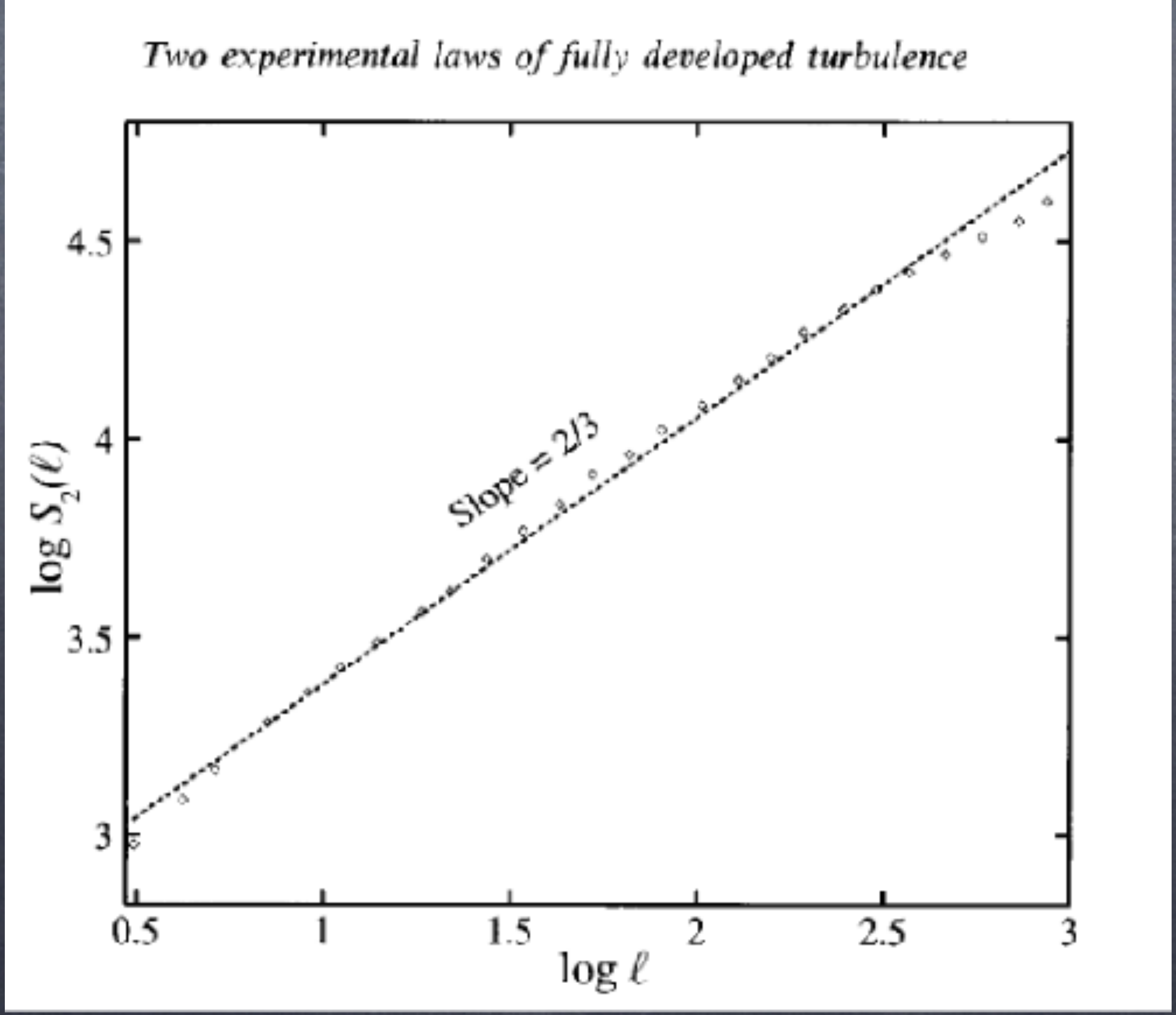
Spatial Scaling

Longitudinal velocity increment

$$\delta v_{\|}(r,l) = [v(r+l) - v(r)] \cdot \frac{l}{|l|}$$

Second order longitudinal Structure Function

$$S_2(\ell) = \langle (\delta v_{\|}(\ell))^2 \rangle$$



Kolmogorov 4/5 law

In the limit of high Reynolds Number

$$\frac{\langle(\delta v_{\|}(\ell))^3\rangle}{\ell} = -\frac{4}{5}\epsilon$$

Onsager conjectured that the physical solution of the Euler equations must have Holder regularity not greater than 1/3. Moreover, regularity 1/3 should imply anomalous dissipation.

$$\epsilon \equiv \epsilon_\nu = \nu\langle\langle\left\|\nabla \mathbf{u}^{(\nu)}\right\|^2\rangle\rangle$$

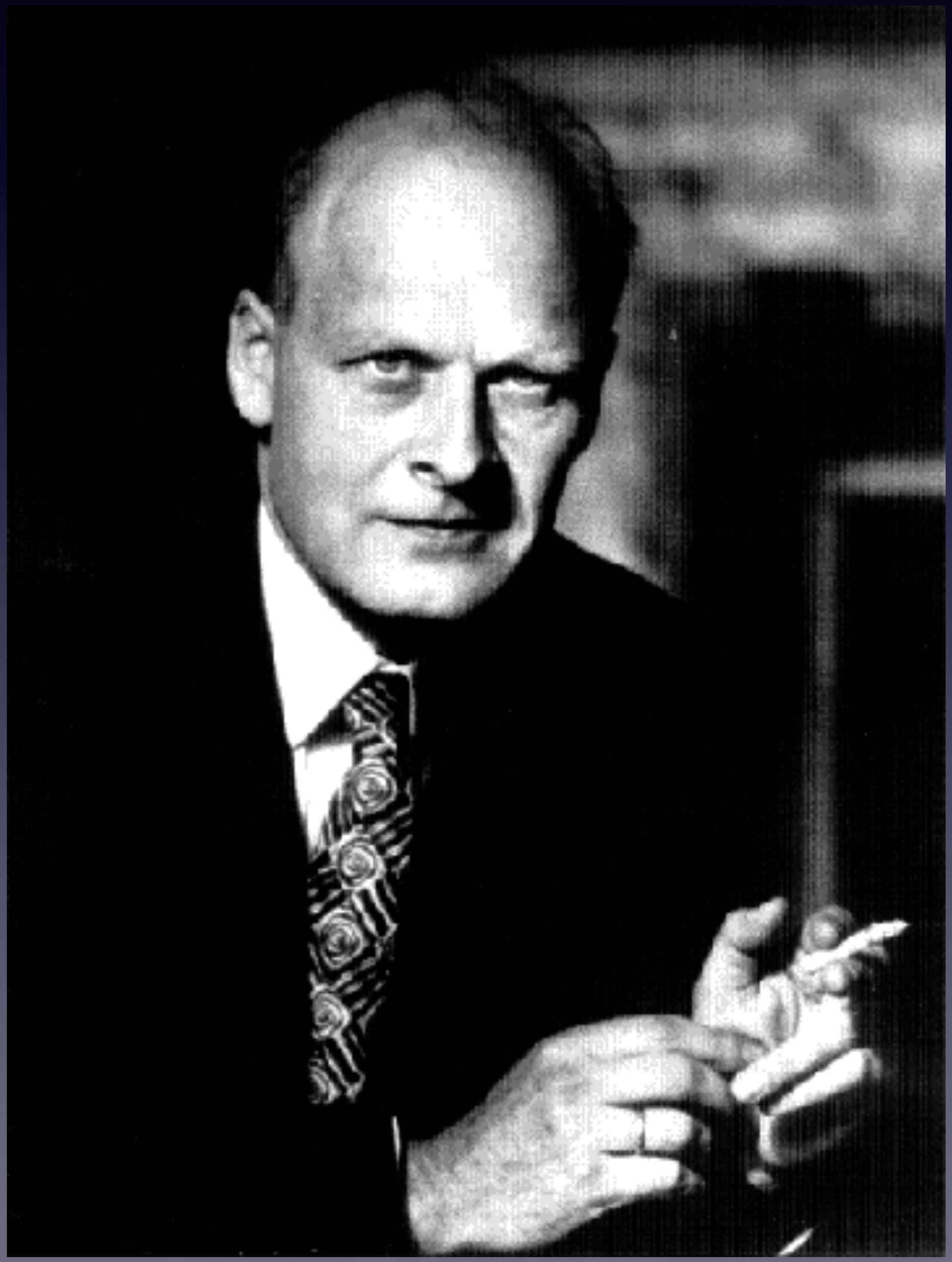
Anomalous dissipation ?

$$\lim_{\nu\to 0} \epsilon_\nu \to \epsilon > 0$$

Lars Onsager

"…in three dimensions a mechanism for complete dissipation of all kinetic energy, even without the aid of viscosity, is available."

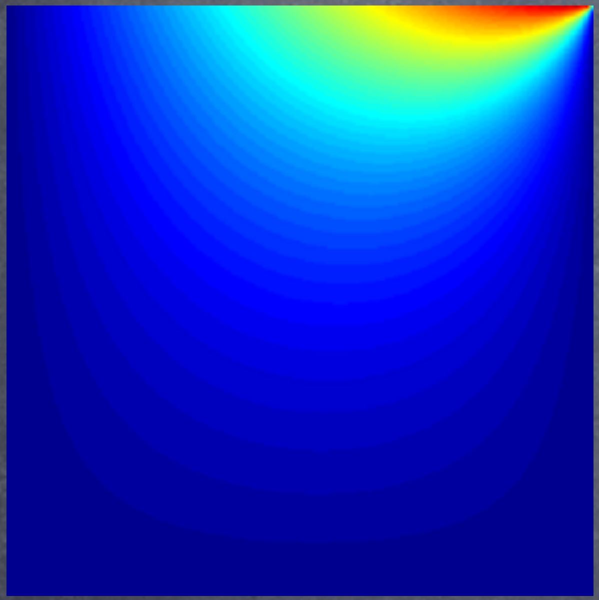
L. Onsager (1949)



Multi-scale phenomena

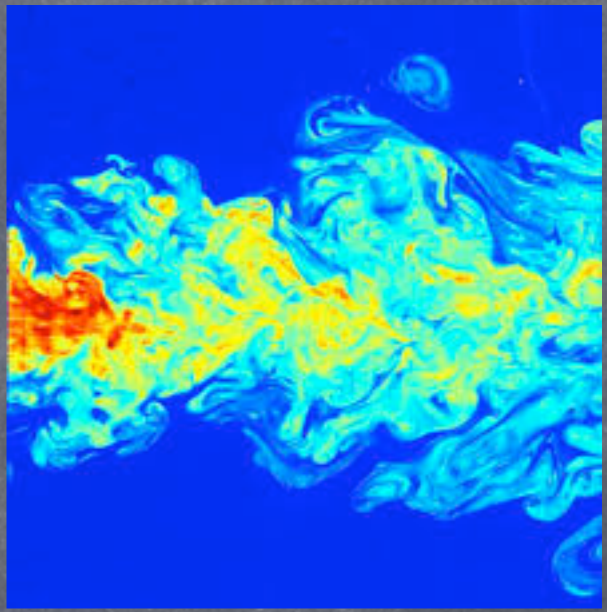
The effect of the nonlinear term

Energy equation $$\frac{d}{dt}\frac{1}{2}\|\mathbf{u}\|_2^2 = -\nu\,\|\nabla u\|_2^2$$



$$\Pi = \int (\mathbf{u}\cdot\nabla)\mathbf{u}\cdot\mathbf{u}\,dx$$

=0 (smoothness)



$$\frac{d}{dt}\theta_k(t) = -k^2\theta_k$$

$$\frac{d\mathbf{u}_k}{dt} = -k^2\mathbf{u}_k + \Pi_k(\mathbf{u})$$

Convolution of all scales

“Inviscid” limit

Incompressible Navier-Stokes Equations

Incompressible Euler Equations

$$\partial_t u + u.\nabla u + \nabla p = \frac{1}{Re}\Delta u \quad \xrightarrow[\mathbf{Re}\to\infty]{?} \quad \partial_t u + u.\nabla u + \nabla p = 0$$

Can we recover enough of turbulence from Euler Equations ?

Minimum regularity
The Onsager Conjecture

$$\Pi = \int (\mathbf{u} \cdot \nabla)\mathbf{u} \cdot \mathbf{u} \, dx$$

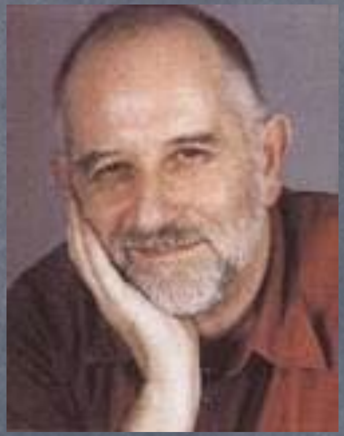
Arguing as Littlewood-Paley

$$\Pi \sim \int (|\nabla|^{1/3}\mathbf{u})^3 \, dx$$

If u has Holder regularity 1/3, we can at least make sense of the flux

Any better regularity yields: $\Pi = 0$

Singular Limits



Sir Michael Berry

What do we know ?

Classical results - Onsager's conjecture

1994

Eyink

Constantin, Titi, E

→ Besov spaces

$L^3(0,T,B^{\alpha}_{3,\infty}),\quad \alpha > 1/3$

Conservation of energy

Wild solutions of 3 Euler Equations

Scheffer

Schnirelmann

De Lellis, Székelyhidi

→ Energy not conserved

$L^2 \cap L^\infty$

Unidimensional Gas

Burgers Equations

$$u_t + uu_x = \nu u_{xx}$$

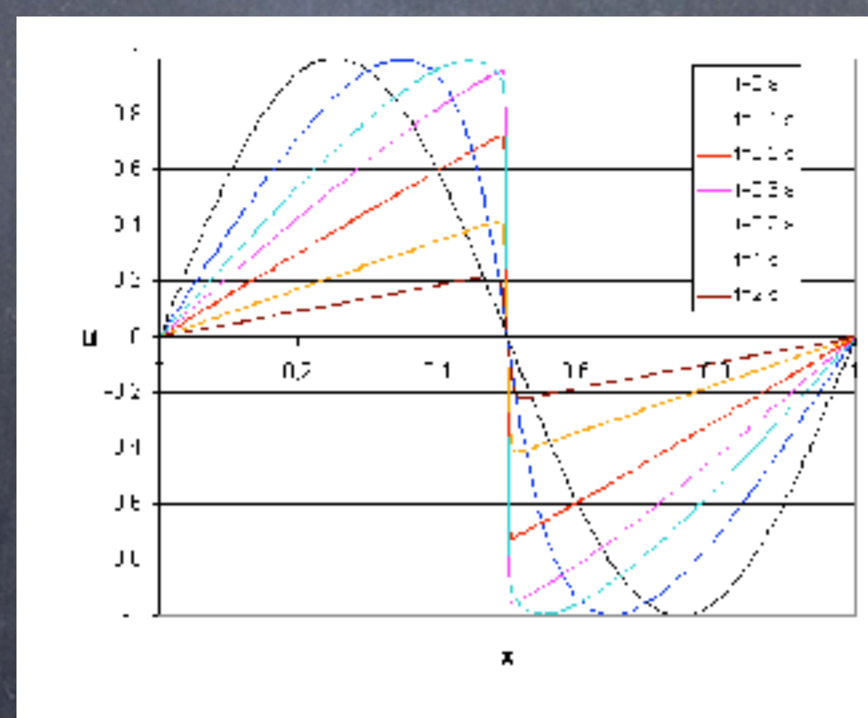
J.M. Burgers.

Mathematical examples illustrating relations
occuring in the theory of turbulent fluid motion. Verhand.
Kon. Neder. Akad. Wetenschappen, Afd. Natuurkunde,
Eerste Sectie, 17:1–53, 1939.

Inviscid Burgers Equations

$$u_t + uu_x = 0$$

Peter Lax

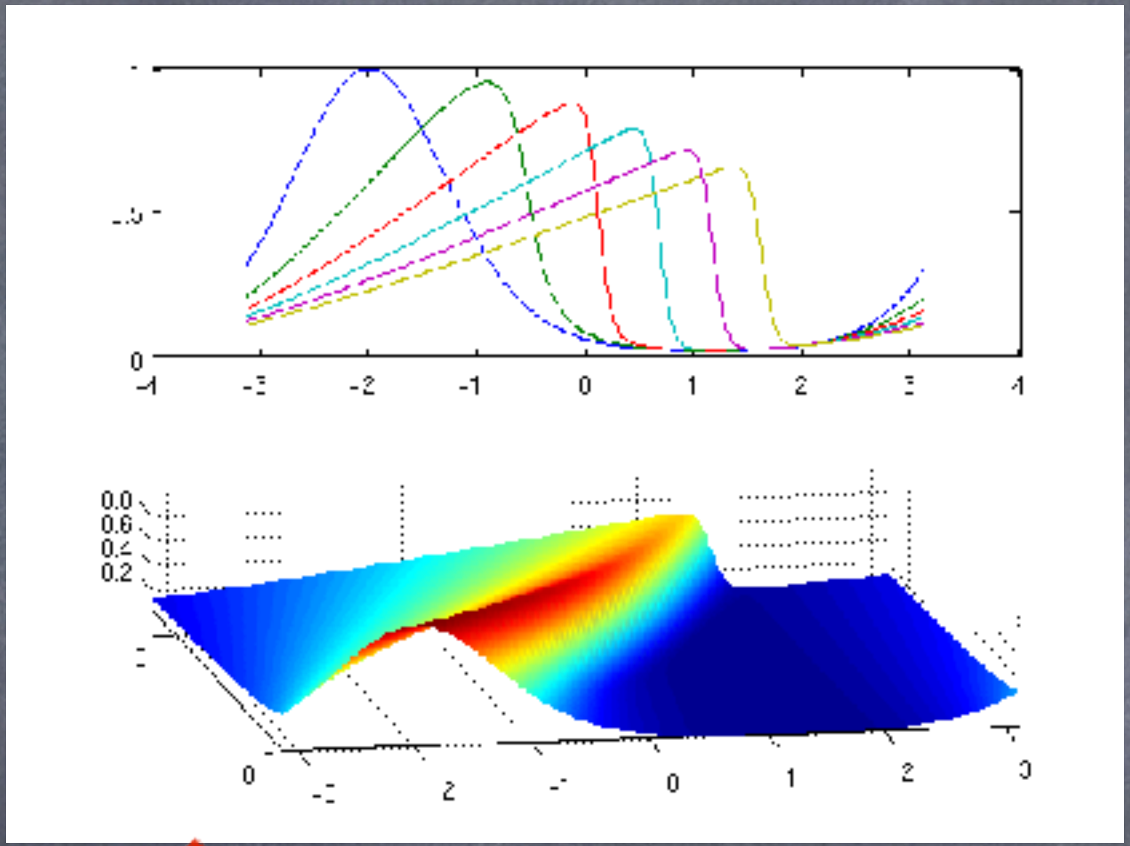


Back to Burgers

$$u_t + uu_x = \nu u_{xx}$$

$$u \equiv u_\nu$$

$$E_\nu = \int |u_\nu|^2 \, dx$$



$$\frac{dE_\nu}{dt} = D_\nu(t)$$

$$D_\nu(t) = -\nu \int |u_x|^2 \, dx$$

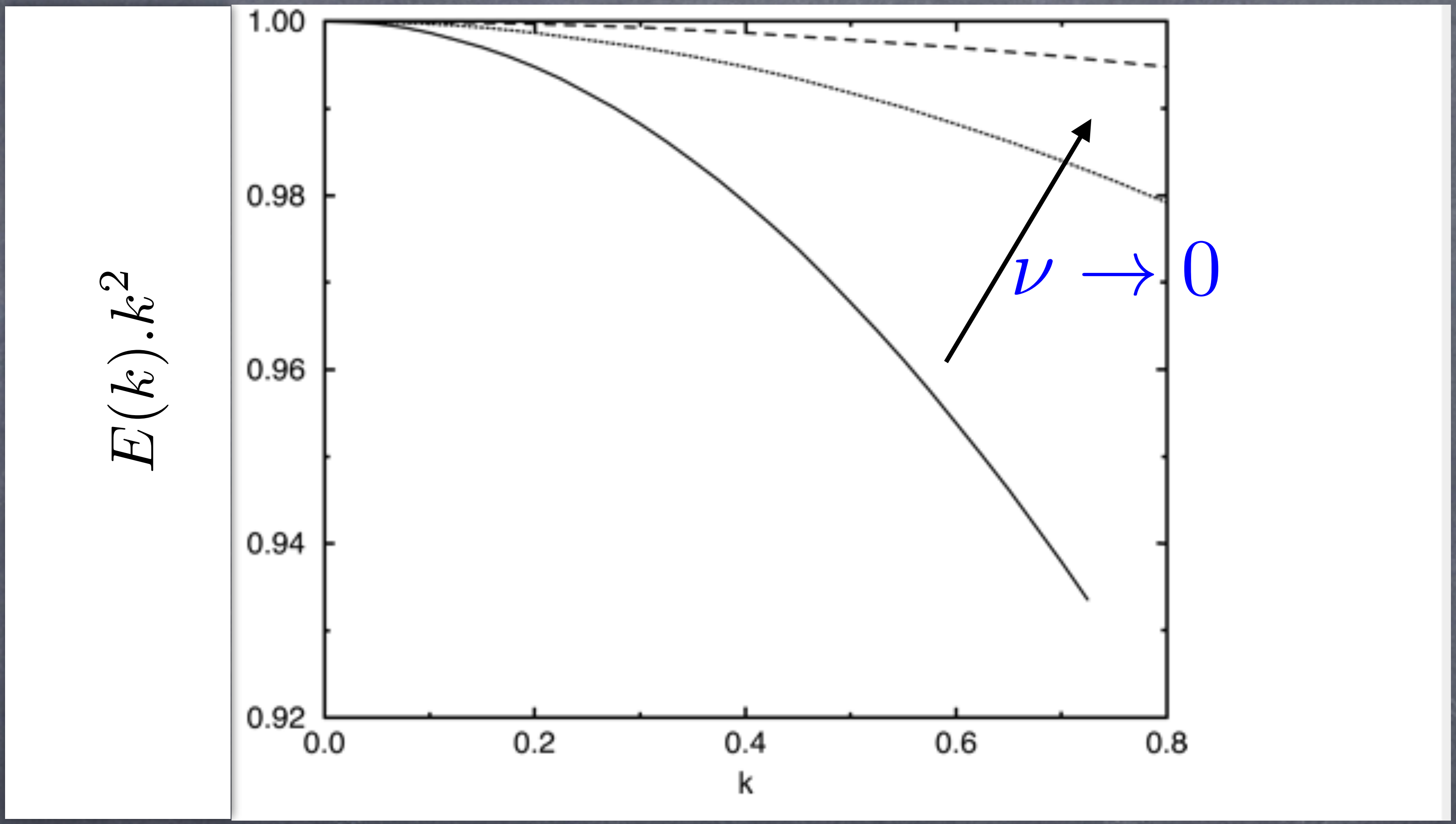
$$\nu = 0 \longrightarrow \frac{dE_0}{dt} \equiv 0 \longrightarrow D_\nu \underset{\nu \to 0}{\longrightarrow} 0$$

Inviscid Singularities

Burgers

Burgers Equations

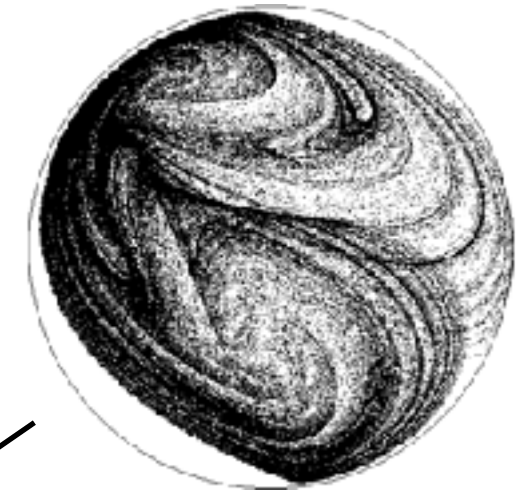
The singularity smile



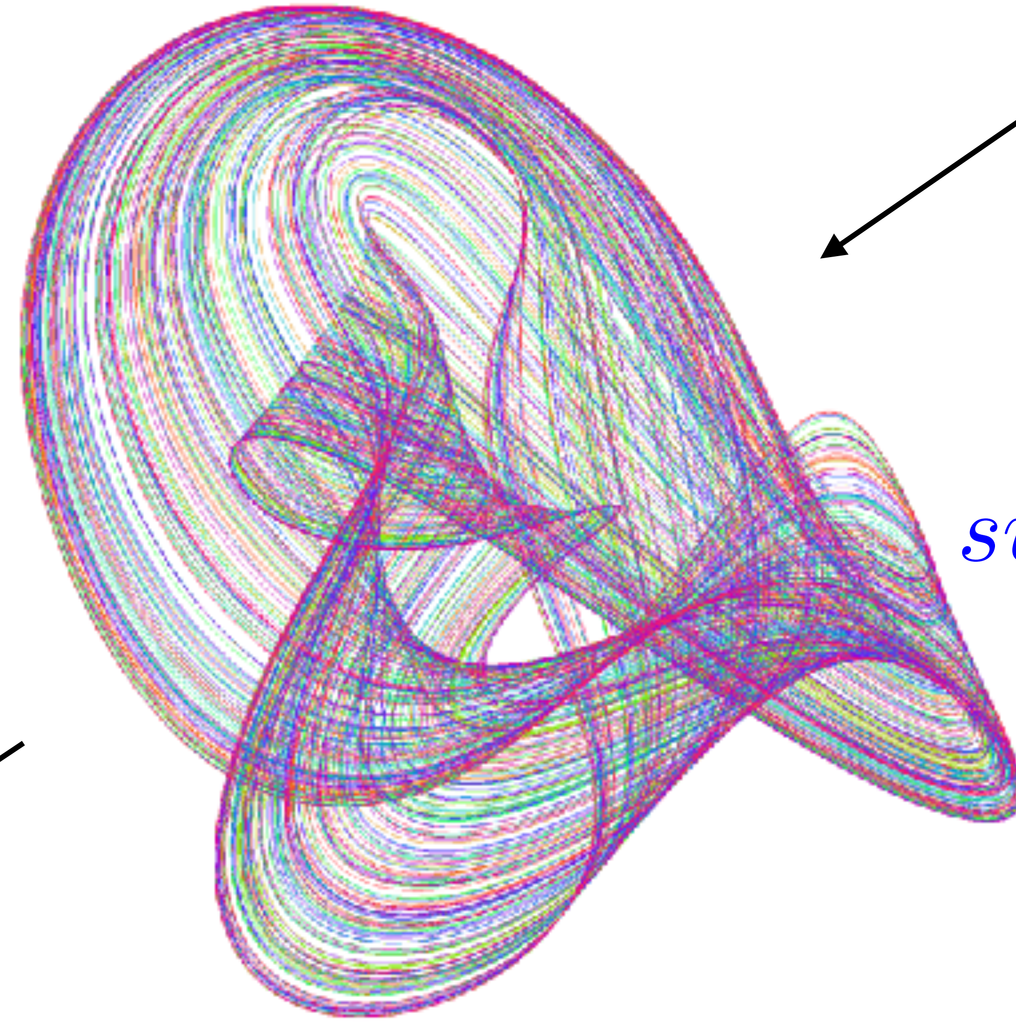
“'All right,' said the Cat; and this time it vanished quite slowly, beginning with the end of the tail, and ending with the grin, which remained some time after the rest of it had gone.” - Alice in the Wonderland

(by Marshall Slemrod)

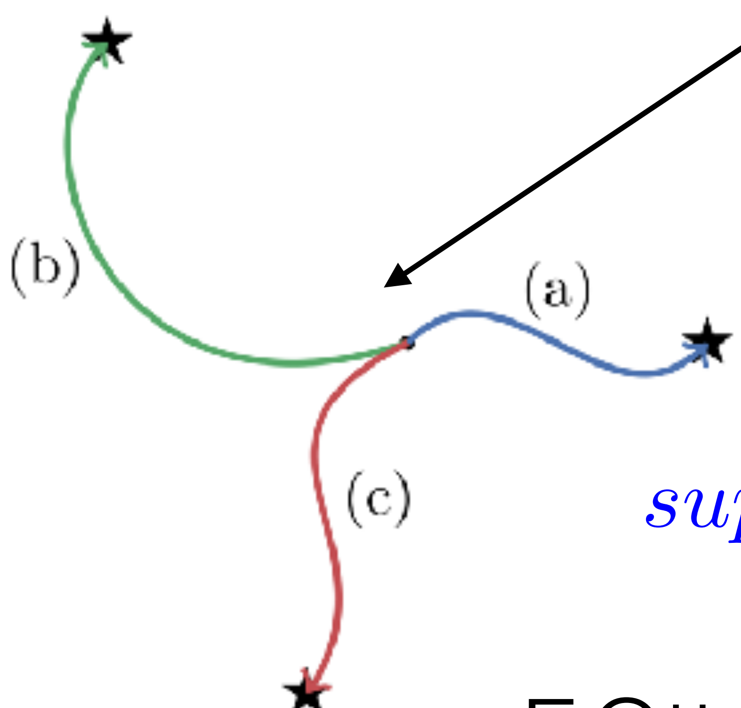
Convergence of attractors
and physical invariant measures
with increasing Re number



$supp\,\mu_{\nu_1}$



$supp\,\mu_{\nu_{1000}}$

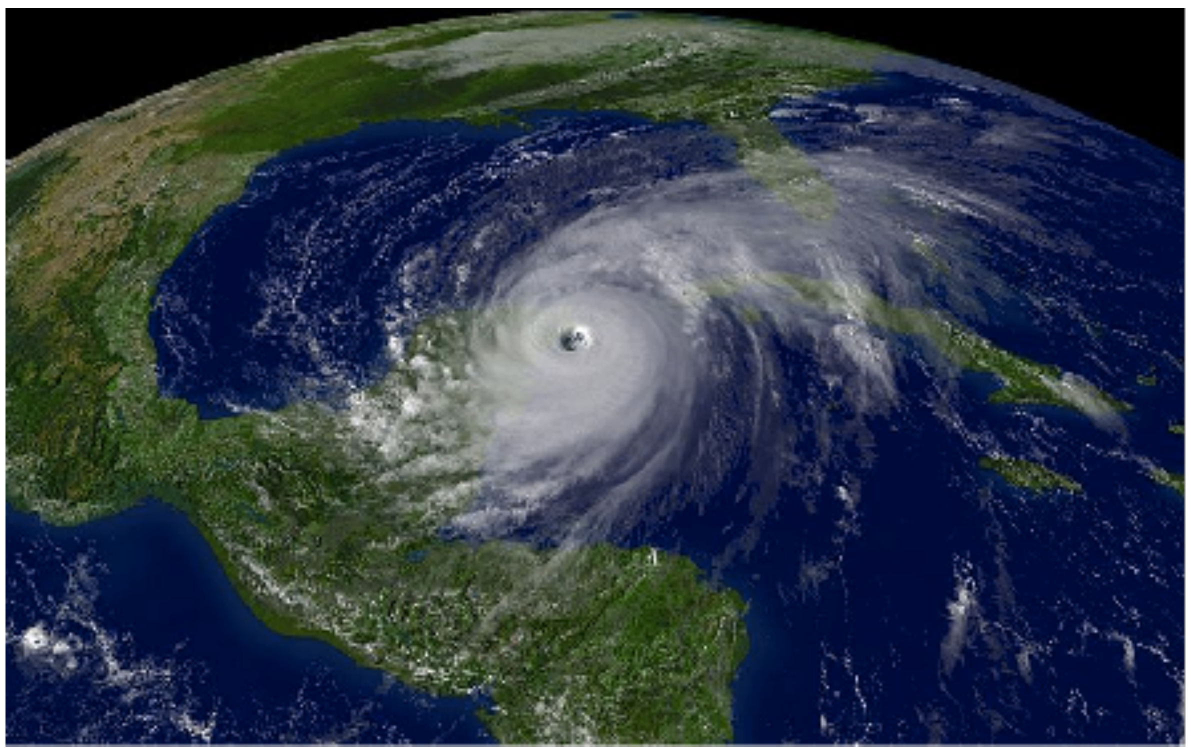


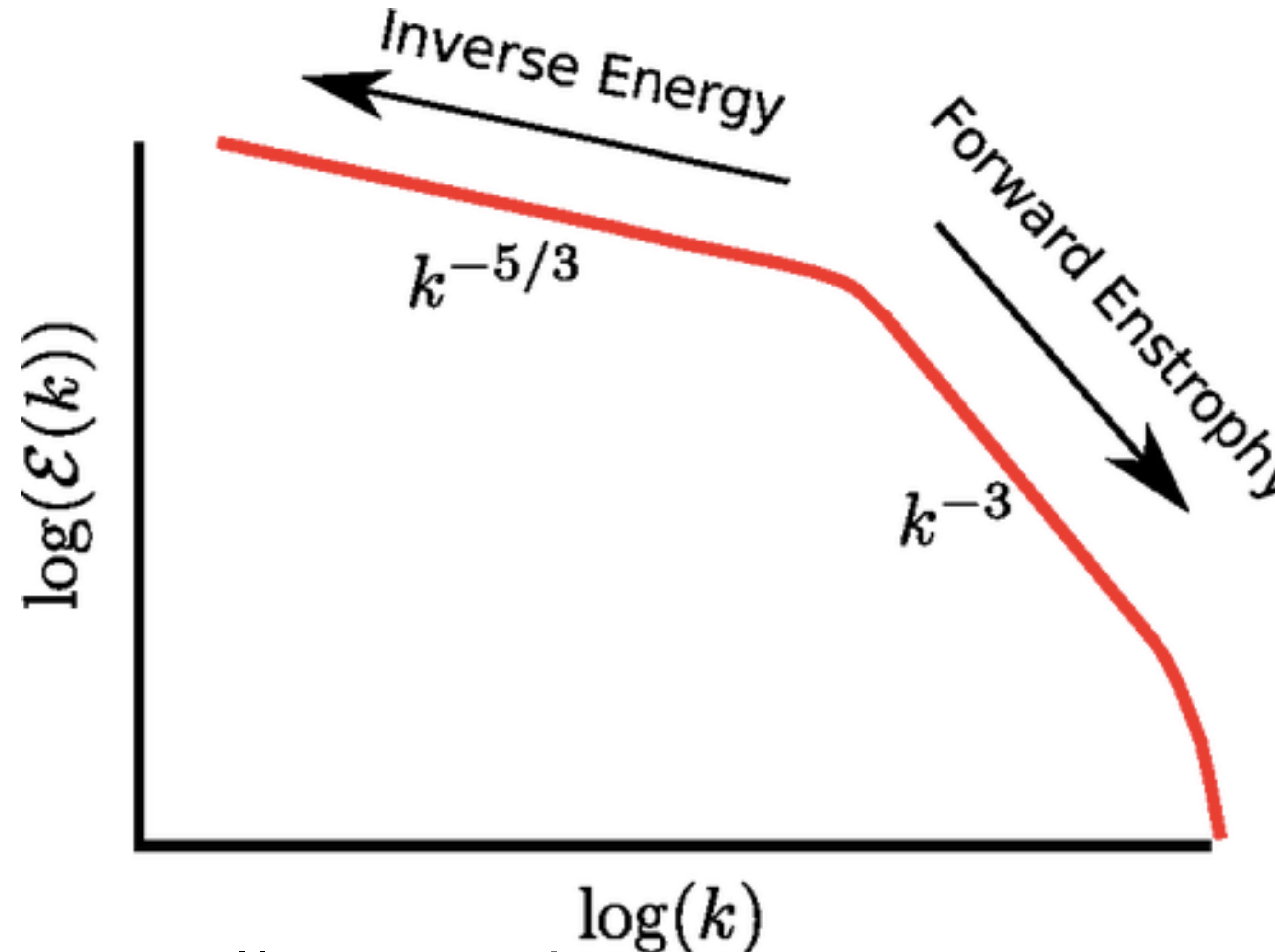
$supp\,\mu_E$

F.Otto,F.R. (Burgers)

Wild solutions of
Euler equations
in the limit support ?

3D->2D->1D (other inviscid invariants)





Solutions are well-posed

$$\eta \equiv \eta_\nu = \nu \langle\langle \left\| \nabla \left(\nabla \times \mathbf{u}^{(\nu)} \right) \right\|^2 \rangle\rangle$$

$$\lim_{\nu \to 0} \eta_\nu \to \eta > 0$$

No anomalous dissipation of enstrophy with linear damping (Constantin, F.R. CMP 2007)

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NIDF

Núcleo Interdisciplinar de Dinâmica dos Fluidos

Reynolds Decomposition

$$u_i = \bar{u}_i + u_i'$$

$$\rho \frac{Du_i}{Dt} = -\frac{\partial p}{\partial x_i} + \mu \left(\frac{\partial^2 u_i}{\partial x_j \partial x_j} \right)$$

Incompressible Navier-Stokes

$$\rho \left[\frac{\partial(\bar{u}_i + u_i')}{\partial t} + (\bar{u}_j + u_j') \frac{\partial(\bar{u}_i + u_i')}{\partial x_j} \right] = -\frac{\partial(\bar{p}_i + p')}{\partial x_i} + \mu \frac{\partial^2 (\bar{u}_i + u_i')}{\partial x_j \partial x_j}$$

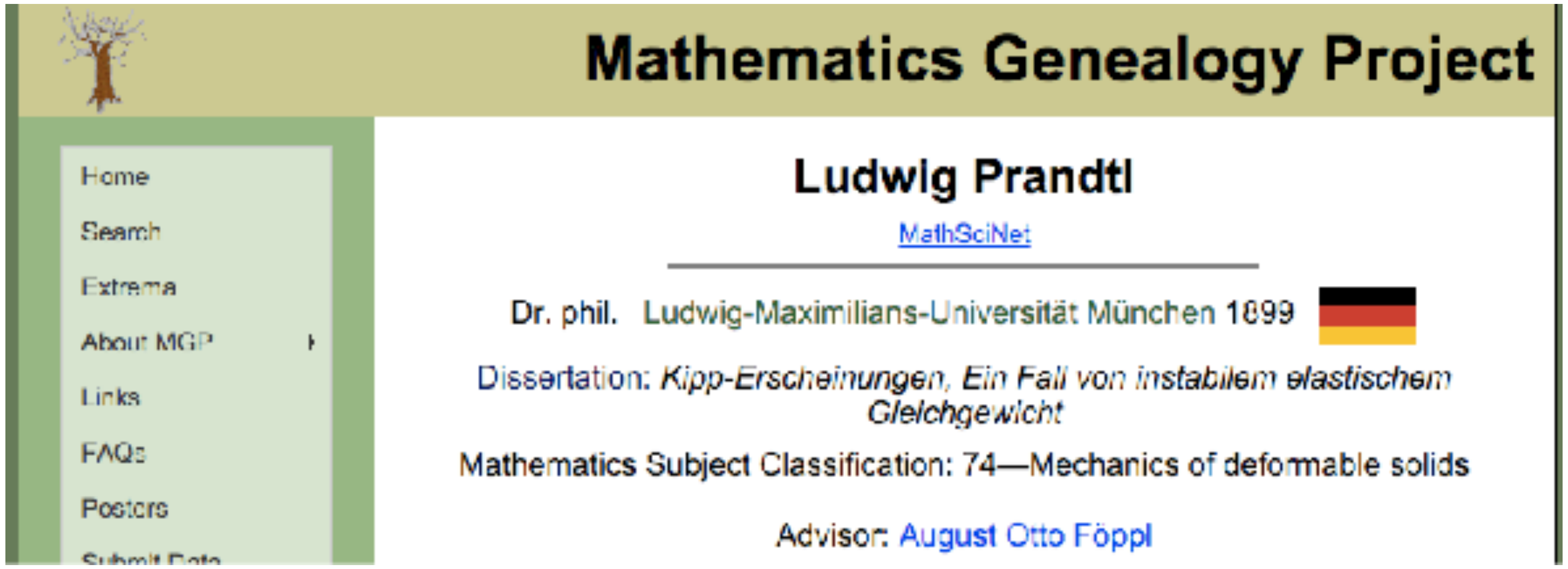
Reynolds Equations

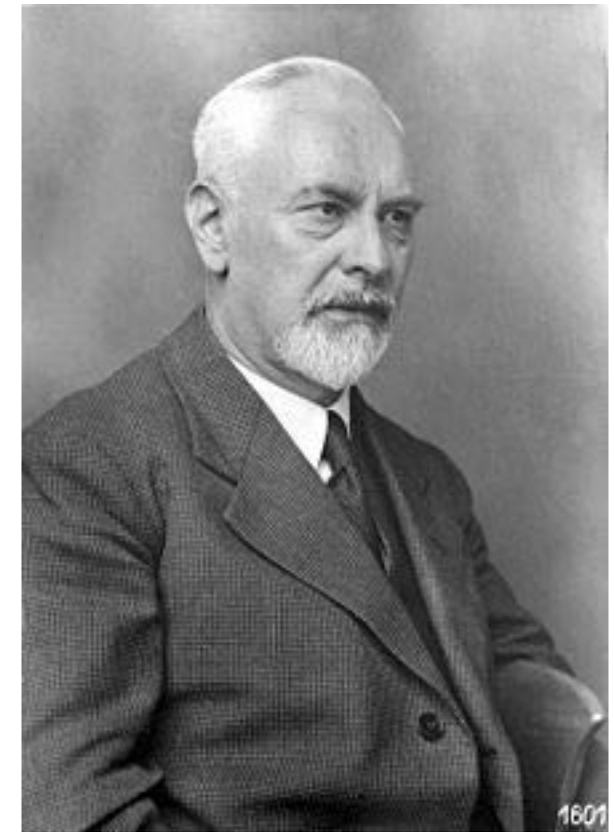
$$\rho \left[\frac{\partial \bar{u}_i}{\partial t} + \bar{u}_j \frac{\partial \bar{u}_i}{\partial x_j} \right] = -\frac{\partial \bar{p}}{\partial x_i} + \frac{\partial}{\partial x_j} \left(\mu \frac{\partial \bar{u}_i}{\partial x_j} - \rho \overline{u_i' u_j'} \right)$$

Total Shear Stress

Reynolds stress tensor

$$\tau_R = \tau_{ij}' = \rho \overline{u_i' u_j'}$$



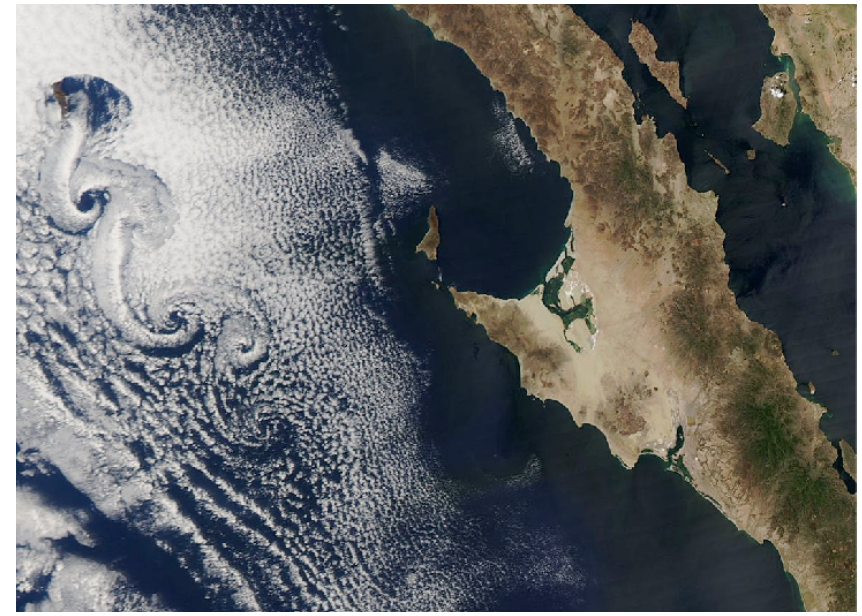


According to our current on-line database,
Ludwig Prandtl has 87 students and 3483 descendants.

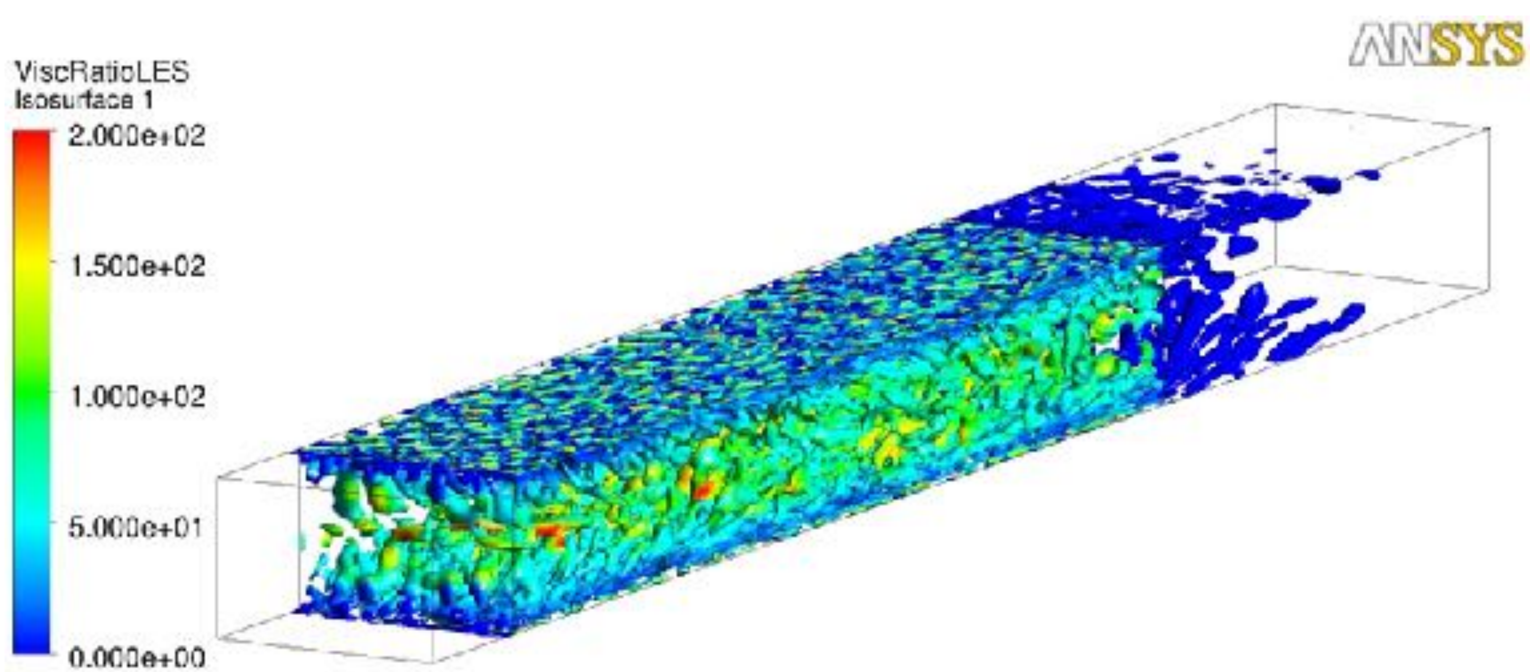
ICM Plenary Speaker 1904
(invited by D. Hilbert)

"Your talk was the most beautiful one of the whole conference" - F. Klein

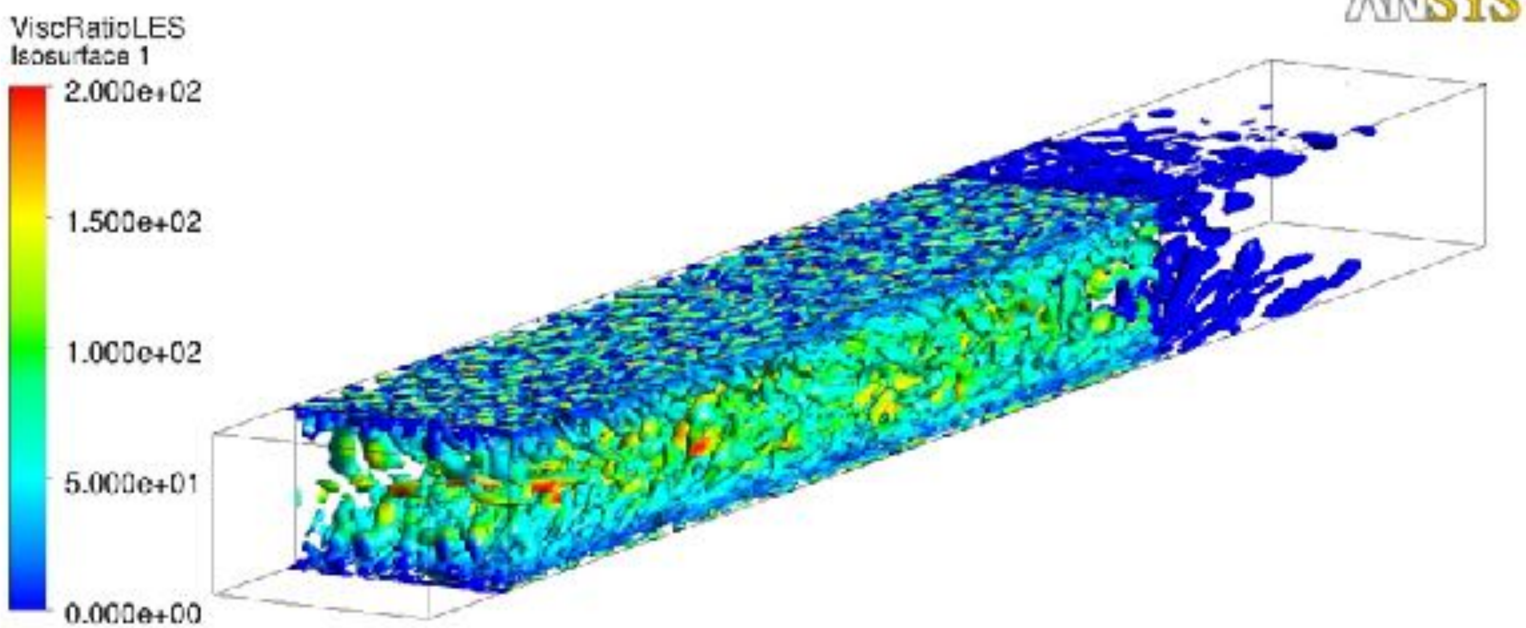
What is the most dramatic kinematic effect of a high Reynolds Number?



Large deviations of velocity gradients, mainly near the boundary



$$\frac{\partial \langle p \rangle}{\partial x} = \frac{dp_w}{dx}$$



$$\tau = \rho\nu \frac{d\langle U \rangle}{dy} - \rho \langle uv \rangle$$

Total Shear Stress

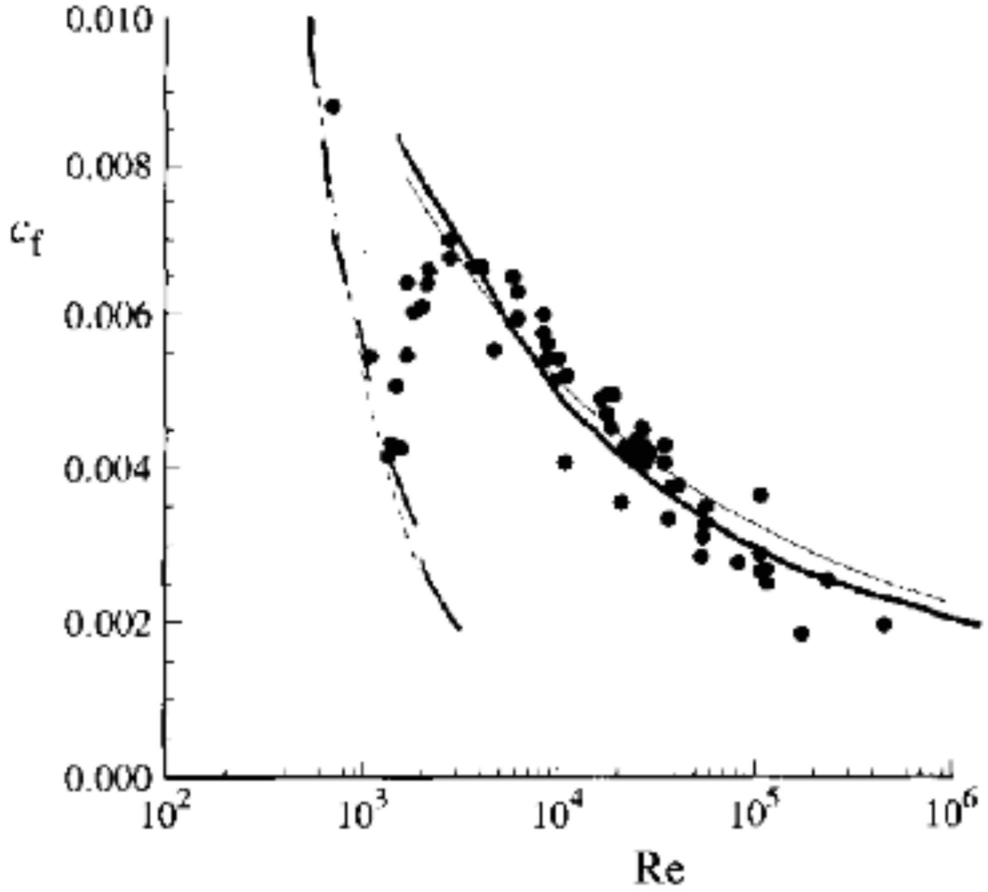
$$\tau_w \equiv \tau_0 = \rho\nu \frac{d\langle U \rangle}{dy} - \rho \langle uv \rangle$$

Wall shear stress

Momentum balance $-\frac{dp_w}{dx} = \frac{\tau_w}{\delta}$

Friction Coefficient

$$C_f = \frac{\tau_w}{\rho \langle U \rangle^2}$$



Blasius

$$C_f \sim \frac{1}{Re^{1/4}}$$

Comparing Life far from and near the wall
Prandtl's Revolution

$$u_\tau = \sqrt{\frac{\tau_w}{\rho}}$$

Friction Velocity

$$\delta_\nu = \frac{\nu}{u_\tau}$$

Wall viscous length-scale

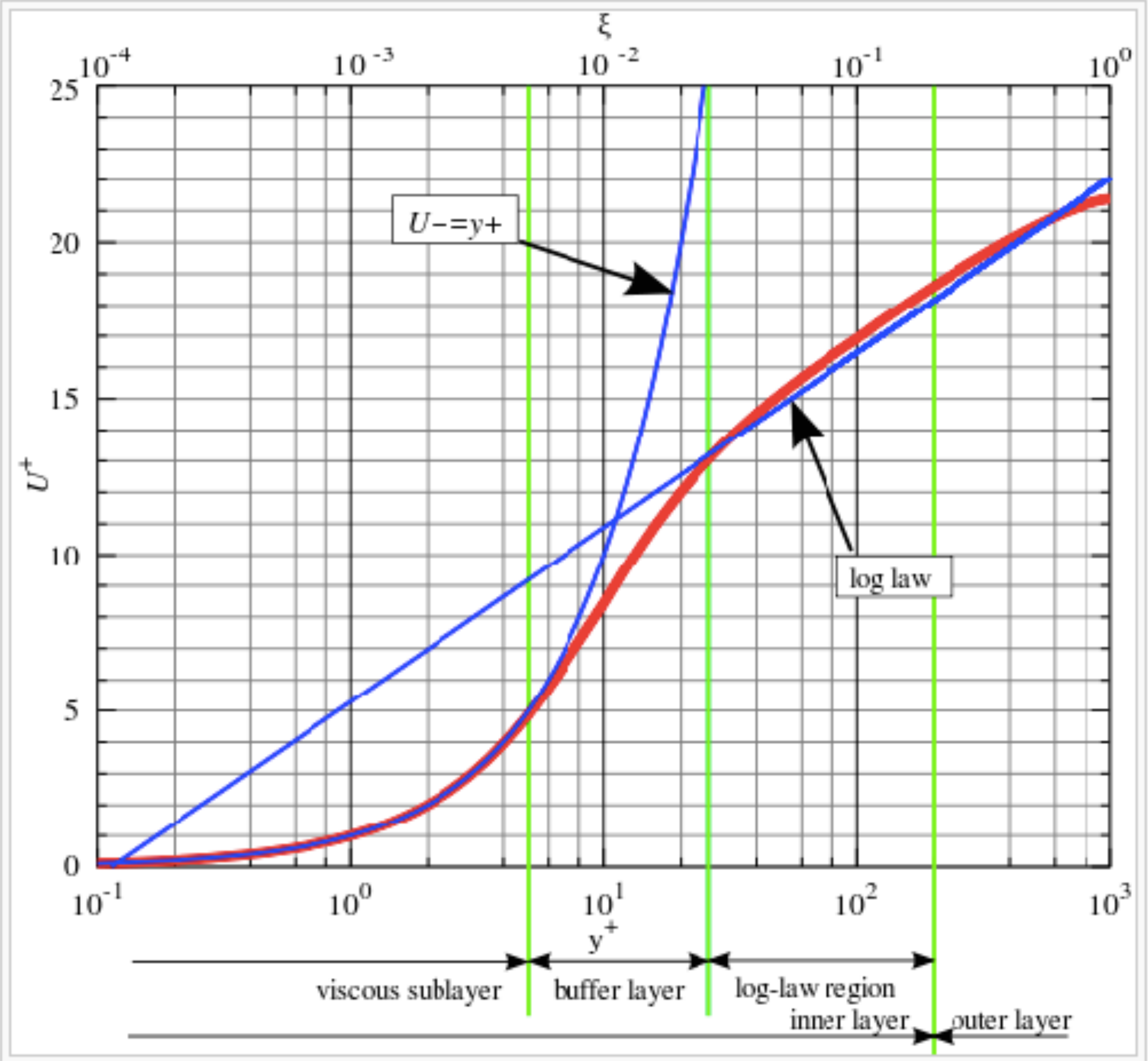
$$y^+ = \frac{y\,u_\tau}{\nu}$$

Wall Units

$$u^+ = \frac{u}{u_\tau}$$

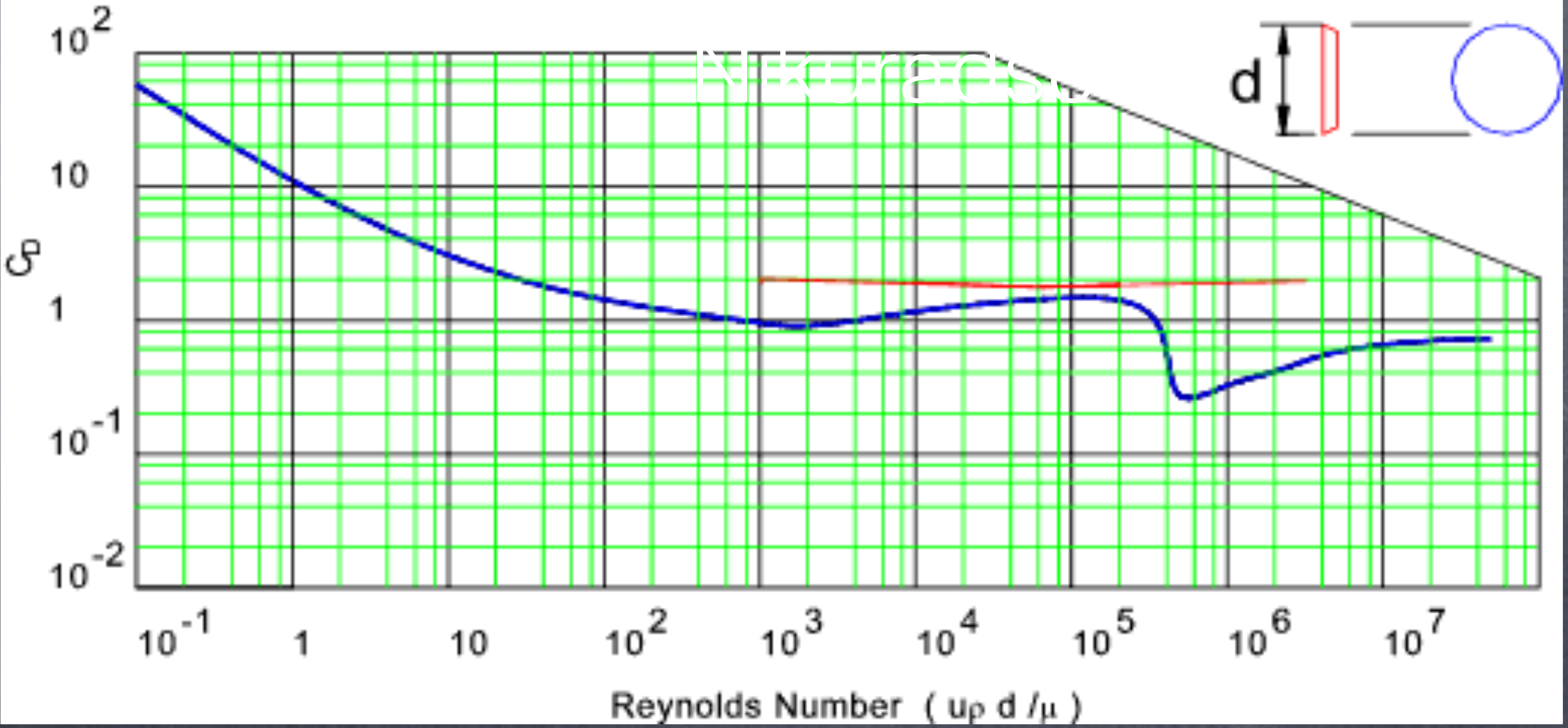
Wall velocity

$$u^+ = \frac{1}{\kappa}\ln y^+ + C^+$$

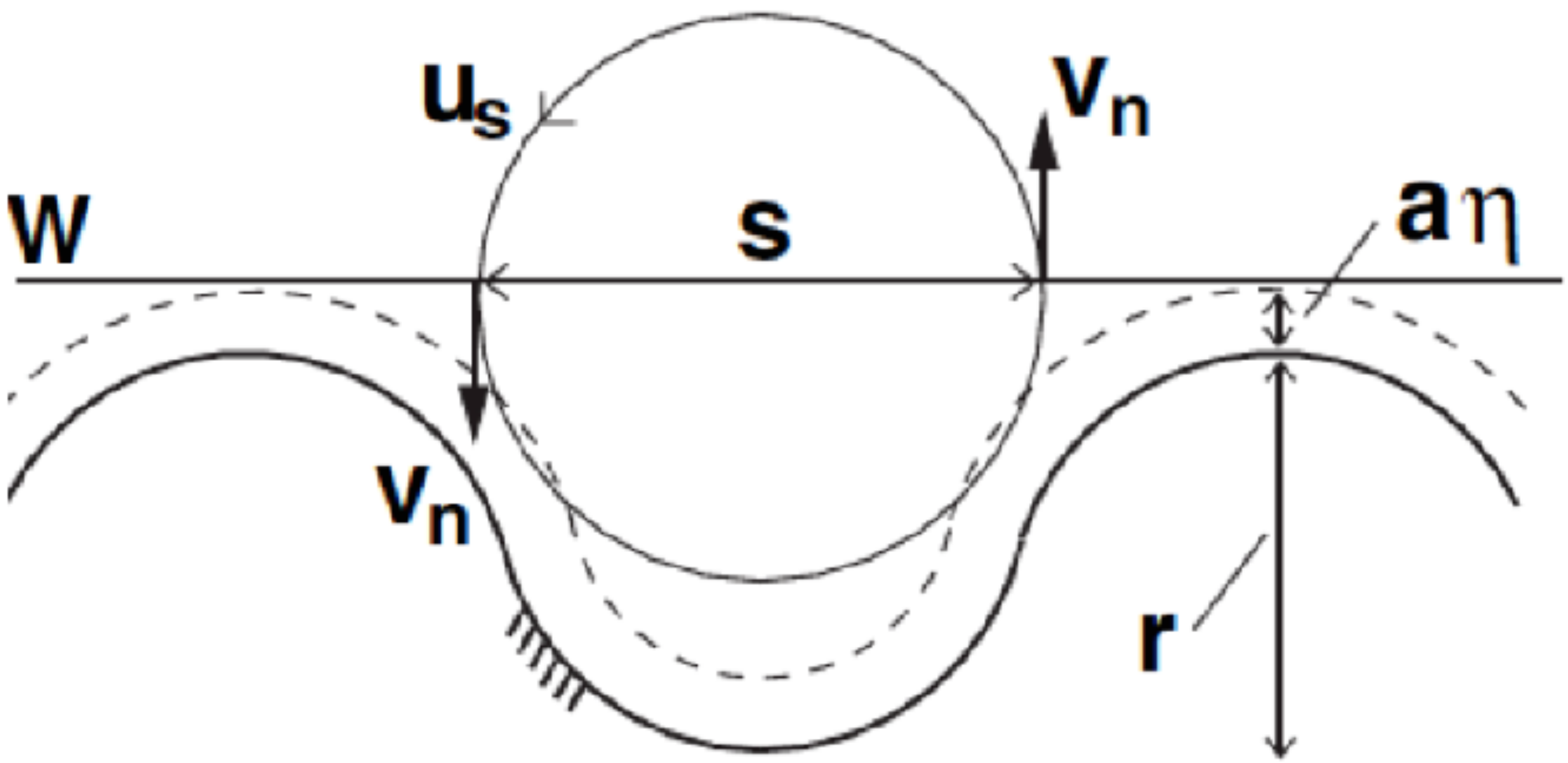


Singularity again?
Drag coefficient for rough walls

Nikuradse

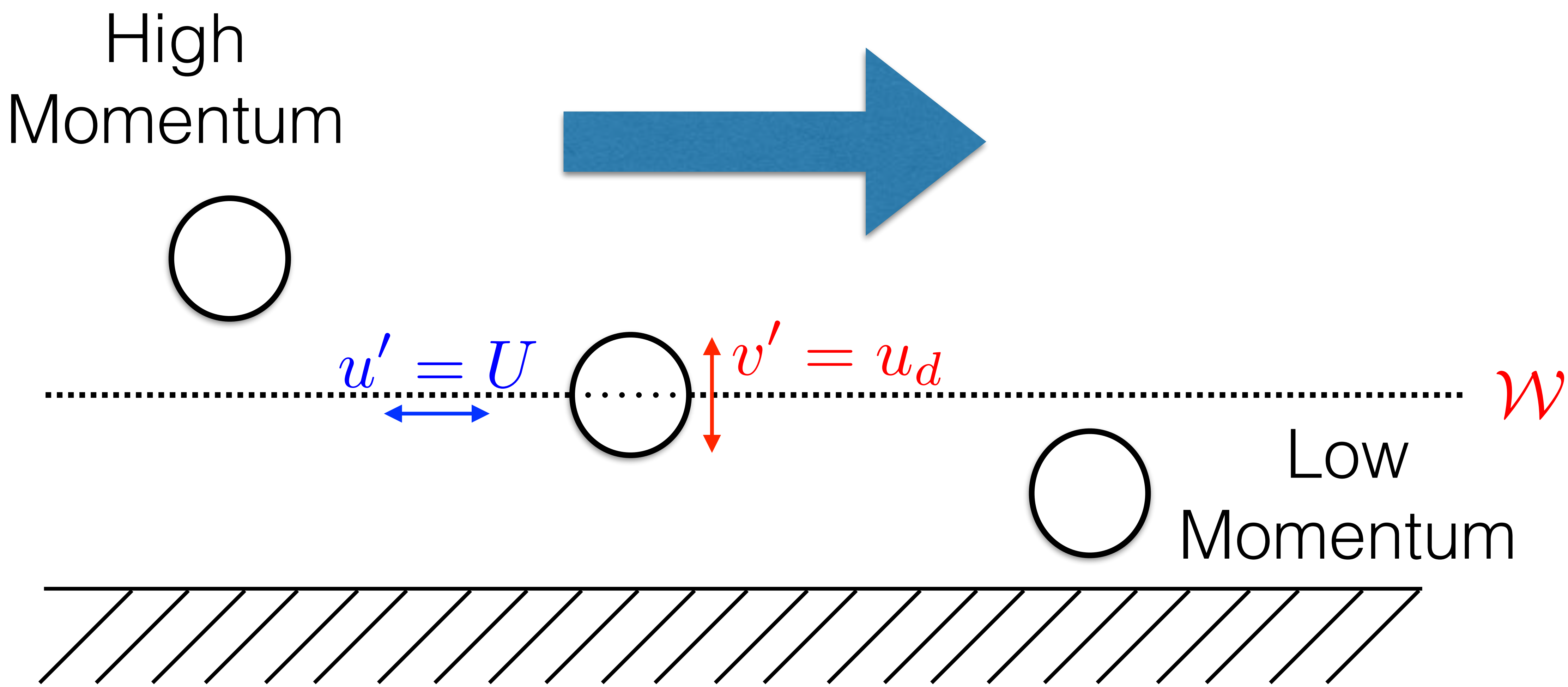


G. Gioia and P. Chakraborty's Phenomenology



$$\tau \sim \rho V v_n$$

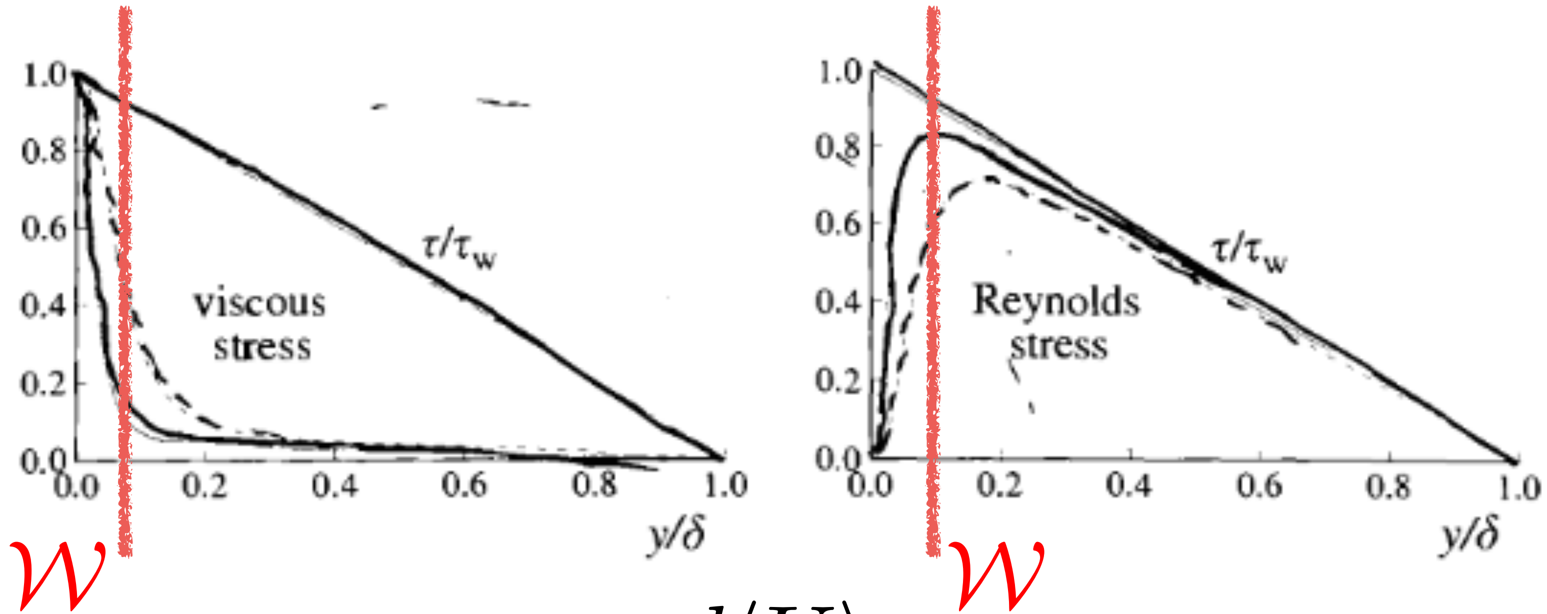
A new phenomenology: Kolmogorov meets Prandtl
A Traveling vortex carrying momentum imbalances



$$\tau = \rho\nu \frac{d\langle U\rangle}{dy} - \rho\langle uv\rangle$$

Inspired on
G. Gioia and P. Chakraborty
P.R.L. 2006

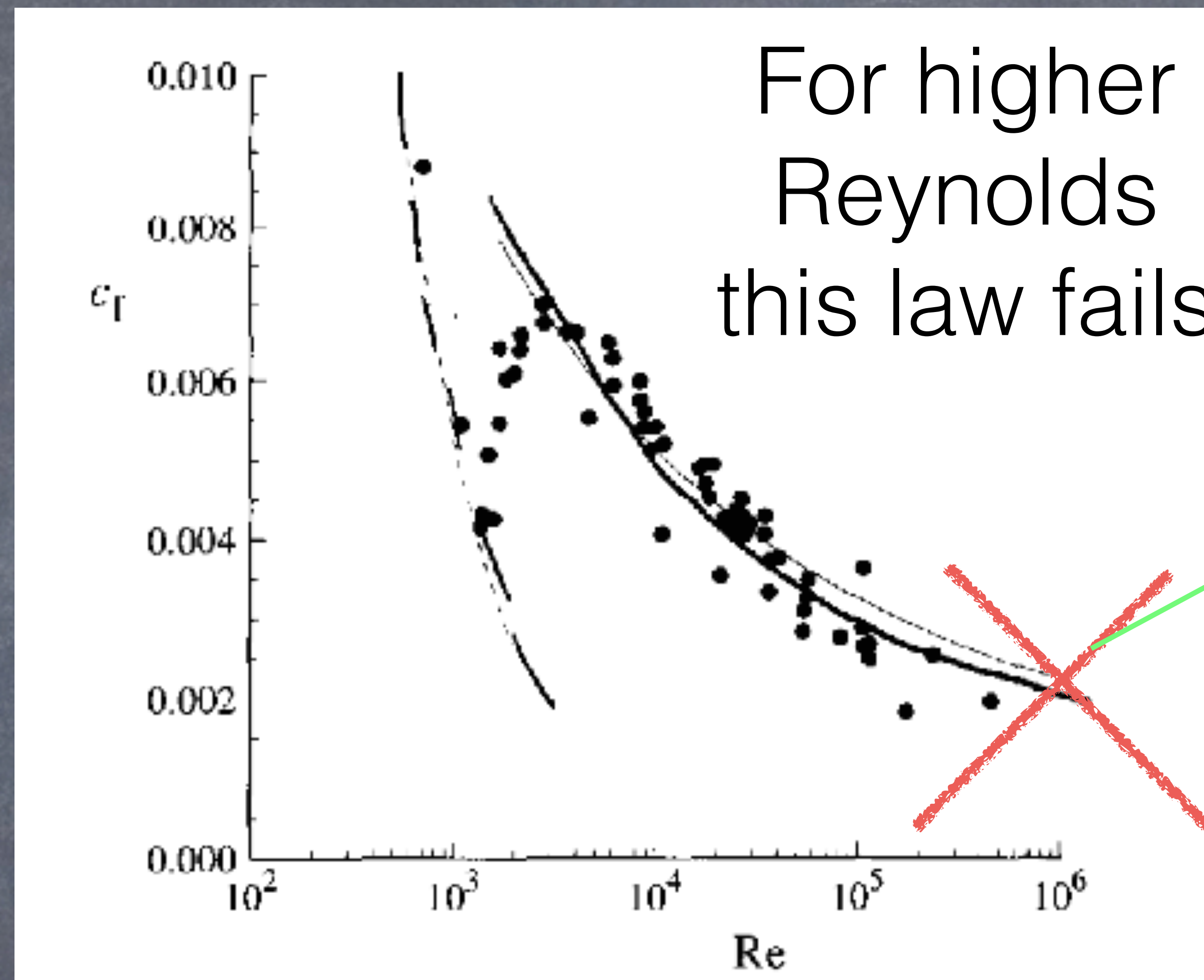
A Brand New Phenomenology



$$\tau = \rho\nu \frac{d\langle U \rangle}{dy} - \rho\langle uv \rangle$$

$$\tau_w \sim \tau_R|_{\mathcal{W}} \sim \rho U v' \sim \rho U u_d$$

Friction coefficient



Prandtl's log-law

$$C_f \sim \frac{1}{(\ln Re)^2}.$$

$$f = \frac{\tau_w}{\frac{1}{2}\rho U^2} \sim \frac{\rho U u_d}{\frac{1}{2}\rho U^2} \sim \frac{u_d}{U} \sim \frac{1}{Re^{1/4}}$$

Blasius Friction Law

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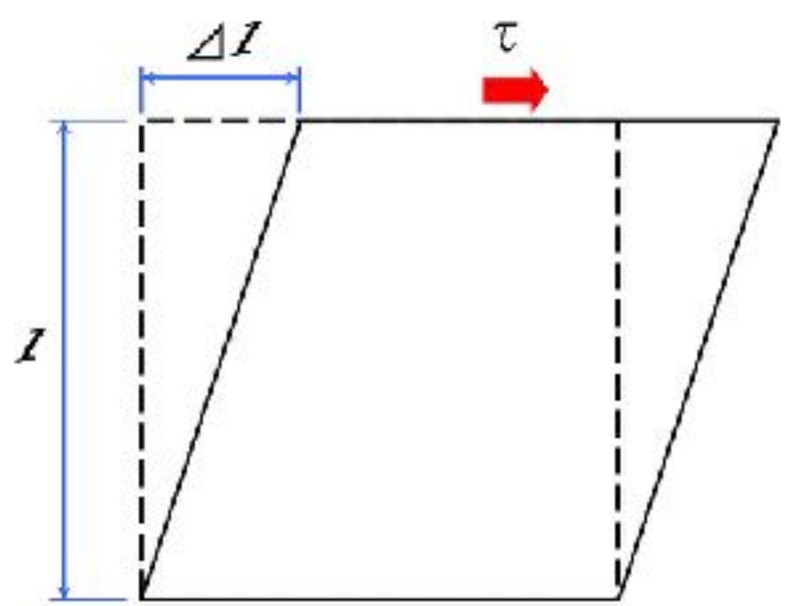
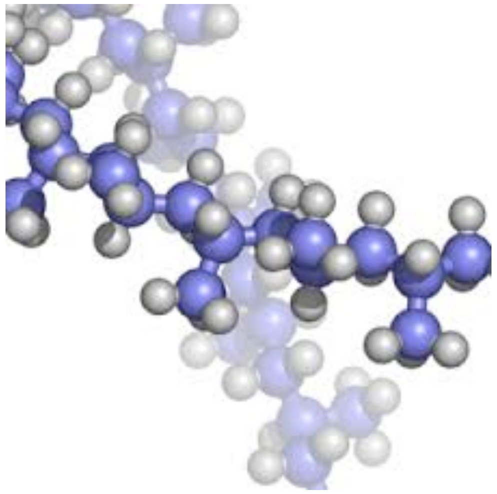
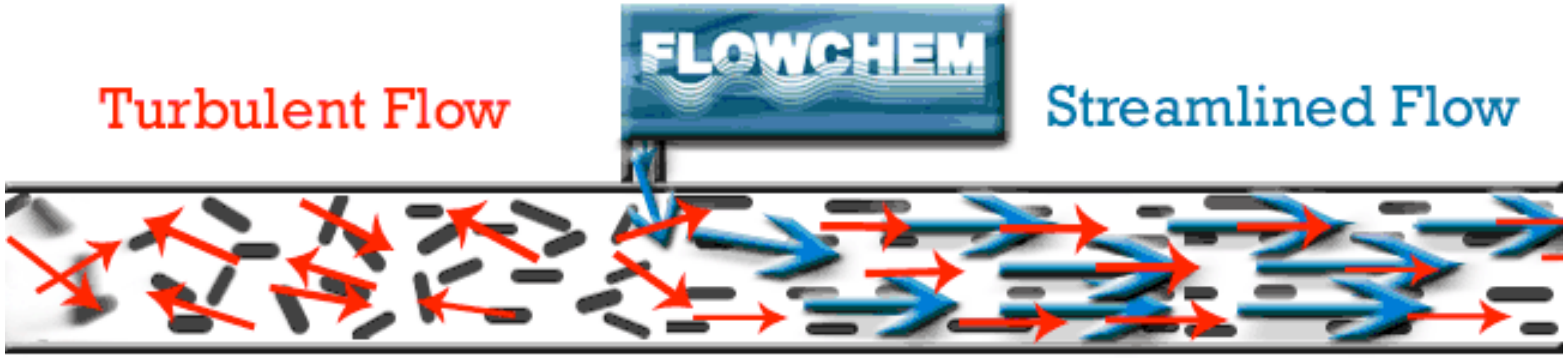
Newton



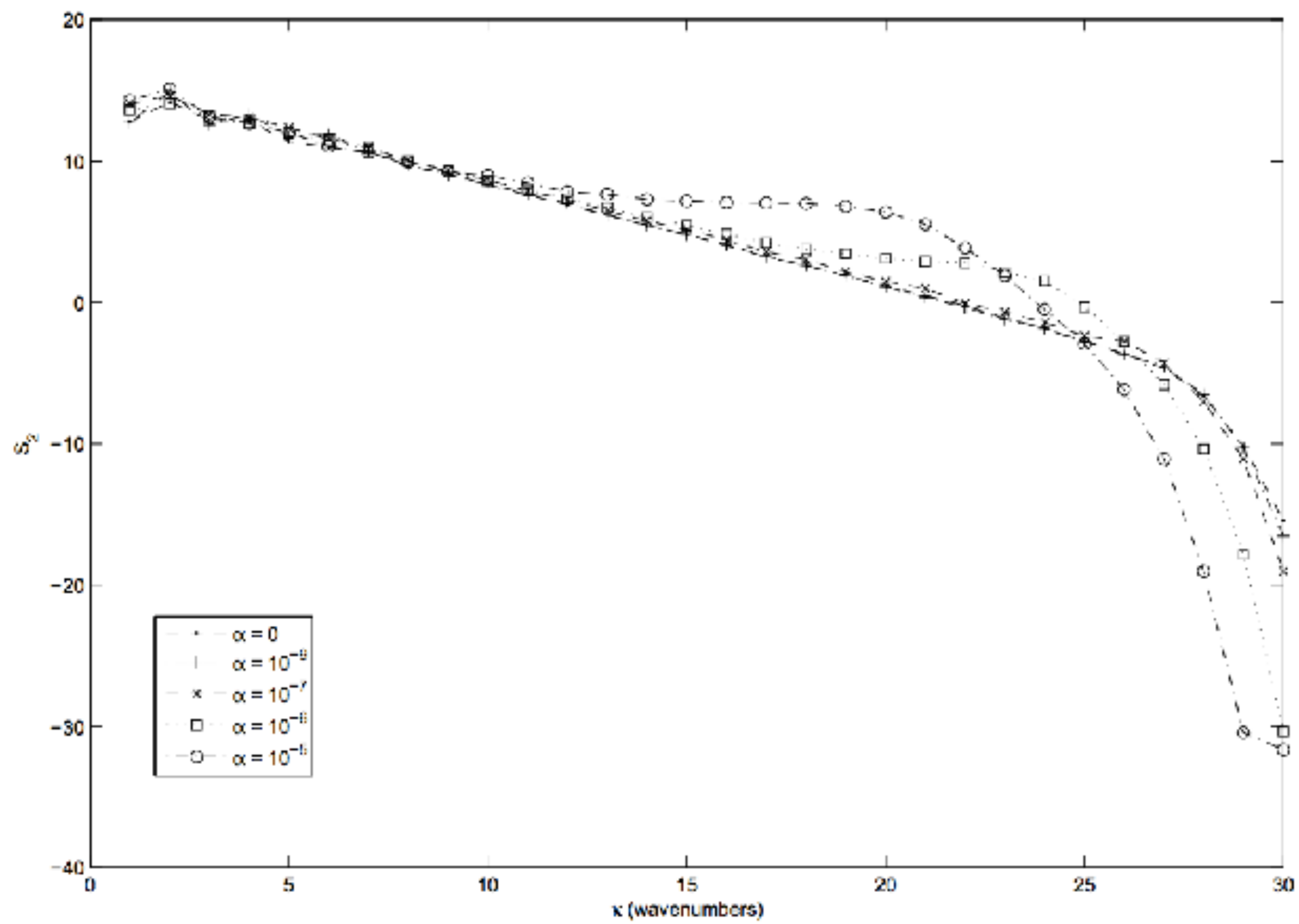
$$\tau = \mu \frac{dU}{dx}$$

Complex Flows - Drag Reduction



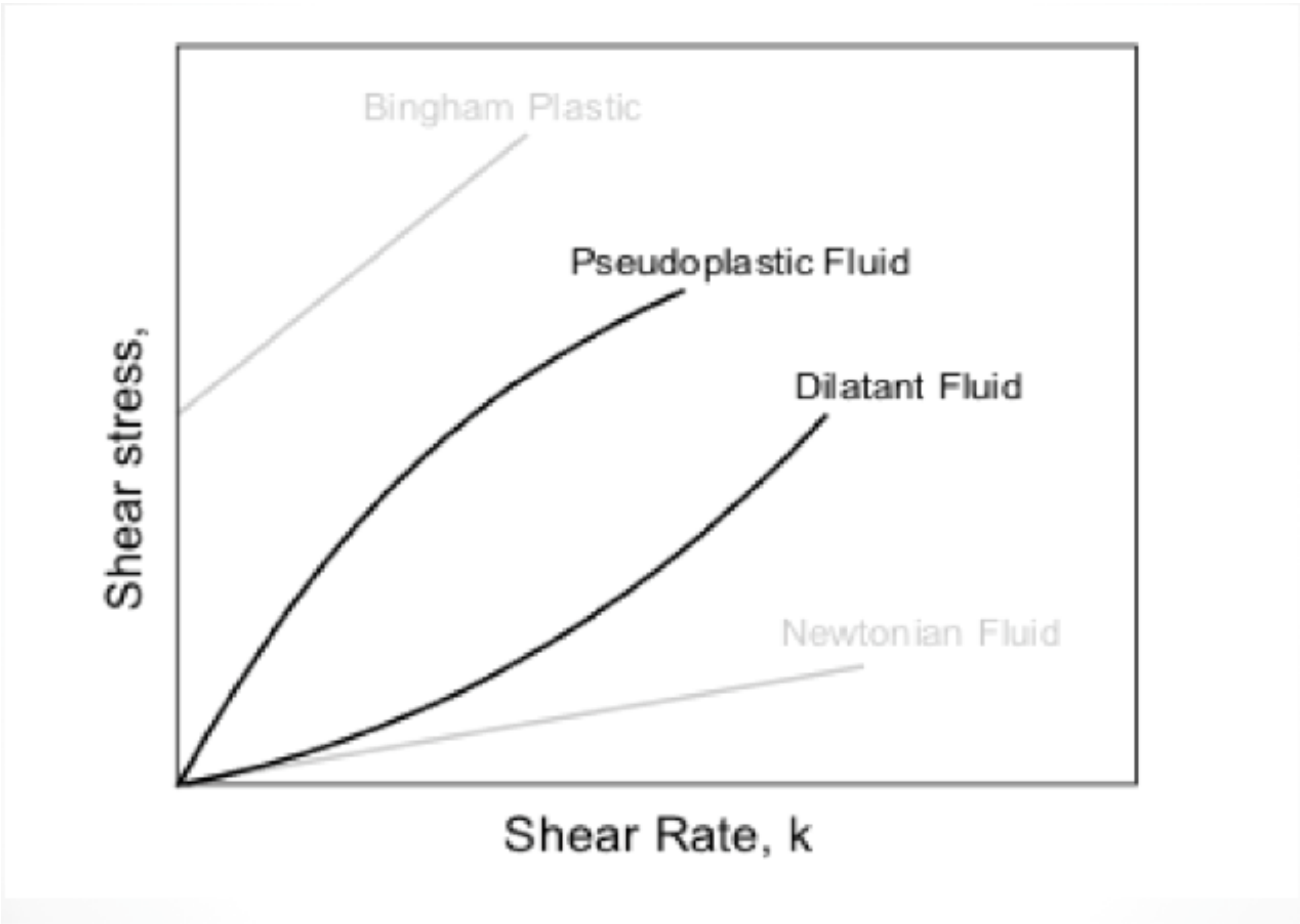


Voigt viscoelastic regularization
(Titi, Levant, F.R.)



$$\partial_t(\mathbf{u} - \alpha^2 \Delta \mathbf{u}) - \nu \Delta \mathbf{u} + \mathbf{u} \cdot \nabla \mathbf{u} + \nabla p = \mathbf{f},$$

Non-Newtonian Rheology



by D. Dennis

Power-law model

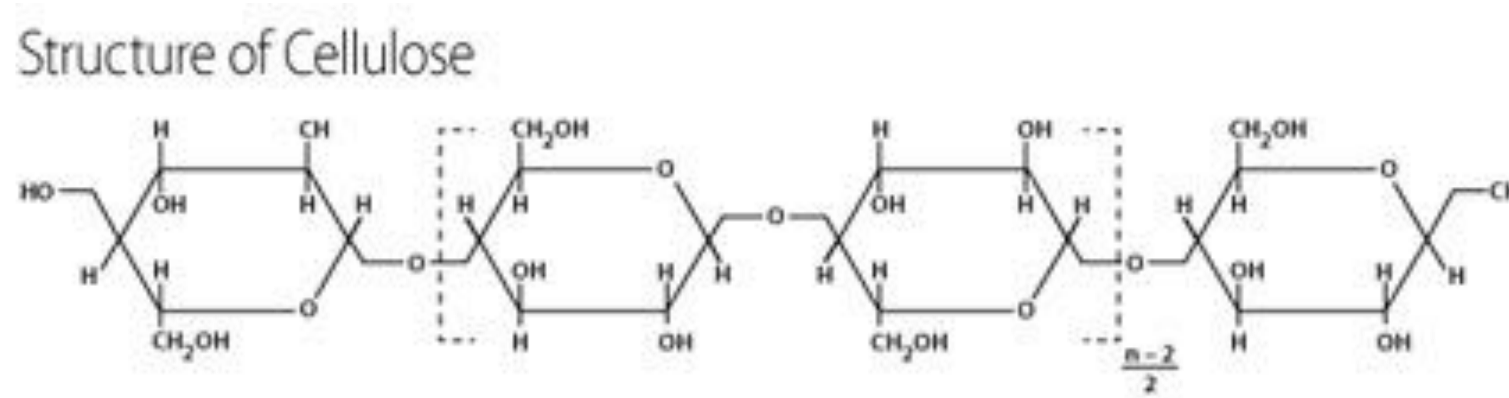
Viscosity is a function of the flow

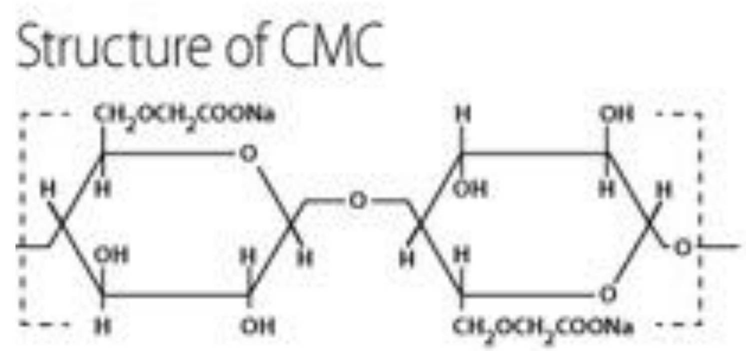
$$\dot{\gamma} \equiv (\nabla \mathbf{u} + \nabla \mathbf{u}^t)$$

$$\tau = \eta(|\dot{\gamma}|)\,\dot{\gamma}$$

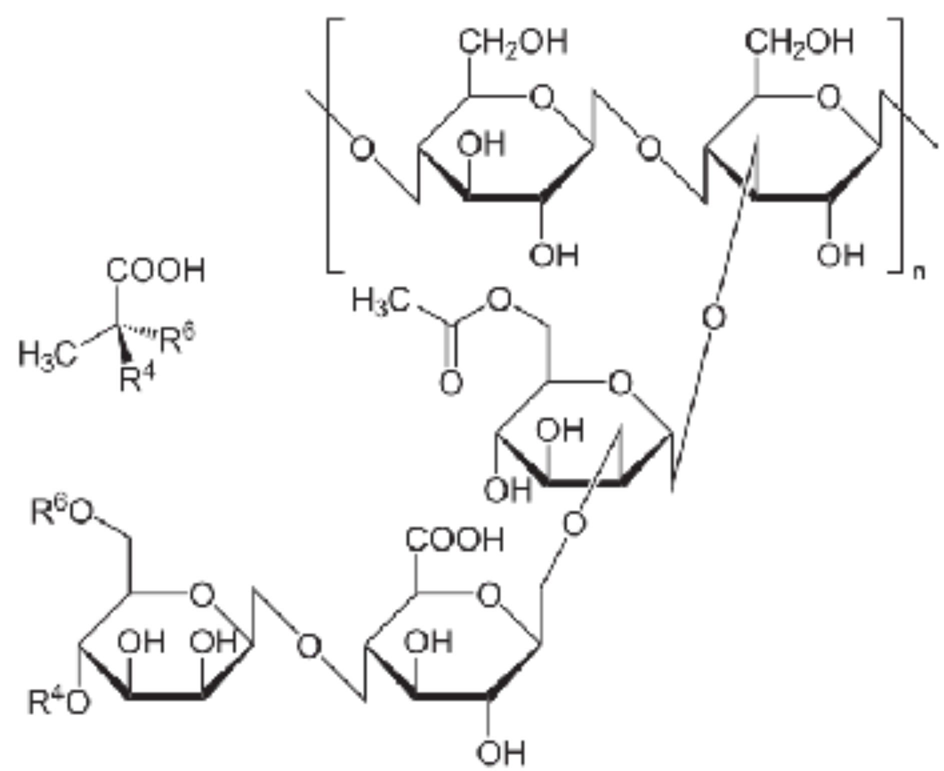
$$\eta(|\dot{\gamma}|) = K|\dot{\gamma}|^{n-1}$$

sodium carboxymethyl cellulose (CMC)



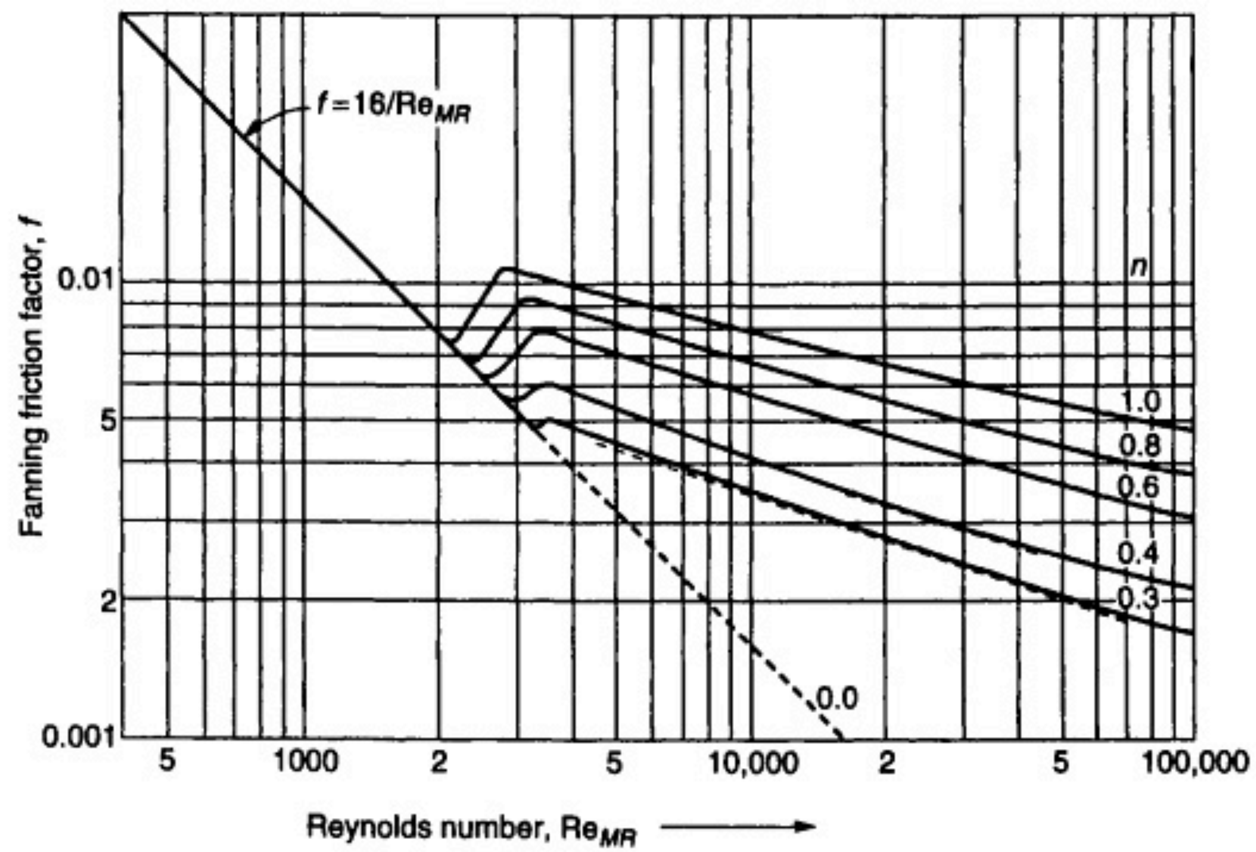


Xanthan gum



Dodge-Metzner Relation

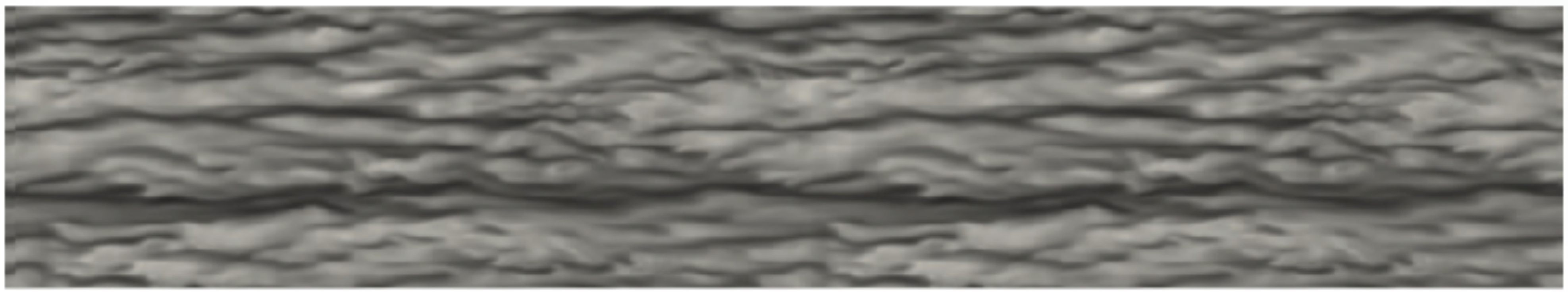
$$f = \frac{\tau_w}{\frac{1}{2}\rho U^2}$$



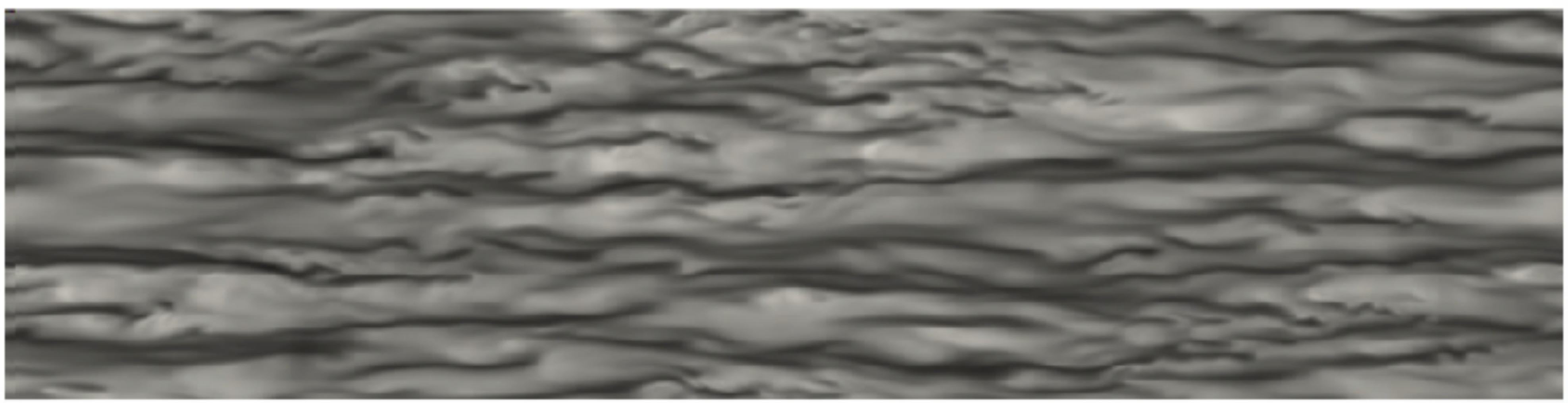
$$Re = Re_{MR} = \frac{\rho U^{2-n} L^n}{K((3n+1)/(4n))^n 8^{n-1}}.$$

$$\frac{1}{\sqrt{f}} = \frac{4}{n^{0.75}} \log\left(\frac{Re}{f^{\frac{n-2}{2}}}\right) - \frac{0.4}{n^{1/2}}$$

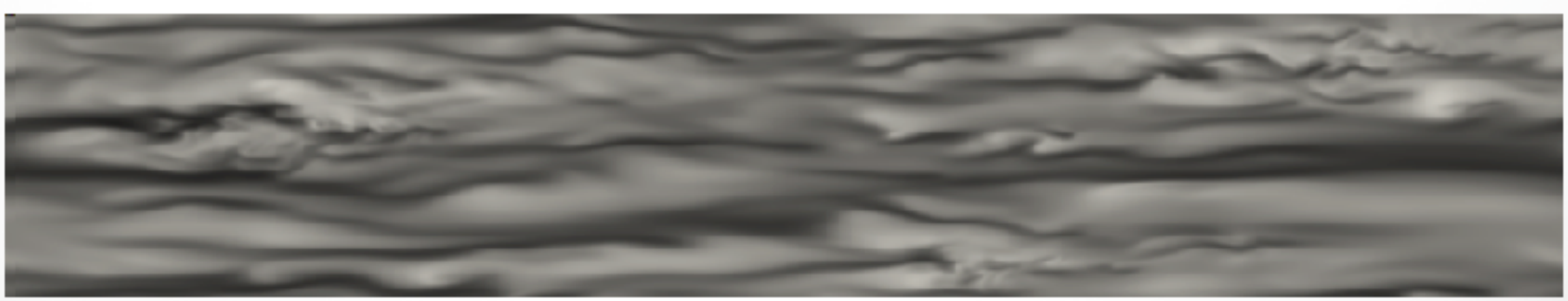
DNS – Re_t ~ 400



n=1.0



n=0.75



n=0.5

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Kolmogorov's scaling for Power-law flows

$$\ell_d^{(n)} \sim k^{\frac{3}{2(n+1)}} \epsilon^{\frac{n-2}{2(n+1)}}, \quad t_d^{(n)} \sim K^{\frac{1}{(n+1)}} \epsilon^{\frac{-1}{(n+1)}}.$$

$$\frac{u_d^{(n)}}{U} \sim \left(\frac{\ell_d^{(n)}}{L} \right)^{1/3} \sim \frac{1}{Re^{\frac{1}{2(n+1)}}}.$$

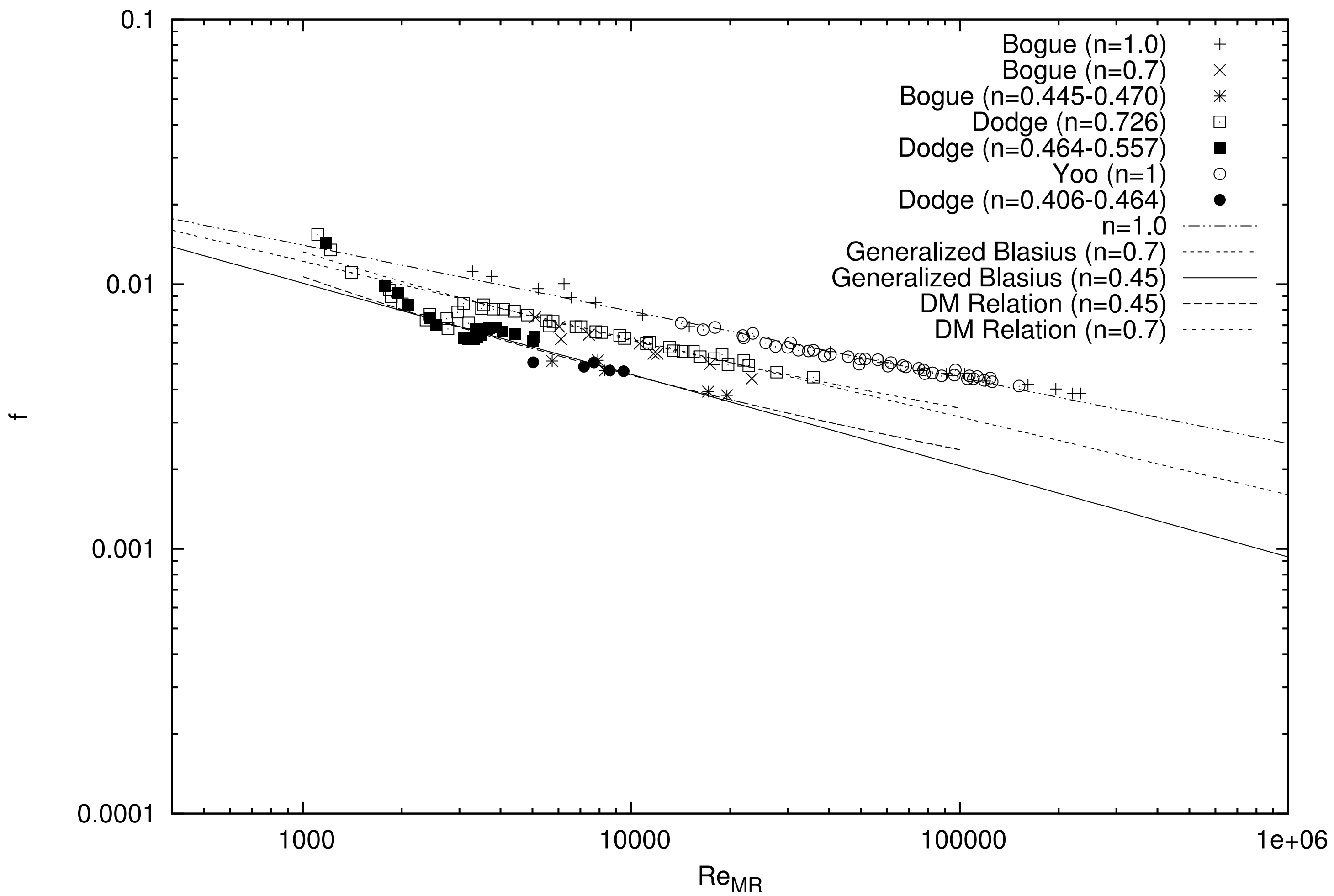
Generalized Blasius' Friction Factor

$$f = \frac{\tau_w}{\frac{1}{2}\rho U^2} = g(n)\frac{u_d^{(n)}}{U} = g(n)\frac{1}{Re^{\frac{1}{2(n+1)}}}.$$

$$C_f = \frac{0.316}{Re^{\frac{1}{2(n+1)}}}$$

H. Anbarlooei, A. P.S. Freire, D. Cruz, F.R.
PRE, 2015

Generalized Blasius



$$C_f = \frac{0.316}{Re^{\frac{1}{2(n+1)}}}$$

H. Anbarlooei, A. P.S. Freire, D. Cruz, F.R.
PRE, 2015

Comparison with Previous Empirical Relations

Hanks and Rick's (1975)

f = $0.0682 n^{-1/2} Re^{-1/(1.87+2.39n)}$

Generalized Blasius

$$C_f = \frac{0.316}{Re^{\frac{1}{2(n+1)}}}$$

A new phenomenology: Kolmogorov meets Prandtl
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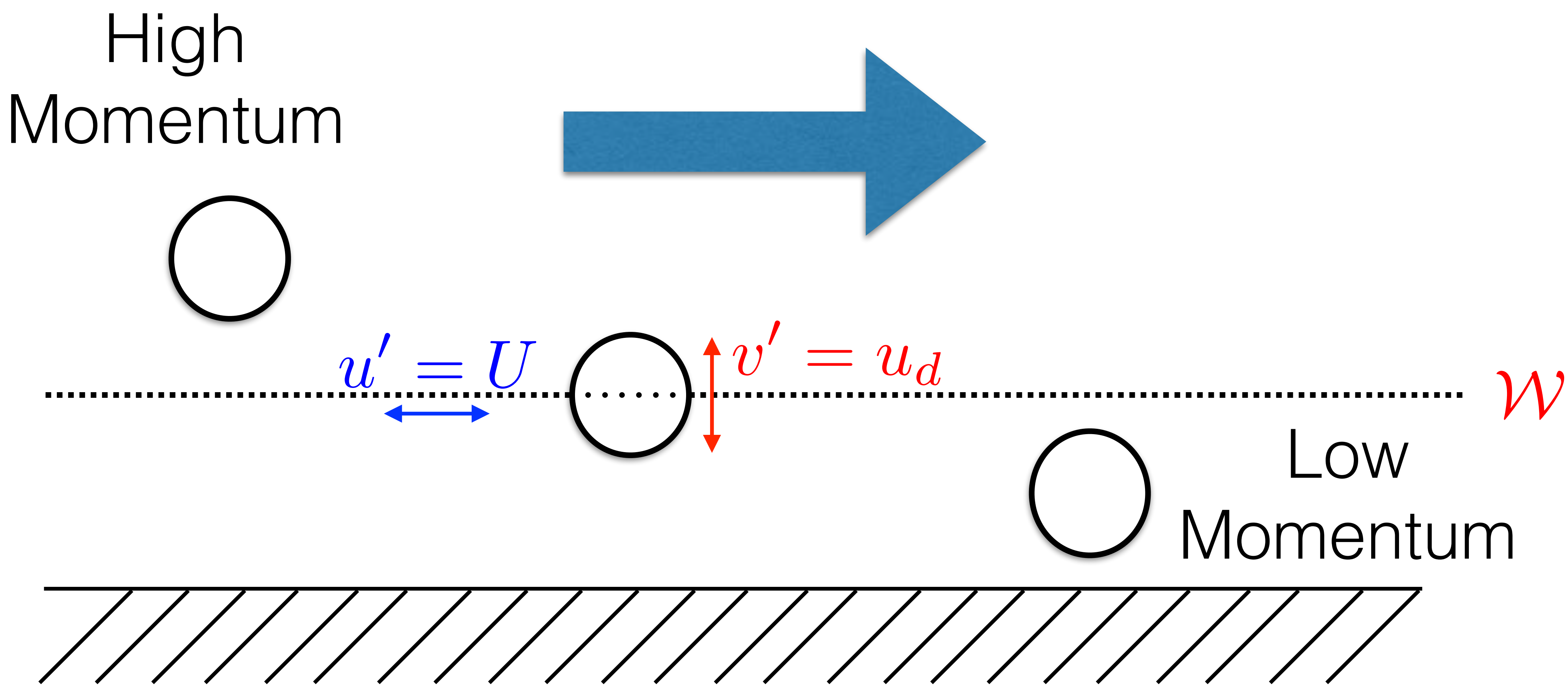
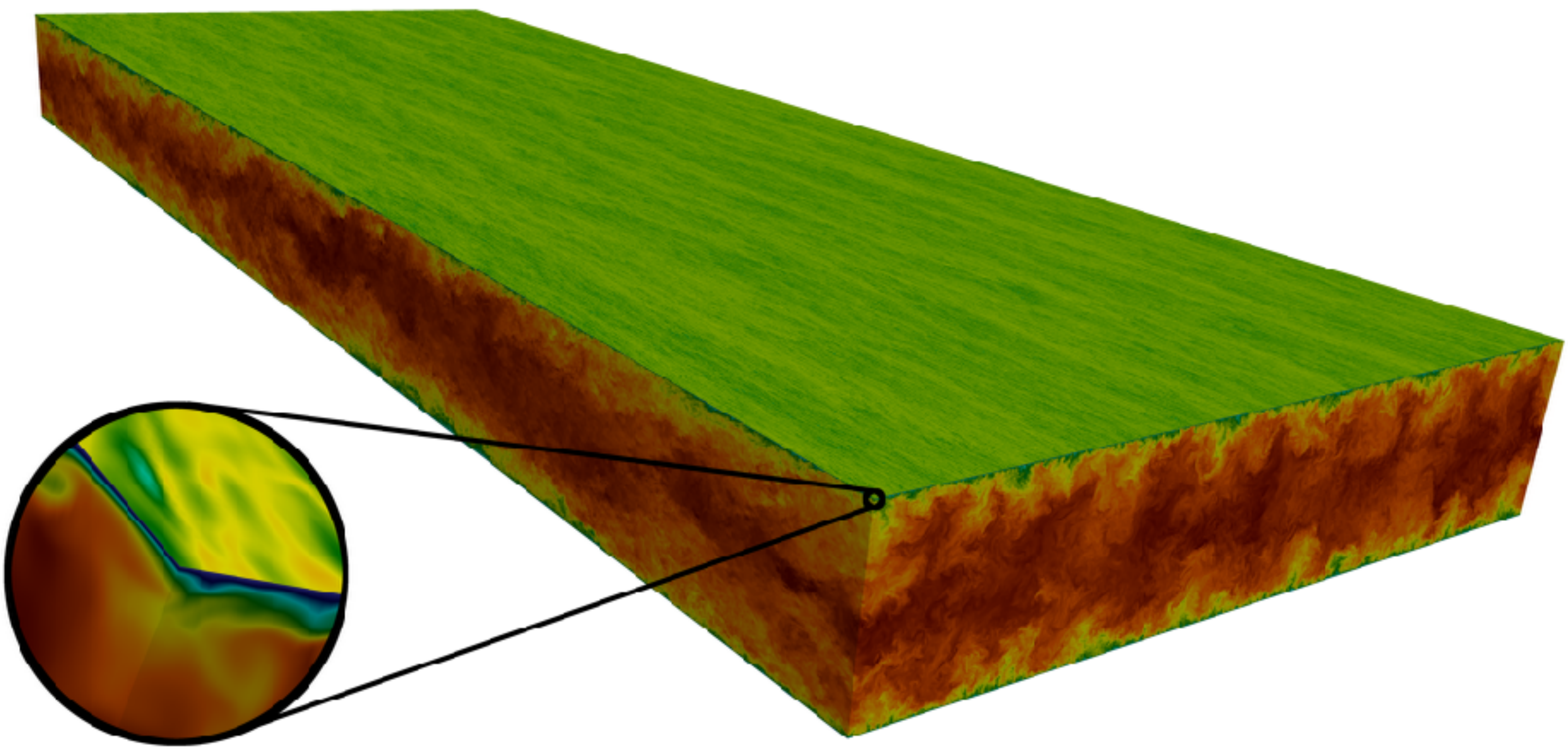


$$\tau = \rho\nu \frac{d\langle U \rangle}{dy} - \rho \langle uv \rangle$$

Inspired on
G. Gioia and P. Chakraborty
P.R.L. 2006

Blasius Domain of validity

$$\frac{\ell_d^{(n)}}{L} \sim \frac{1}{Re^{\frac{3}{2n+2}}}, \qquad \frac{\delta_\nu^{(n)}}{L} \sim \frac{1}{Re^{\frac{4n-3}{4n+4}}}.$$

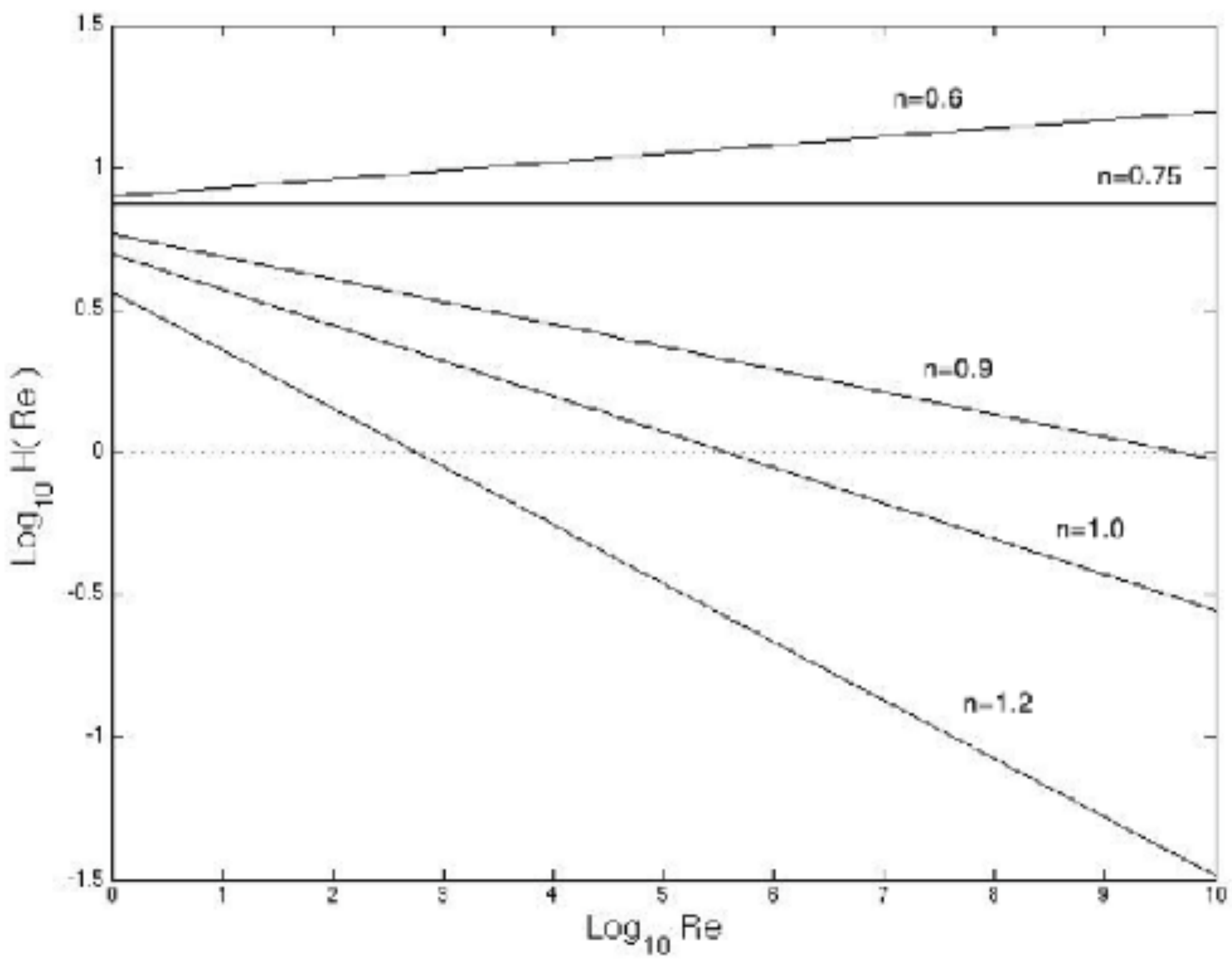


Domain of validity

Comparing Prandtl (wall) and Kolmogorov(smallest eddy) scales

$$\frac{\ell_d^{(n)}}{L} \sim \frac{1}{Re^{\frac{3}{2n+2}}}, \qquad \frac{\delta_\nu^{(n)}}{L} \sim \frac{1}{Re^{\frac{4n-3}{4n+4}}}.$$

$$H(Re,n) \equiv \frac{a_n \delta_\nu^{(n)}}{\ell_d^{(n)}} = \frac{2^{3-n}\sqrt{2}}{(3n+1)^n g^{1/2}(n)} \cdot \frac{a_n}{Re^{\frac{4n-3}{4n+4}}}.$$



Similar results hold for other complex flows

- Bingham
- Viscoelastic
- Herschel-Buckley
- Annular flows (other geometry)
- Transpiration walls (other boundary conditions)

"Prandtl was able to see the solutions of differential equations without calculating them."

W. Heisenberg

Muito obrigado