STOCHASTIC ASPECTS OF QUANTUM THEORY

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In 1920s–1940s the main uncertainty was about whether QM is good for mankind, and it was a source of several controversies and even personal dramas

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In QM, an important rôle is played by eigenvalues λ and (normalized) eigenvectors/eigenfunctions ψ with $\|\psi\| = 1$.

That is, given a linear operator **A**, we analyze this:

$$\mathbf{A}\psi = \lambda\psi, \quad \text{or} \quad (\mathbf{A} - \lambda\mathbf{I})\psi = 0, \quad \text{or} \quad \psi \in \texttt{ker}\,(\mathbf{A} - \lambda\mathbf{I}),$$

or, more generally, we look for values $\lambda \in \mathbb{C}$ where

$$(\mathbf{A} - \lambda\mathbf{I})^{-1} \text{ doesn't exist as a bounded operator.}$$

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The most important operator is a Hamiltonian, as introduced by Schrödinger:

$$\mathbf{H} = \mathbf{K} + \mathbf{U}.$$

Here $\mathbf{K}$ stands for a kinetic energy operator and $\mathbf{U}$ for a potential energy operator.

In a standard form, $\mathbf{K}$ is minus a half of a Laplacian (in continuous or discrete variables, curved or flat):

$$\big(\mathbf{K}\phi\big)(\mathbf{x}) = \frac{-1}{2}\big(\mathbf{\Delta}\phi\big)(\mathbf{x}).$$

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Individually, **K** and **U** are quite tame but as a sum they generate a complex picture.

The eigenvalues (EVs) and eigenfunctions (EFs) of **H** attracted attention (at times controversial) for nearly 100 years. An illustrative quotation:

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Nevertheless, the analysis of the Schrödinger operator **H** is a flourishing and sophisticated area of modern Functional Analysis and Mathematical Physics.

For results before 2000, see reviews by **B. Simon**:

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After 2000 there were many first-class results proved in this field; I have no doubt there will be more in years to come (possibly, at the Fields medal level).

In the case $\mathbf{U} = \mathbf{0}$ (no potential), the Hamiltonian $\mathbf{H} = \mathbf{K} + \mathbf{U}$ is reduced to its kinetic part $\mathbf{K}$:

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In a simple case of single-particle system in one dimension, the Laplacian $\boldsymbol{\Delta}$ becomes the operator of the second derivative, in $L_2(a, b)$ or $L_2(\mathbb{R}^1)$:

$$\boldsymbol{\Delta}\phi(x) = -\frac{1}{2}\,\frac{\mathrm{d}^2\phi}{\mathrm{d}x^2}(x) = -\phi''(x)/2.$$

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Its EFs are exponents:

$$\psi(x)(= \psi(x, \lambda)) = \frac{1}{\sqrt{2\pi}}\mathrm{e}^{\mathrm{i}\lambda x}, \text{ with the EVs } \frac{\lambda^2}{2}.$$

In the case of $L_2(\mathbb{R}^1)$, these are generalized EFs.

Also, λ is taken real: $\lambda \in \mathbb{R}^1$, and $\lambda^2 \geq 0$. Then $|\psi(x)|^2 \equiv 1/\sqrt{2\pi}$. Every unit element $\phi \in L_2(\mathbb{R}^1)$, with $\displaystyle\int_{\mathbb{R}^1} |\phi(x)|^2 \mathrm{d}x = 1$, can be written as the Fourier integral:

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$$\phi(x) = \frac{1}{\sqrt{2\pi}} \int_{\mathbb{R}^1} \mathrm{e}^{\mathrm{i}\lambda x} \widehat{\phi}(\lambda) \mathrm{d}\lambda.$$

Here $\displaystyle\widehat{\phi}(\lambda) = \frac{1}{\sqrt{2\pi}} \int_{\mathbb{R}^1} \mathrm{e}^{-\mathrm{i}\lambda x} \phi(x) \mathrm{d}x$ is the Fourier transform of $\phi(x)$, with $\displaystyle\int_{\mathbb{R}^1} |\widehat{\phi}(\lambda)|^2 = 1$ (a probability density normalization). The variables $x, \lambda \in \mathbb{R}^1$ are related, respectively, to the **position** and **momentum** of the QM particle.

In the case of $L_2(a,b)$ let us take $a=0,\ b=2\pi$. To make our operator $\phi \in L_2(0,1) \mapsto -\phi''/2$ self-adjoint, we impose periodic boundary conditions: we require that the EFs obey

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Then the normalized EFs become

$$\psi_n(x)=\frac{1}{\sqrt{2\pi}}\,\mathrm{e}^{\mathrm{i}nx},\ 0\leq x\leq 2\pi,$$

with the EVs $\lambda_n=\frac{n^2}{2}$, $n\in\mathbb{Z}^1$, $n^2\geq 0$ and $|\psi_n(x)|^2\equiv\frac{1}{2\pi}$. And again, every unit element $\phi\in L_2(0,2\pi)$, with

$\int_0^{2\pi} |\phi(x)|^2 \mathrm{d}x = 1$, admits an expansion

$$\phi(x) = \frac{1}{\sqrt{2\pi}} \sum_{n \in \mathbb{Z}^1} \mathrm{e}^{\mathrm{i}nx} \widehat{\phi}_n$$

into the Fourier series with Fourier coefficients $\widehat{\phi}_n = \int_0^{2\pi} \mathrm{e}^{\mathrm{i}nx} \phi(x) \mathrm{d}x$ where $\sum\limits_{n \in \mathbb{Z}^1} |\widehat{\phi}_n|^2 = 1$. This turns $|\widehat{\phi}_n|^2$ into probabilities. Again, x relates to the **position** and n to the **momentum** of the QM particle.

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Physicists interpret $|\phi(x)|^2$ as a **probability density** for the position of a QM particle in the unit 'wave packet' $\phi \in L_2$ (since $|\phi(x)|^2 \geq 0$ and $\int |\phi(x)|^2 \mathrm{d}x = 1$).

Similarly, $|\widehat{\phi}(\lambda)|^2$ and $|\widehat{\phi}_n|^2$ are interpreted as a probability density and probability mass for the momentum of a QM particle in the unit 'wave packet' $\phi \in L_2$.

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For $\phi = \psi_n \in L_2(0, 2\pi)$ (an EF), the density is uniform in $(0, 2\pi)$, i.e., the QM position carries an extreme uncertainty.

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In a general linear combination $\phi(x) = \frac{1}{\sqrt{2\pi}} \sum_{n\in\mathbb{Z}} \mathrm{e}^{\mathrm{i}nx}\widehat{\phi}_n$, the position density $|\phi(x)|^2$ has peaks/bottoms and carries less uncertainty, but has a more dispersed collection of coefficients $|\widehat{\phi}_n|^2$.

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That is, making the position density more 'localized' we inevitably make its momentum less determined, i.e., more uncertain.

A similar interpretation holds for $L_2(\mathbb{R}^1)$, although here we lack extreme cases (neither the uniform density nor Dirac's delta lies in L_2). Formally, one can write a (very general) lower bound for the product of 'spreads' in position/momentum probability masses in terms of 'expectations' and 'variances' (**Bell's inequalities**).

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A chock produced by this statement (nearly 90 years ago) was huge: viz., it generated an astronomic number of jokes and quotations, and the Internet is full of them.

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(In Cambridge, I knew Lady Jeffreys who told me how in 1920s

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Anyway, before I go further, I'll tell you another quotation:

"*I'm a better physicist than Fock: he uses PDEs; I – only ODEs. However, Frenkel' is a better physicist than me: he doesn't even know ODE's, only straight lines and second-order curves.*" – Attributed to Landau.

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I now want to talk about **teleportation**, another example of uncertainty in QM. This topic is very hot; 2 years ago the paper I will discuss (and its successful applications) nearly missed the Nobel Prize in Physics. And it's a simple 4D Linear Algebra (OK, may be 6D).

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Some of you may know an expression 'the Schrödinger cat' : it's a paradox invented in 1935 and showing how delicately the QT can interact with its classical counterpart. But it is a sad and confusing story (the cat is **both** dead **and** alive), and I don't want to continue this talk on a sad note.

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An example of an inclusion: the term 'an **entangled** state /vector' has been coined by Schrödinger in the cat story.

The aforementioned teleportation paper is 4-page long:

C. H. Bennett, G. Brassard, C. Crépeau, R. Jozsa, A. Peres, W. K. Wooters. Teleporting an unknown quantum state via dual classical and Einstein–Podolsky–Rosen channels. *Phys. Rev. Lett.* **70** (1993), 1895-1899.

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The term **teleportation** was introduced by Charles Fort, an American author, in 1931 and means "making a person or object disappear while an exact replica appears somewhere else".

Fort wrote about 'anomalies' people saw in their lives.

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Following the above paper, I will show that a QM state may be 'transported' intact from one place to another, by a sender who knows neither the state to be transported, nor the location of the receiver.

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Definition. A quantum measurement in a (finite-dimensional) Hilbert space $\mathcal{K}$ is identified with a collection of matrices

$\{\mathbf{M}_\alpha\}$ in $\mathcal{K}$ such that

$$\sum_\alpha \mathbf{M}_\alpha \mathbf{M}_\alpha^* = \mathbf{I}.$$

Matrices $\mathbf{F}_\alpha = \mathbf{M}_\alpha \mathbf{M}_\alpha^*$ are positive definite; they form a so -called positive-definite partition of unity, or a **positive -definite operator-valued measure** (POVM) in $\mathcal{K}$.

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An important example is where each $\mathbf{M}_\alpha$ is an orthoprojection (i.e., a matrix $\mathbf{P}_\alpha$ such that $\mathbf{P}_\alpha = \mathbf{P}_\alpha^* = \mathbf{P}_\alpha^2$).

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In this case $\mathbf{F}_\alpha$ coincides with $\mathbf{P}_\alpha$, and the POVM is identified with an orthogonal decomposition of $\mathcal{K}$ into linear subspaces to which operators $\mathbf{P}_\alpha$ project.

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I will start with a simplest example of a space $\mathcal{K}$ where $\mathcal{K} = \mathcal{H}$, the 2D complex Hilbert space $\mathbb{C}^2$ (called a single-**qubit** space), with a fixed basis

$$|0\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \quad |1\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}.$$

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I use here the so-called Dirac's bra-ket notation $\langle \ \ |$ and $| \ \ \rangle$. In this notation, $\langle\phi|\phi'\rangle$ stands for the inner product of unit vectors $|\phi\rangle$ and $|\phi'\rangle$ while $|\phi\rangle\langle\phi|$ stands for the rank one

orthoprojection on $|\phi\rangle$.

Here

$$|\phi\rangle = a_0|0\rangle + a_1|1\rangle, \ |\phi'\rangle = a_0'|0\rangle + a_1'|1\rangle,$$

and $a_0, a_1, a_0', a_1' \in \mathbb{C}$, with

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Note: I label the co-ordinates by 0 and 1, not by 1 and 2.

The rank one projection $|\phi\rangle\langle\phi|$ is associated with a **pure state** of a QM system in space $\mathcal{H}$. (The same holds for a general Hilbert space $\mathcal{K}$.)

Another feature of space $\mathcal{H}$ is that there are 3 fundamental 2×2 Hermitian matrices (4 if you count the unit matrix as well). They are called **Pauli matrices**:

$$\sigma_X = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \ \sigma_Y = \begin{pmatrix} 0 & \mathrm{i} \\ -\mathrm{i} & 0 \end{pmatrix}, \ \sigma_Z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

and play an important rôle in theory and applications.

We will use the relations

$$\sigma_X^2 = \sigma_Y^2 = \sigma_Z^2 = \mathbf{I}.$$

Suppose that the pure state of the system is described by a rank one orthoprojection $|\phi\rangle\langle\phi|$, and $\{\mathsf{P}_\alpha\}$ is an orthogonal measurement in $\mathcal{K}$. Setting

$$\begin{aligned}\pi(\alpha) &= \mathrm{tr}\,\Big(\big|\mathsf{P}_\alpha\phi\big\rangle\big\langle\phi|\mathsf{P}_\alpha\big|\Big)\\ &= \langle\mathsf{P}_\alpha\phi|\mathsf{P}_\alpha\phi\rangle = \|\mathsf{P}_\alpha\phi\|^2\end{aligned}$$

generates a probability distribution on the set of outcomes $\mathtt{A} = \{\alpha\}$, with $\pi(\alpha)\geq 0$ and $\sum\limits_\alpha \pi(\alpha) = 1$.

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We will assume that the following postulates hold:

Postulate One.

The outcome of an orthogonal quantum measurement of a system in a pure state $|\phi\rangle\langle\phi|$ in $\mathcal{K}$, with a POVM $\{\mathbf{P}_\alpha\}$, is a value of classical 'observable' $\alpha \in \mathtt{A}$ with probability $\pi(\alpha)$.

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Postulate Two.

If an orthogonal quantum measurement with a POVM $\{\mathbf{P}_\alpha\}$ is performed on a quantum system in state $|\phi\rangle\langle\phi|$ and the result is $\alpha \in \mathtt{A}$ then the state becomes $\dfrac{1}{\pi(a)}\big|\mathbf{P}_\alpha\phi\big\rangle\big\langle\mathbf{P}_\alpha\phi\big|$.

Now I take the tensor product of two copies of $\mathcal{H}$ which I denote by $\mathcal{H}_{\rm A}$ and $\mathcal{H}_{\rm B}$, respectively:

$$\mathcal{H}_{\rm AB} = \mathcal{H}_{\rm A} \otimes \mathcal{H}_{\rm B} \simeq \mathbb{C}^4.$$

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It has 4 'disentangled' (tensor-product) basis vectors $|\alpha\beta\rangle$ $= |\alpha\rangle \otimes |\beta\rangle$, with $\langle\alpha_1\beta_1|\alpha_2\beta_2\rangle = \langle\alpha_1|\alpha_2\rangle\langle\beta_1|\beta_2\rangle$:

$$|00\rangle,\ |01\rangle,\ |10\rangle,\ |11\rangle.$$

These vectors can be represented by the (standard) columns of height 4:

$$|00\rangle = \begin{pmatrix}1\\0\\0\\0\end{pmatrix}, |01\rangle = \begin{pmatrix}0\\1\\0\\0\end{pmatrix}, |10\rangle = \begin{pmatrix}0\\0\\1\\0\end{pmatrix}, |11\rangle = \begin{pmatrix}0\\0\\0\\1\end{pmatrix}.$$

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Here I label the co-ordinates by 00, 01, 10, 11, not by 1, 2, 3, 4. In other words, a general unit vector from $\mathbb{C}^4$ is written as

$$\begin{pmatrix} a_{00} \\ a_{01} \\ a_{10} \\ a_{11} \end{pmatrix}, \quad \text{with} \quad |a_{00}|^2 + |a_{01}|^2 + |a_{10}|^2 + |a_{11}|^2 = 1.$$

These labels yield the binary decomposition of numbers 0, 1, 2, 3. (For instance, $3 = 2^1 \times 1 + 2^0 \times 1 \sim 11$.)

In this notation, a tensor-product unit vector has

$$a_{\alpha\beta} = b_\alpha c_\beta, \quad \alpha, \beta = 0, 1,$$

where $|b_0|^2 + |b_1|^2 = |c_0|^2 + |c_1|^2 = 1.$

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Next, I fix a pure state $|\psi_{\rm AB}\rangle\langle\psi_{\rm AB}|$, where

$$\psi_{\rm AB} = \frac{1}{\sqrt{2}}\Big(|01\rangle - |10\rangle\Big) = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ 1 \\ -1 \\ 0 \end{pmatrix}.$$

This is **not** a tensor-product vector. (Quite the opposite.)

Here A refers to **Alice** and B to **Bob**, two characters from the literature (there will be also C: **Charlie**). Vector $\psi_{\rm AB}$ yields an **entangled** state of 2 particles, A and B (not a tensor product). (Say photons kept in a magnetic bottle or something.)

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The story is that once Alice & Bob were in love.
They 'prepared' this state, $\psi_{\rm AB}$, for future use.

Sadly, they separated and have lived different lives since, geographically and socially. When they parted they promised to remember each other for the rest of their lives and generated a kind of a secret code:

$$\mathbf{P}_+ \sim 00,\ \mathbf{P}_- \sim 11,\ \mathbf{Q}_+ \sim 01,\ \mathbf{Q}_- \sim 10$$

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A took away qubit (or particle) A and B qubit B. But these qubits (particles) 'feel' each other: their joint state is entangled (**not** a tensor product).

A & B have not kept their promises until now. A is Ok (works in a bank), but B lives in hiding. A wants to transmit to B a vitally important single-qubit quantum 'message' brought to her by C. In this experiment, C is B's trusted friend, but not particularly close to A: she never liked him.

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(In more complex experiments, a host of other characters appear: E (**Eva**), F (**Freddy** or **Frank**) and so on, ending up with V (**Victor**) and W (**Walter**). And every protagonist plays his/her specific role: they are called *placeholders*.)

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NB. I can't help thinking about strange aspects of the

Schroedinger cat experiment. Einstein suggested blowing the cut with a cask of gunpowder while Schroedinger himself used the Preussic acid, one of the strongest known poisons.

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Back to A & B: the message is represented by a unit vector

$$\psi_{\rm C} = a|0\rangle_{\rm C} + b|1\rangle_{\rm C} = \begin{pmatrix} a \\ b \end{pmatrix}$$

'prepared' in C's 'private' single-qubit Hilbert space

$$\mathcal{H}_{\rm C} \sim \mathcal{H}, \ \text{ with } ||\psi_{\rm C}||^2 = |a|^2 + |b|^2 = 1.$$

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According to the story, A doesn't (and perhaps doesn't want to) know the content of the message (i.e., the coefficients $a, b \in \mathbb{C}$). And C knows $\psi_{\rm C}$ but doesn't know $\psi_{\rm AB}$ or the secret code.

A performs an orthogonal quantum measurement, formally in the three-qubit space $\mathcal{H}_{\rm CAB}=\mathcal{H}_{\rm C}\otimes\mathcal{H}_{\rm AB}$, but effectively in the two-qubit space $\mathcal{H}_{\rm CA}=\mathcal{H}_{\rm C}\otimes\mathcal{H}_{\rm A}$.

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A uses 4 operators $\mathbf{P}_\pm=\left|\phi^\pm_{\rm CA}\right\rangle\left\langle\phi^\pm_{\rm CA}\right|$ and $\mathbf{Q}_\pm=\left|\psi^\pm_{\rm CA}\right\rangle\left\langle\psi^\pm_{\rm CA}\right|$ that are orthoprojections in space $\mathcal{H}_{\rm CA}$ to entangled vectors $\phi^\pm_{\rm CA}=\dfrac{1}{\sqrt{2}}\Big(|00\rangle\pm|11\rangle\Big)$ and $\psi^\pm_{\rm CA}=\dfrac{1}{\sqrt{2}}\Big(|01\rangle\pm|10\rangle\Big)$ (the **Bell states**):

$$\phi^\pm_{\rm CA}=\frac{1}{\sqrt{2}}\begin{pmatrix}1\\0\\0\\\pm1\end{pmatrix},\quad \psi^\pm_{\rm CA}=\frac{1}{\sqrt{2}}\begin{pmatrix}0\\1\\\pm1\\0\end{pmatrix}.$$

In $\mathcal{H}_{\rm CAB}$, A uses 4 rank two orthoprojections

$$\widetilde{\mathsf{P}}_\pm = \mathsf{P}_\pm \otimes \mathsf{I}_{\rm B},\ \widetilde{\mathsf{Q}}_\pm = \mathsf{Q}_\pm \otimes \mathsf{I}_{\rm B}.$$

Here $\{\mathsf{P}_\pm, \mathsf{Q}_\pm\}$ is a POVM in $\mathcal{H}_{\rm CA}$ (and $\{\widetilde{\mathsf{P}}_\pm, \widetilde{\mathsf{Q}}_\pm\}$ in $\mathcal{H}_{\rm CAB}$).

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Then A & C manage to deliver this classical 2-bit string to B.

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(It does not matter how; we simply assume that B gets it intact, perhaps, on a piece of paper.)

NB: It's a cheap price for delivery: $\epsilon\,\delta \in \{0,1\}^2$ for $(a,b) \in \mathbb{C}^2$.

This is why we want quantum computers!

Before the measurement, the pure state in $\mathcal{H}_{\rm CAB}$ is described by $|\psi_{\rm CAB}\rangle\langle\psi_{\rm CAB}|$ where

$$\psi_{\rm CAB} = \frac{a}{\sqrt{2}}\Big(|001\rangle - |010\rangle\Big) + \frac{b}{\sqrt{2}}\Big(|101\rangle - |110\rangle\Big)$$

$$= \frac{1}{\sqrt{2}}\Big[\phi^+_{\rm CA}\otimes\big(a|1\rangle_{\rm B} - b|0\rangle_{\rm B}\big)$$

$$+\phi^-_{\rm CA}\otimes\big(a|1\rangle_{\rm B} + b|0\rangle_{\rm B}\big)$$

$$+\psi^+_{\rm CA}\otimes\big(-a|0\rangle_{\rm B} + b|1\rangle_{\rm B}\big)$$

$$+\psi^+_{\rm CA}\otimes\big(-a|0\rangle_{\rm B} - b|1\rangle_{\rm B}\big)\Big].$$

After the measurement, according to the postulates, qubits C and A collapse in one of four classical states

However, qubit B is either in state $\big(a|1\rangle_{\rm B} - b|0\rangle_{\rm B}\big)$ (if the delivered string is 00)

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or finally $\big(-a|0\rangle_{\rm B} - b|1\rangle_{\rm B}\big)$ (if it is 11).

Thus, Bob has no problem with reconstructing $\psi_{\rm C}$: if he receives 00, he knows that his qubit is $-{\rm i}\sigma_Y\psi_{\rm C}$. Then, by applying the matrix ${\rm i}\sigma_Y$, he gets $\psi_{\rm C}$. Etc.

This miracle shows the (frightening) power of QM technology.

The modern equipment allows one to teleport photons at distances $\sim$150km (between Spanish isles in the Atlantic). I would put my money on that these guys (some of them at least) will finally get their prize.

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I would put my money on that these guys (some of them at least) will finally get their prize.
To diffuse the tension, let me tell you another joke.
A&B walk in a bar.The barmen asks: 'Are you two together? '
A (or B) answers: 'Together? We are **non-separable**.'
(An alternative version: **non-core-related**. And so on.)

So what can we do, normal academic people? We can't compete with those geeks: they have technology.

My answer is asymmetric:

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My answer is asymmetric:

PUBLISH BOOKS, with jokes.

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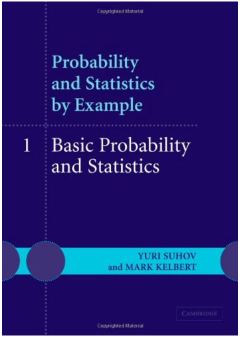
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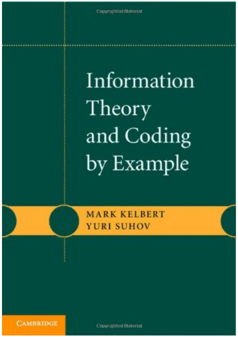
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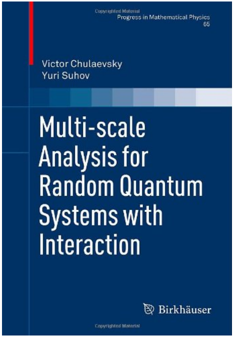
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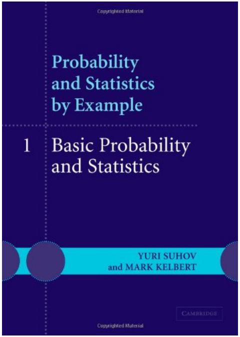
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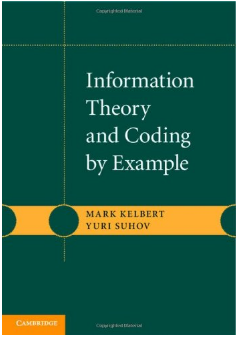
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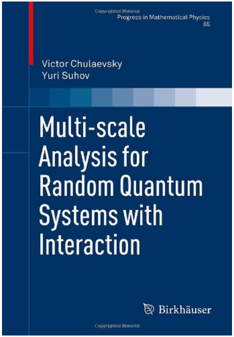
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